PROPAGATION OF PLANE LONGITUDINAL WAVES IN A MICROPOLAR ELASTIC SOLID WITH VOIDS

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Abstract: In this paper, the plane longitudinal waves in a micropolar elastic solid with voids have been studied. We observed that two sets of coupled plane longitudinal waves in void solid and one set of longitudinal wave in a non-void solid exist. All these waves are dispersive in nature. The phase speeds are shown graphically against the various non-dimensional frequency ratios. Further, the comparative phase speeds of waves in micropolar solid and classical case are shown graphically.

Keywords: Micropolar elasticity, Longitudinal plane waves, Voids.

Introduction: Plane longitudinal wave propagation in elastic solids with voids is an important generalization of theory of micropolar elasticity. The theory is used for investigating various types of biological and geological materials for which classical theory of elasticity is inadequate to describe the behaviour of such materials possessing the microstructure. The theory of linear elastic materials with voids deals with the materials with a distribution of small porous or voids, where the volume of void is included among the kinematics variables. The theory reduces to the classical theory in the limiting case of volume of void tending to zero.
The non-linear theory of simple micro-elastic solids was developed by Eringen and Suhubi[1], [2] and the linear theory of micropolar elasticity was developed by Eringen [3]. The basic difference between the Eringen'stheory of micropolar elasticity and that of the classical theory of elasticity is the introduction of an independent micro-rotation vector. Thus, in the theory of micropolar elasticity the motion in a body is characterized by six independent functions, namely, three components of displacement vector and three components of micro-rotation vector whereas in the classical theory of elasticity, the motion is described by a displacement vector only. The interaction between two parts of a micropolar body is transmitted not only by a force vector but also by a couple resulting in asymmetric force stress tensor and couple stress tensor. Many problems on wave propagating in micropolar elastic media have been investigated by several authors in the past like Tomar*et al.* [4] and Tomar and Gogna [5], [6]. Plane waves in thermo-elastic materials with voids were studied by Singh and Tomar [7] but plane waves in rotating solid with voids were investigated by Jaswant Singh and Tomar [8].
In this paper, we study the plane longitudinal waves in a micropolar elastic solid with voids. It is observed that two sets of coupled longitudinal waves in void solid and one set of coupled longitudinal wave in non-void solid exist and all these waves are dispersive in nature.

Basic equations: The basic equations in terms of displacementvector $\vec{u}$, micro-rotation vector $\vec{\phi}$ for micropolarelastic solid with voids under the absence of body forces and body couples are given by Eringen [3]

$$(\mu+\kappa)\nabla^2\vec{u}+(\lambda+\mu)\nabla\nabla\cdot\vec{u}+\kappa\nabla\times\vec{\phi}=\rho\frac{\partial^2\vec{u}}{\partial t^2} \quad (1)$$

$$\gamma\,\nabla^2\vec{\phi}+(\alpha+\beta)\nabla\nabla\cdot\vec{\phi}+\kappa\nabla\times\vec{u}-2\kappa\vec{\phi}=\rho\, j\frac{\partial^2\vec{\phi}}{\partial t^2} \quad (2)$$

where λ,μ are Lame's constants, κ is an elastic constant, ρ is the density of the medium, j is the moment of micro-inertia and α,β,γ are micropolar void parameters.

Formulation and solution of the problem:
Equations (1) and (2) rewrite as

$$C_1^2\ \nabla^2\vec{u}+C_2^2\ \nabla\nabla\cdot\vec{u}+\omega_0^2\ \nabla\times\vec{\phi}=\frac{\partial^2\vec{u}}{\partial t^2} \quad (3)$$

$$C_4^2\ \nabla^2\vec{\phi}+C_5^2\ \nabla\nabla\cdot\vec{\phi}+C_6^2\ \nabla\times\vec{u}-C_7^2\ \vec{\phi}=\frac{\partial^2\vec{\phi}}{\partial t^2} \quad (4)$$

where

$$C_1^2=\frac{\mu+\kappa}{\rho},\ C_2^2=\frac{\lambda+\mu}{\rho},\ \omega_0^2=\frac{\kappa}{\rho},\ C_4^2=\frac{\gamma}{\rho\, j},$$

$$C_5^2=\frac{\alpha+\beta}{\rho\, j},\ C_6^2=\frac{\kappa}{\rho\, j}=\frac{\omega_0^2}{j},\ C_7^2=\frac{2\kappa}{\rho\, j}=\frac{2\omega_0^2}{j}.$$

For plane wave propagation in the positive direction of unit vector $\hat{n}$, we may seek the solution of the equations (3) and (4) as

$$\left[\vec{u},\vec{\phi}\right]=\left[\vec{A},\vec{B}\right]\exp\left\{ik(\hat{n}\cdot\vec{r}-vt)\right\} \quad (5)$$

where $\vec{A}$, $\vec{B}$ are vector constants, k is the wave number, $\vec{r}$ is the position vector and v is the phase velocity. Thus,

$$k = \frac{2\pi}{l},\ \vec{r} = x_k i_k,\ \omega = kv \tag{6}$$

Where l is the wave length, x_k are components of position vector and ω is angular frequency.

On using the equation (5) in equation (3) and (4), we obtain

$$k^2\left(C_1^2 - v^2\right)\vec{A} + C_2^2 k^2\, \hat{n}\left(\hat{n}\cdot\vec{A}\right) - \omega_0^2\, ik\left(\hat{n}\times\vec{B}\right) = 0 \tag{7}$$

and

$$\left[k^2\left(C_4^2 - v^2\right) + C_7^2\right]\vec{B} + C_5^2 k^2\, \hat{n}\left(\hat{n}\cdot\vec{B}\right) - C_6^2\, ik\left(\hat{n}\times\vec{A}\right) = 0 \tag{8}$$

Taking the scalar product of equation (7) with vector $\vec{A}$, we obtain

$$k^2\left(C_1^2 - v^2 + C_2^2\right)A^2 - \omega_0^2\, ik\,\vec{A}\cdot\left(\hat{n}\times\vec{B}\right) = 0 \tag{9}$$

where $\vec{A}\cdot\vec{A} = A^2$

Solving the equation (8) for $\vec{B}$, we obtain

$$\vec{B} = \frac{iC_6^2 k}{\left(C_4^2 + C_5^2 - v^2\right)k^2 + C_7^2}\left(\hat{n}\times\vec{A}\right) \tag{10}$$

Where $\hat{n}\left(\hat{n}\cdot\vec{B}\right) = \vec{B}$

Substituting the equation (10) in equation (9), we get

$$\left(C_1^2 + C_2^2 - v^2\right)\left(C_4^2 + C_5^2 - v^2\right)k^4 + \left[2C_1^2\frac{\omega_0^2}{j} + 2C_2^2\frac{\omega_0^2}{j} - \frac{2\omega_0^2}{j}v^2 + \frac{(\omega_0^2)^2}{j}\right]k^2 = 0 \tag{11}$$

For plane longitudinal wave take $\kappa = \frac{\omega}{v}$ in equation (11), we obtain the quadratic equation in v^2

$$P\left(v^2\right)^2 + Qv^2 + R = 0 \tag{12}$$

where

$$P = \omega_0^2 - \frac{\omega^2}{\omega_0^2},$$

$$Q = C_1^2 + C_2^2 + \frac{2}{j} + \frac{\omega_0^2}{j} - \left(C_1^2 + C_2^2 + C_4^2 + C_5^2\right)\frac{\omega^2}{\omega_0^2},$$

$$R = \left(C_1^2 + C_2^2\right)\left(C_4^2 + C_5^2\right)\frac{\omega^2}{\omega_0^2}.$$

The roots of the equation (12) are given by

$$v_{R_1}^2 = \frac{-Q + \left(Q^2 - 4PR\right)^{1/2}}{2P},$$

$$v_{R_2}^2 = \frac{-Q - \left(Q^2 - 4PR\right)^{1/2}}{2P} \tag{13}$$

The equation (13) represents the speed of two sets of dispersive coupled longitudinal micro-rotational waves and they are not encountered in classical theory of elasticity and they are influenced by micro-rotational void parameters $\alpha,\ \beta,\ \gamma$.

Particular case: If the material have no micro-voids that is $\alpha = \beta = \gamma = 0$ then the phase speed v_R of longitudinal micro-rotational waves are given by

$$v_R^2 = \frac{-Q_1}{P} \tag{14}$$

where

$$Q_1 = \left(\frac{2 + \omega_0^2}{j}\right) - \left(C_1^2 + C_2^2\right)\left(\frac{\omega^2}{\omega_0^2} - 1\right)$$

The longitudinal wave given in the equation (14) also dispersive in nature.

The phase speed v_R^* of longitudinal micro-rotational wave in classical case can be obtained by

$$v_R^{*\,2} = \left[\left(\frac{\lambda + 2\mu}{\rho}\right)\left(\frac{\omega^2}{\omega_0^2} - 1\right) - \left(\frac{2 + \omega_0^2}{j}\right)\right]\left(\omega_0^2 - \frac{\omega^2}{\omega_0^2}\right)^{-1} \tag{15}$$

as a particular case of κ tending to zero.

Numerical results and discussions: In order to examine this study in detail, we have taken the following values of relevant parameters $\lambda = 7.59\times10^{10}\ dynes/cm^2, \mu = 1.89\times10^{10}\ dynes/cm^2$, $\kappa = 0.0149\times10^{10}\ dynes/cm^2$, $j = 0.196\times10^{-2}\ cm^2$, $\rho = 2.192\ gm/cm^3$, $\alpha = 2.14\times10^8\ dynes$, $\beta = 2.26\times10^8\ dynes$ and $\gamma = 2.263\times10^8\ dynes$. For computing square phase speeds at different values of $\frac{\omega^2}{\omega_0^2}$ ranging from 0.002 to 0.01 assume that non-dimensional value $\omega_0^2 = 0.0068$. We have computed the square phase speeds $v_{R_1}^2$ (curve I) and $v_{R_2}^2$ (curve II) of the existing waves given by equation (13) and depicted in Fig. 1. We see that these waves are symmetric at frequency ratio 0.006. The phase speeds $v_{R_1}^2$ and $v_{R_2}^2$ are suddenly downwards and upwards respectively and they are tends to $-\infty$ and $+\infty$ as $\frac{\omega^2}{\omega_0^2}$ approaches to 0.0068. In Fig. 2, we have plotted all the phase speed curves of void and non-void solids. The phase speed curves for micropolar and classical cases are shown in the Fig. 3 to Fig. 5.

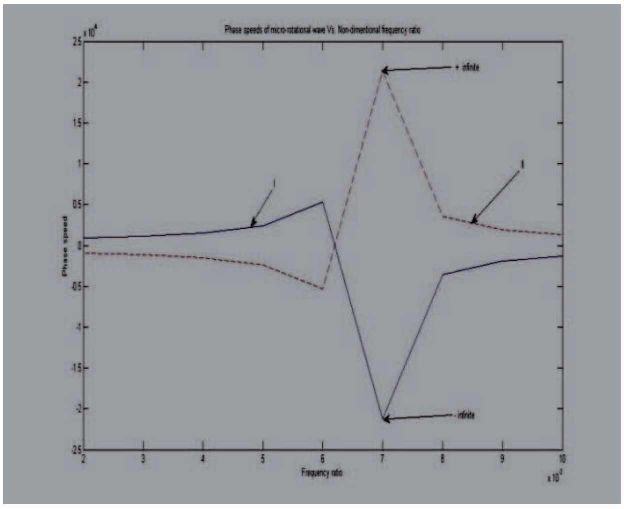

Fig. 1

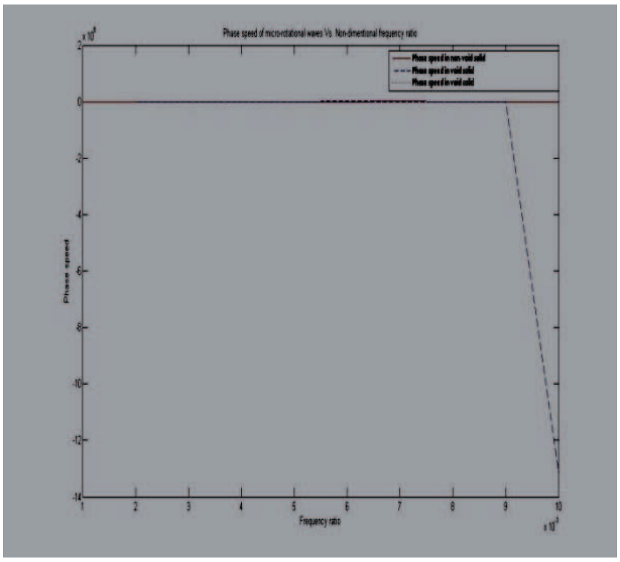

Fig. 2

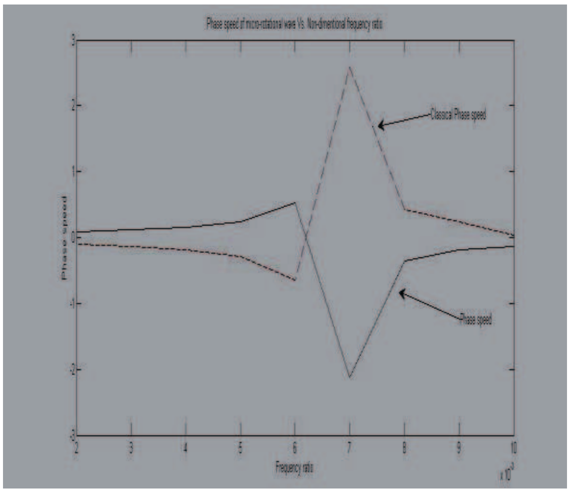

Fig. 3

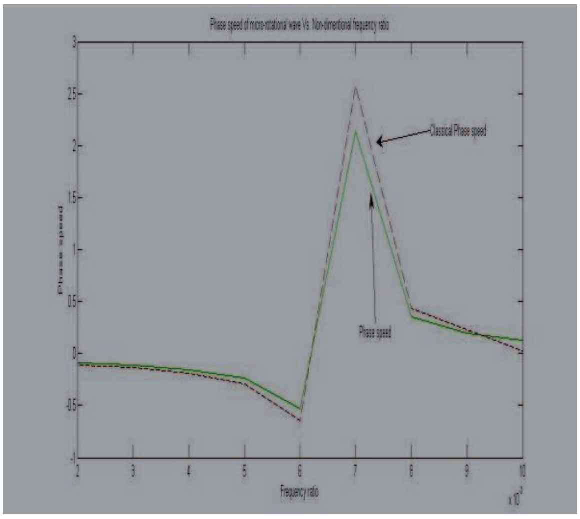

Fig. 4

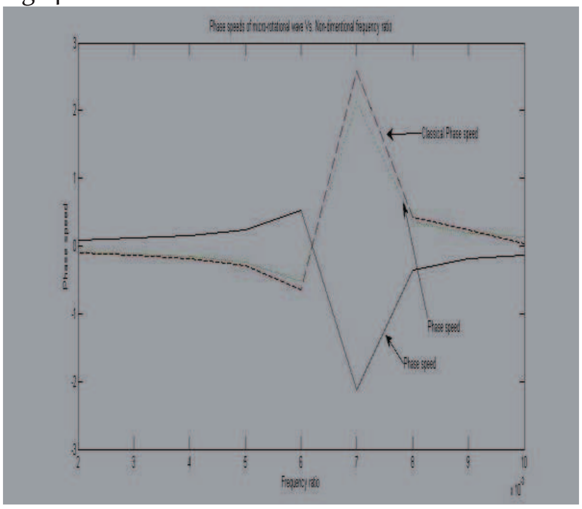

Fig. 5

Conclusions: In this paper, we have studied the plane longitudinal micro-rotational waves in a micropolar elastic solid with voids and the axis of rotation of the body is parallel to the displacement vector and we conclude the results as follows.

(i). It is found that there are two sets of plane longitudinal waves are propagate in a micropolar elastic solid with voids.

(ii). There exist a plane longitudinal wave in non-void solid also all these waves are dispersive in nature.

(iii). These waves are also obtained as a particular case

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