CONCEPTS ON STRONGLY Γ-CANCELLATIVE REGULAR Γ-SEMIGROUPS

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Abstract: In this paper, some preliminaries and basic concept of regular Γ-semigroups were presented and proved that a strongly cancellative Γ-semigroup S is left(right) regular Γ-semigroup if and only if it is a: (i) completely regular Γ-semigroup (ii) Clifford Γ-semigroup (iii) E-inversive Γ-semigroup (iv) g-regular Γ-semigroup.

Keywords: Regular Γ-semigroups, E-inverse, strongly Γ-cancellative, g-regular, Clifford Γ-semigroups.

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1. Introduction: As a generalization of a semigroup, SEN [11], introduced the notion of Γ-semigroup in 1981 and developed some theory on Γ-semigroups [12], [13], [14] and [15]. JIROJKUL, SRIPAKORN, CHINRAM [7], extended many classical notions of semigroups to Γ-semigroups. DUTTA and CHATTERZEE [2] also studied the properties of Green's relations in Γ-semigroups and generalized the notions; idempotent elements, regular elements and semisimple elements in Γ-semigroups. MADHUSUDHANA RAO, ANJANEYULU, and GANGADHARA RAO [8] made a study on primary and semiprimary Γ-ideals, and special elements in Γ-semigroups. Further they [3] extended the results to duo chained Γ-semigroups.

2. Preliminaries:

Definition 2.1[11]: Let S and Γ be two non-empty sets. Then S is said to be a *Γ-semigroup* if there exist a mapping from $S\times\Gamma\times S$ to S which maps $(a,\alpha,b)\rightarrow a\alpha b$ satisfying the condition : $(a\gamma b)\mu c = a\gamma(b\mu c)$ for all $a,b,c\in S$ and $\gamma,\mu\in\Gamma$.

Note2.2 [8]: Let S be a Γ-semigroup. If A and B are two subsets of S, we shall denote the set $\{ a\gamma b : a\in A, b\in B \text{ and } \gamma\in\Gamma \}$ by AΓB.

Definition 2.3[8]: A Γ-semigroup S is said to be *commutative* provided$a\gamma b = b\gamma a$ for all $a,b\in S$ and $\gamma\in\Gamma$.

Note 2.4[8]: If S is a commutative Γ-semigroup then $a\Gamma b = b\Gamma a$ for all $a, b\in S$.

Definition 2.5 [8]: An element a of a Γ-semigroup S is said to be *left regular* provided $a = a\alpha a\beta x$, for some $x\in S$ and $\alpha,\beta\in\Gamma$. i.e, $a\in a\Gamma a\Gamma S$.

Definition 2.6 [8]: An element a of a Γ-semigroup S is said to be *right regular* provided $a = x\alpha a\beta a$, for some $x\in S$ and $\alpha,\beta\in\Gamma$. i.e, $a\in S\Gamma a\Gamma a$.

Definition 2.7 [8]:An element a of a Γ-semigroup S is said to be *regular* provided $a = a\alpha x\beta a$, for some $x\in S$ and $\alpha,\beta\in\Gamma$. i.e, $a\in a\Gamma S\Gamma a$.

Definition 2.8 [8]: A Γ-semigroup S is said to be *regular* if every element of S is regular.

Example 2.9. (i) Every Γ-group is regular. (ii) Every inverse Γ-semigroup is regular.

Definition 2.10[8]: An element a of a Γ-semigroup S is said to be *completely regular* provided, there exists an element $x\in S$ such that $a = a\alpha x\beta a$ for some $\alpha,\beta\in\Gamma$ and

$a\alpha x = x\beta a$ i.e., $a\in a\Gamma x\Gamma a$ and $a\Gamma x = x\Gamma a$.

Definition2.11 [8]: A Γ-semigroup S is said to be*completely regular Γ-semigroup*provided every element of S is completely regular.

Theorem 2.12[8]: If 'a' is a completely regular element of a Γ- semigroupS, then a is both a left regular element and a right regular element.

Definition2.213 [3]: An element a of a Γ-semigroup S is said to be*α-cancellative* provided for$\alpha\in\Gamma$, $a\alpha b = a\alpha c$ implies $b = c$ and $b\alpha a = c\alpha a$implies $b = c$.

Definition2.14 [3]: An element a of a Γ-semigroup S is said to be *Γ-cancellative* provided for all $\alpha\in\Gamma$, $a\alpha b = a\alpha c$ implies $b = c$ and $b\alpha a = c\alpha a$implies $b = c$.

Definition 2.15[3]: An element a of a Γ-semigroup S is said to be *strongly Γ-cancellative* provided $a\Gamma b= a\Gamma c$ implies $b = c$ and $b\Gamma a = c\Gamma a$implies $b = c$.

Note2.16 [3]: An element a of a Γ-semigroup S is said to be strongly Γ-cancellative provided $a\alpha b = a\beta c$, implies $b = c$ and $b\alpha a = c\beta a$ implies $b = c$ for $\alpha,\beta\in\Gamma$.

Theorem 2.17[3]: Every **Γ**-group is strongly **Γ**-cancellative **Γ**-semigroup.

3. Main Results:

Theorem 3.1: A strongly **Γ**-cancellative left regular **Γ**-semigroup is commutative.

Proof: Let S be a stronglyΓ-cancellative left regular Γ-semigroup.

Let $a,b\in S \Rightarrow (a\Gamma b\Gamma)^1 a\Gamma b = a\Gamma b\Gamma a\Gamma b \Rightarrow a\Gamma a\Gamma b\Gamma b = a\Gamma b\Gamma a\Gamma b \Rightarrow a\Gamma b\Gamma b = b\Gamma a\Gamma b$. Since S is Γ-cancellative, $a\Gamma b\Gamma b = b\Gamma a\Gamma b \Rightarrow a\Gamma b = b\Gamma a$. Hence S is commutative. Therefore, a Γ-cancellative left regular Γ-semigroup is commutative.

Theorem 3.2: A strongly **Γ**-cancellative right regular **Γ**-semigroup is commutative.

Theorem 3.3: A strongly **Γ**-cancellative regular **Γ**-semigroup is commutative

Theorem3.4: A strongly **Γ**-cancellative**Γ**-semigroup is left (right) regular **Γ**-semigroup if and only if it is completely regular.

Proof: Let S be a strongly Γ-cancellative Γ-semigroup. Assume that S is left regular Γ-semigroup. Then for any $a \in S$, there exist $x \in S$ such that $x\Gamma a\Gamma a = a$ $\Rightarrow x\Gamma x\Gamma a\Gamma a = x\Gamma a$ $x\Gamma a\Gamma x\Gamma a = x\Gamma a$. Since S is strongly Γ-cancellative, $x\Gamma a\Gamma x\Gamma a = x\Gamma a \Rightarrow a\Gamma x\Gamma a = a \Rightarrow a\Gamma x\Gamma a = a$. Therefore a is a regular for every $a \in S$. Hence S is a regular Γ-semigroup. From Theorem 3.3, S is commutative. Thus, $a\Gamma x = x\Gamma a$. Therefore, S is completely regular Γ-semigroup.

Conversely, let S be a completely regular Γ-semigroup. Then for any $a \in S$ there exist $x \in S$ such that $a\Gamma x\Gamma a = a$ and $x\Gamma a = a\Gamma x$. By theorem 2.12.,a is left regular. Hence S is a left regular Γ-semigroup.

Theorem 3.5: A strongly Γ-cancellative Γ-semigroup is a regular if and only if it is completely regular.

Proof: Similar to 3.4.

Corollary 3.6: A Γ-cancellative Γ-semigroup is a regular if and only if it is completely regular.

Corollary 3.7: A α-cancellative Γ-semigroup is a regular if and only if it is completely regular.

Definition 3.8: Let S be a multiplicative commutative Γ-semigroup and Id(S) the Γ-subsemigroup of idempotent elements of S. If S = Id(S), then S is called a *Boolean Γ-semigroup*. For e ∈Id(S), let S_e denote the set of all $x \in S$ such that $x\alpha e = x$ and $x\alpha y = e$ for some $y \in S$, $\alpha \in \Gamma$. Then the Γ-semigroup S is called a *Clifford* **Γ**-*semigroup* if

$$S = \bigcup_{e \in Id(S)} S_e .$$

Theorem 3.9: A strongly **Γ**-cancellative **Γ**-semigroup is left (right) regular if and only if it is Clifford **Γ**-semigroup.

Proof: Let S be a strongly Γ-cancellative Γ-semigroup. Suppose S is left regular Γ-semigroup then from Theorem 3.4, S is a regular Γ-semigroup. Since S is a regular Γ-semigroup ⇒S is an E-inversive Γ-semigroup. For any $a \in S$ there exist $x \in S$ such that ax, $xa \in E(S)$. Again from Theorem 3.1., S is commutative. Therefore, all the idempotent elements commutes. Hence S is a Clifford Γ-semigroup.

Conversely, assume that S is a Clifford Γ-semigroup. Since S is a Clifford Γ-semigroup, S is regular and idempotent elements commutes. Since S is regular, then from Theorem 3.1 S is a left regular Γ-semigroup.

Corollary 3.10: A **Γ**-cancellative **Γ**-semigroup is left (right) regular if and only if it is Clifford semigroup.

Corollary 3.11: A α-cancellative **Γ**-semigroup is left (right) regular if and only if it is Clifford semigroup.

Definition 3.12: An element a of a Γ-semigroup S is said to be *g-regular* if there exist an element $x \in S$, α, $\beta \in \Gamma$ such that $x = x\alpha a\beta x$. i.e., $x \in x\Gamma a\Gamma x$.

Note 3.13: An element a of a Γ-semigroup S is said to be g-regular if and only $x \in x\Gamma a\Gamma x$ for $x \in S$, $\alpha, \beta \in \Gamma$.

Definition3.14: A Γ-semigroup S is said to be *g-regular Γ-semigroup* if every element of S is g-regular.

Example3.15:(i) Every regular semigroup is g-regular. (ii) Every inverse semigroup is g-regular.

Theorem 3.16: A strongly **Γ**-calcellative**Γ**-semigroup is regular if and only if it is g-regular.

Proof: LetSbea strongly Γ-cancellative Γ-semigroup. Assume that S is aregular Γ-semigroup. For any $a \in S$ there exist an element $x \in S$ such that $a = a\Gamma x\Gamma a \Rightarrow a\Gamma x = a\Gamma x\Gamma a\Gamma x$. Since S is cancellative, $a\Gamma(x) = a\Gamma(x\Gamma a\Gamma x) \Rightarrow x = x\Gamma a\Gamma x \Rightarrow a$ is g-regular for every $a \in S$. Therefore S is g-regular Γ-semigroup.

Conversely, Assume that S is g-regular Γ-semigroup. For any $a \in S$ there exist an element $x \in S$ such that $x = x\Gamma a\Gamma x \Rightarrow x\Gamma a = x\Gamma a\Gamma x\Gamma a$ $\Rightarrow a = a\Gamma x\Gamma a \Rightarrow a$ is regular, for all $a \in S$. Therefore, S is a regular Γ-semigroup.

Corollary 3.17: A **Γ**-calcellative**Γ**-semigroup is regular if and only if it is g-regular.

Corollary 3.18: A α-calcellative**Γ**-semigroup is regular if and only if it is g-regular.

Theorem 3.19: A strongly **Γ**-cancellative**Γ**-semigroup S is left(right) regular if and only if it is g-regular.

Proof: Similar to Theorem 3.16.

Definition3.20: An element a is said to be an *E-inversive* of Γ-semigroupS if there exist an element $x \in S$ such that $a\Gamma x\Gamma a\Gamma x = a\Gamma x$ and $x\Gamma a\Gamma x\Gamma a = x\Gamma a$.

Definition3.21: A Γ-semigroup S is said to be *E-inversive Γ-semigroup* if every element of S is E-inversive.

Example3.22: (i) Every regular Γ-semigroup is an E-inversive Γ-semigroup. (ii) Every inverse Γ-semigroup is an E-inversive Γ-semigroup.

Theorem 3.23: A strongly **Γ**-cancellative left regular **Γ**-semigroup is an E-inversive **Γ**-semigroup.

Proof: Let S be a strongly Γ-cancellative left regular Γ-semigroup then by Theorem 3.1, S is commutative. Let $a \in S$. Then there exist $x \in S$ such that $x\Gamma a\Gamma a = a$ $\Rightarrow x\Gamma x\Gamma a\Gamma a = x\Gamma a$
$\Rightarrow x\Gamma x\Gamma a\Gamma a = x\Gamma a \Rightarrow (x\Gamma a)\Gamma(x\Gamma a) = x\Gamma a$ and we have $(a\Gamma x)\Gamma(a\Gamma x) = a\Gamma x$ and $(x\Gamma a)\Gamma(x\Gamma a) = x\Gamma a \Rightarrow a\Gamma x$ and $x\Gamma a$ are elements of E(S) ⇒a is an E-inversive element of S. Therefore S is an E-inversive Γ-semigroup.

Conversely, assume that S is an E-inversive Γ-semigroup. For any $a \in S$, there exist $x \in S$ such that $(a\Gamma x)\Gamma(a\Gamma x) = a\Gamma x$, $(x\Gamma a)\Gamma(x\Gamma a) = x\Gamma a$. To prove that S is a left regular, consider $(a\Gamma x)\Gamma(a\Gamma x) = a\Gamma x$ $\Rightarrow a\Gamma x\Gamma a\Gamma x = a\Gamma x$. Since S is a strongly Γ-cancellative, $a\Gamma x = a\Gamma x\Gamma a\Gamma x$

$\Rightarrow a = a\Gamma x\Gamma a \Rightarrow a = (a\Gamma x)\Gamma a \Rightarrow a = (x\Gamma a)\Gamma a \Rightarrow a = x\Gamma a\Gamma a$ $\Rightarrow a$ is left regular. Hence S is a left regular Γ-semigroup.

Corollary 3.24: A **Γ**-cancellative left regular **Γ**-semigroup is an E-inversive**Γ**-semigroup.

Corollary 3.25: A α-cancellative left regular **Γ**-semigroup is an E-inversive**Γ**-semigroup.

Theorem 3.26:A stronglyΓ-cancellativeΓ-semigroup is right regular Γ-semigroup if and only if it is an E-inversiveΓ-semigroup.

Proof: Similar to 3.23.

Conclusion: In this paper, we proved that every *g*-regular **Γ**-semigroup is regular**Γ**-semigroup by taking the strongly **Γ**-cancellative law but in general it is not possible.

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