PROPAGATION OF WAVES IN A ROTATING GENERALIZED ELASTIC SOLID WITH A CYLINDRICAL HOLE

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Abstract: This paper deals with the propagation of waves in a rotating generalized elastic solid having a cylindrical hole. We derive the frequency equation for the propagation of waves in a rotating elastic solid. The effect of the rotation on the dispersion curve and on the phase velocity depicted graphically for non-dimensional wave number. The absence of rotation also discussed as a particular case of this paper.

Keywords: Propagation of waves, rotation, cylindrical hole.

Introduction: Wave propagation along a rotating solid with a cylindrical hole is of great importance due to its manifold applications. The cylindrical hole may be realized by a bore hole or a mine gallery. The studies in bore hole are of great help in exploration of gases, hydrocarbons and oils. In the oil industries, acoustic bore hole logging is commonly practiced. Problems of cylindrical bore have been studied by several researchers like Kumar and Deswal [1], Vashishth and Khurana [2], Arora and Tomar [3], and Bhujanga Rao and Ramamurthy [4]. Longitudinal wave propagation in a cylinder was discussed within the frame work of classical theory of elasticity by Pochhammer [5] and Chree [6]. Bancroft [7] investigated the velocity of longitudinal waves in a cylindrical bars. Lamb [8] investigated the propagation of waves in a liquid cylinder enclosed by a metal-walled tube. Recently, Abd-Alla [9] studied the effect of the rotation on waves in a cylindrical bore hole filled with micropolar fluid. Somaiah and Sambaiah [10] studied the elastic waves due to a time dependent force in an elastic sold having a cylindrical hole.

In this paper, we have investigated the propagation of waves in a rotating generalized elastic solid with a cylindrical hole. First we have derive the displacements of the rotating elastic solid and then frequency equation. The effect of the rotation on dispersive curves and phase velocity against non-dimensional wave number are shown graphically. The absence of the rotation is a particular case [10] of this paper.

Basic Equations: With the usual convention for symbols the constitutive relation [11] for displacement $\vec{u}$ and the field equations of classical continuum in the absence of body forces are given by

$$(\lambda+2\mu)\nabla\cdot\nabla\vec{u}-\mu\nabla\times\nabla\vec{u}=\rho\left[\frac{\partial^2 u}{\partial t^2}+\vec{\Omega}\times\left(\vec{\Omega}\times\vec{u}\right)\right] \qquad (1)$$

and

$$t_{ij}=\lambda u_{r,r}\delta_{ij}+\mu\left(u_{i,j}+u_{j,i}\right) \qquad (2)$$

where λ,μ are Lame's constants, ρ is density of the solid, δ_{ij} is Kronecker's delta and $\vec{\Omega}$ is angular velocity of the solid.

Formulation of the Problem and its Solution: Consider a homogeneous infinite elastic solid having a cylindrical hole of circular cross-section with radius $r=a$. We use the cylindrical coordinate system and (r,θ,z) represent the coordinates of a point. The z-axis is taken along the cylindrical hole. The medium is to assumed be rotating at a constant rate with constant angular velocity $\vec{\Omega}=(0,0,\Omega)$ about z-axis, where Ω is the component of rotation. In case of axial symmetry all the displacements and stresses are independent of θ. The displacement vector $\vec{u}$ can be written as $\vec{u}=(u, 0, w)$, where *u, w* are functions of *r z* and *t*. Now the equations of motion (1) can be reduces to

$$(\lambda+2\mu)\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}+\frac{\partial^2 w}{\partial z\partial r}\right]+\mu\left[\frac{\partial^2 u}{\partial z^2}-\frac{\partial^2 w}{\partial z\partial r}\right]=\rho\left[\frac{\partial^2 u}{\partial t^2}-\Omega^2 u\right] \qquad (3)$$

$$(\lambda+2\mu)\left[\frac{\partial^2 u}{\partial r\partial z}+\frac{1}{r}\frac{\partial u}{\partial z}+\frac{\partial^2 w}{\partial z^2}\right]+\mu\left[\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}-\frac{\partial^2 u}{\partial z\partial r}\right]=\rho\left[\frac{\partial^2 w}{\partial t^2}-\Omega^2 w\right] \qquad (4)$$

and the boundary conditions (2) reduces to

$$t_{rr}=(\lambda+2\mu)\frac{\partial u}{\partial r}+\frac{\lambda}{r}u+\lambda\frac{\partial w}{\partial z} \qquad (5)$$

$$t_{rz}=\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial r}\right) \qquad (6)$$

Where t_{rr} and t_{rz} are respectively the radial and the tangential stress components. We seek the solutions of equations (3) and (4) are

$$u(r,z,t)=AJ_1(hr)e^{i(\omega t-kz)} \qquad (7)$$

$$w(r,z,t)=BJ_0(hr)e^{i(\omega t-kz)} \qquad (8)$$

where $J_0(\ )$, $J_1(\ )$ denotes the modified Bessel functions of order zero and one respectively, k is the wave number. The phase velocity in the z-direction is $c = \frac{\omega}{k}$.

Substituting equations (7) and (8) in equations (3) and (4), we obtain the following homogeneous system in *A* and *B*,

$$\left[(\lambda+2\mu)h^2+\rho\left(\omega^2+\Omega^2\right)-\mu k^2\right]A-(\lambda+\mu)ikhB=0$$

and (9)

$$-(\lambda+\mu)ikhA+\left[\rho\left(\omega^2+\Omega^2\right)-(\lambda+2\mu)k^2+\mu h^2\right]B=0$$

For the existence of non-trivial solution of the system (9), the determinant of the coefficient matrix is zero i.e., $|a_{ij}|=0$ (10)

where $a_{11}=(\lambda+2\mu)h^2+\rho\left(\omega^2+\Omega^2\right)-\mu k^2$;

$a_{12}=a_{21}=-(\lambda+\mu)ikh$;

(11)

$a_{22}=-(\lambda+2\mu)k^2+\rho\left(\omega^2+\Omega^2\right)+\mu h^2$;

For solving the determinant (10), we obtain the following quadratic equation in h^2,

$$\left(h^2\right)^2+Qh^2+R=0 \tag{12}$$

where

$$Q=\left[\rho\left(\omega^2+\Omega^2\right)(\lambda+3\mu)-2\mu(\lambda+2\mu)k^2\right]\left[(\lambda+2\mu)\mu\right]^{-1} \tag{13}$$

$$R=\left[\rho^2\left(\omega^2+\Omega^2\right)^2-\rho\left(\omega^2+\Omega^2\right)(\lambda+\mu)k^2+(\lambda+2\mu)\mu k^4\right]\times\left[(\lambda+2\mu)\mu\right]^{-1}$$

Since eq. (12) is quadratic in h^2 so assuming two roots say h_1^2, h_2^2. As h_1 and h_2 are roots of an equation corresponding to the simultaneous differential equations (3) and (4), the solution of these equations are in the form of

$$u(r,z,t)=\sum_{j=1}^{2}a_j\, J_1\left(h_j r\right)e^{i(\omega t-kz)} \tag{14}$$

$$w(r,z,t)=\sum_{j=1}^{2}b_j\, J_0\left(h_j r\right)e^{i(\omega t-kz)} \tag{15}$$

On substituting equations (14) and (15) in equations (3) and (4), we obtain the constants b_1, b_2 in terms a_1, a_2 and radial and tangential displacements $u(r,z,t)$ and $w(r,z,t)$ are as follows:

$$u(r,z,t)=\left[a_1J_1\left(h_1r\right)+a_2J_1\left(h_2r\right)\right]e^{i(\omega t-kz)} \tag{16}$$

$$w(r,z,t)=\left[a_1\delta_1J_0\left(h_1r\right)+a_2\delta_2J_0\left(h_2r\right)\right]e^{i(\omega t-kz)} \tag{17}$$

where

$$\delta_1=\frac{\delta_3k^2-\delta_4\left(\omega^2+\Omega^2\right)-h_1^2}{(\delta_3-1)ikh_1};\delta_2=\frac{ikh_2(1-\delta_3)}{\delta_3h_2^2+\delta_4\left(\omega^2+\Omega^2\right)-k^2};$$

$$\delta_3=\frac{\mu}{\lambda+2\mu};\delta_4=\frac{\rho}{\lambda+2\mu}; \tag{18}$$

and $h_1^2, h_2^2=\frac{1}{2}\left[-Q\pm\left(Q^2-4R\right)^{\frac{1}{2}}\right]$

Derivation of the Frequency Equation:

At the surface $r=a$, the appropriate boundary conditions are:

$$t_{rr}=t_{rz}=0 \tag{19}$$

Making use of equations (16), (17) in boundary conditions (19), we obtain two homogeneous equations in two unknowns a_1 and a_2. The elimination of these unknowns gives the frequency equation

$$D(\omega,\Omega,k)=\det\left(b_{ij}\right)=0 \tag{20}$$

where

$b_{11}=(\lambda+2\mu)h_1J_1'\left(h_1a\right)+\frac{\lambda}{a}J_1\left(h_1a\right)-ik\lambda\delta_1J_0\left(h_1a\right)$

$b_{12}=(\lambda+2\mu)h_2J_1'\left(h_2a\right)+\frac{\lambda}{a}J_1\left(h_2a\right)-ik\delta_2J_0\left(h_2a\right); b_{21}=h_1\delta_1J_0'\left(h_1a\right)-ikJ_1\left(h_1a\right);$

(21)

$b_{22}=h_2\delta_2J_0'\left(h_2a\right)-ikJ_1\left(h_2a\right)$.

In case of absence of rotation, i.e., $\Omega=0$, all these results are coincide with the results [10].

Numerical Results and Discussion: Here we shall investigate the dispersion relation given by eq. (20) numerically. Since this equation is an implicit functional relation of wave number and natural frequency, therefore, on one can proceed to find the variation of natural frequency with wave number. For numerical computations in detail, we assume that the derivative of Bessel functions are vanishes and take the material values [9] $\lambda=7.59\times10^{10}\,dyne/cm^2$; $\mu=1.89\times10^{10}\,dyne/cm^2; \rho=2.192\,gm/cm^3$ and radius *a*=10 cm, which is not mentioned. Corresponding to rotation, $\Omega=0$, 0.5, 1.5, the dispersion curves and square phase velocities $c^2=\frac{\omega^2}{k^2}$ are plotted against the non-dimensional wave number k^2 with $1.5\le k\le3.5$. Fig. 1 and Fig.2 shows the variation of natural frequency and the square phase velocities with the wave number for all the rotations, ($\Omega=0$, 0.5, 1.5), respectively. The frequency and phase velocities are inverse proportional to the rotation.

Conclusions: The propagation of waves in a rotating generalized elastic solid with a cylindrical hole has been investigated after deriving the secular eq. (20). The modified Bessel functions have been used to study the problem. It is seen that:

i) The natural frequency of wave propagation depends on wave number, showing that frequency equation is dispersive curves.

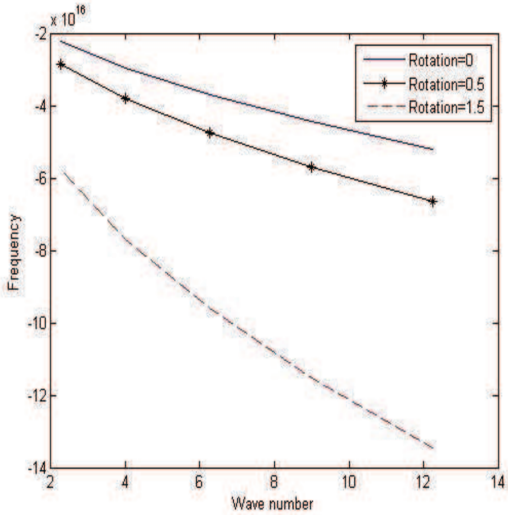

Fig.1. Variation of frequency versus non- dimensional wavenumber for $\Omega =$ 0, 0.5, 1.5.

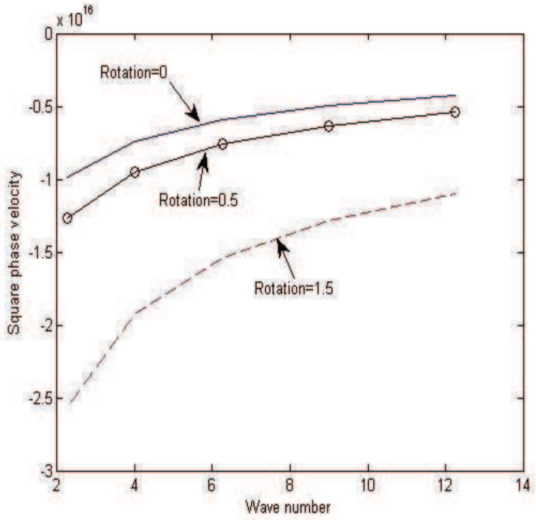

Fig. 2. Variation of square phase velocity versus non-dimensional wave number for $\Omega =$ 0, 0.5, 1.5.

ii) The dispersive relations coincide with the relation [10] for a non- rotating elastic solid.

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