INSERTION AND DELETION CLOSED LANGUAGES RECOGNIZED BY A MONOID

A.R.RAJAN, LEELAMMA K.V

Abstract: For a finite set A the free monoid on A is denoted by A* whose elements are called words over A. A subset L of A^* is called a language over the alphabet A. Let L be a language over the alphabet A and $w\epsilon A^*$. Define $L \leftarrow w = \{u_1 w u_2 : u_1, u_2 \epsilon A^* \text{ and } u_1u_2 \epsilon L\}$ and $L \rightarrow w = \{u_1u_2 : u_1 w u_2 \epsilon L \text{and} u_1, u_2 \epsilon A^*\}$. A language L is insertion closed if $L \leftarrow w \subseteq L$ for all $w\epsilon L$. Similarly L is deletion closed if $L \rightarrow w \subseteq L$ for all $w\epsilon L$. We combine insertion and deletion operations to define a hyper operation o on words of A^* as follows. For $w_1, w_2 \epsilon A^*$, $w_1 \circ w_2 = (w_1 \leftarrow w_2) \cup (w_1 \rightarrow w_2)$. Then a language L is both insertion and deletion closed if and only if $L\circ L\subseteq L$. When L is a language recognized by a monoid M, then these operations can be described in terms of certain relations on this monoid. We characterize insertion deletion closed languages in terms of properties of the recognizing sets. Some properties of these operations are derived.

Keywords: deletion, insertion, morphism, recognizable language.

1. Introduction: In this section we recall basic definitions, results and notations that will be used in the sequel. All undefined terms and notations are as in Lallement[3] and Howie[1]. Let A be a non empty set which is called alphabet and elements of A are called letters. A word over A is a finite sequence $a_1a_2...a_n$ of elements of A. The set of all words over A is denoted by A^* . The empty word is defined as word of length zero and is denoted by 1. Thus A^* is a monoid under the catenation of words. It is the free monoid over A. A language L is a subset of A^*. L is called recognizable if there exists a monoid M and an onto morphism $\phi:A^*\rightarrow M$ such that $L=P\phi^{-1}$ for some $P\subseteq M$. In this case, we say that L is recognized by M[6].

Definition 1.1: Let $L\subseteq A^*$ be a language. The syntactic congruence of L denoted by P_L is a congruence on A^* defined as $P_L=\{(u,v)$:for all $x,y\in A^*, xuy\in L\Leftrightarrow xvy\in L$ } [3]

Remark: It is easy to see that L is a union of P_L classes. That is if u $\in L$ and $v\notin L$ then $(u,v)\notin P_L$.

2. Insertion and Deletion Operations: Insertion and deletion operations on languages have been introduced by Lila Kari[4] and are defined as follows:

Definition 2.1: Let L and L' be languages over the alphabet A and let $w\in L$, $w'\in L'$. Then insertion of w' into w denoted by $(w\leftarrow w')$ is defined as

$w\leftarrow w' =\{u_1w'u_2 : w=u_1u_2$ with $u_1, u_2\in A^*$ }

Definition 2.2: For any language L, $L_w=\{u_1w'u_2 : u_1,u_2\in A^*$and $u_1u_2\in L$ } . L_w is usually denoted by $L\leftarrow w$

Definition 2.3: For any two languages L and L',

$$L \leftarrow L' = \bigcup_{w\in L'} (L \leftarrow w)$$

Definition 2.4: Consider $w,w'\in A^*$. The deletion of w' from w denoted by $(w\rightarrow w')$ is defined as

$$(w \rightarrow w') = \begin{cases} u_1\, u_2 : \text{ if } w = u_1 w' u_2 \text{ for some } u_1, u_2 \in A^* \\ w \quad \text{otherwise} \end{cases}$$

Definition 2.5: For any two languages L and L' over the alphabet A ,

$$L \rightarrow L' = \bigcup_{w\in L, w'\in L'} (w \rightarrow w')$$

Definition 2.6: A language L is insertion closed if $L\leftarrow w\subseteq L$for all$w\in L$

Definition 2.7: A language L is deletion closed if $L\rightarrow w\subseteq L$for all$w\in L$

Definition 2.8: A hyper operation on a set S is a mapping ∘ from S×S to the set of subsets of S. A pair (S,∘) where S is a set and ∘ is a hyper operation on S is called a hyper algebra.[2]

Definition 2.9: Let (S,∘) be a hyper algebra. A subset T of S is called a sub algebra of (S,∘) if $T\circ T=\cup(x\circ y : x,y\in T\}\subseteq T$

Definition 2.10: Define an operation '∘' on A^* as follows for $w_1,w_2\in A^*$,

$w_1\circ w_2= (w_1\leftarrow w_2) \cup (w_1\rightarrow w_2)$

Theorem 2.1: A Language L is insertion and deletion closed iff L is a sub algebra of the hyper algebra $(A^*,\circ)$.

Proof: Assume that a language L is insertion and deletion closed.

Let $w_1,w_2\in L$

Then $(w_1\leftarrow w_2)\subseteq L$ and $(w_1\rightarrow w_2)\subseteq L$.

So $w_1\circ w_2\subseteq L$. Hence $L\circ L\subseteq L$.

Thus L is a subalgebra of $(A*,\circ)$.

Conversely let $L\circ L\subseteq L$,

Then for $w1,w2 \in L$, $w_1\circ w_2\subseteq L$.

So $(w_1\leftarrow w_2)\subseteq L$ and $(w_1\rightarrow w_2)\subseteq L$.

Hence $L\leftarrow w_2\subseteq L$ and $L\rightarrow w_2\subseteq L$.

Thus L is insertion and deletion closed

Theorem 2.2: Let L be a Language recognized by a monoid M and let $L= P\phi^{-1}$ for some $P\subseteq M$ where $\phi:A^*\rightarrow M$ is an onto morphism. Then L is a subalgebra of $(A^*,\circ)$ iff the following hold: $xy\in P\Leftrightarrow xPy\subseteq P$

Proof: Assume that L is a subalgebra of $(A^*,\circ)$. That is $L\circ L\subseteq L$.

So if $w_1,w2\in L$, then $w_1\circ w2\subseteq L$.

That is $(w_1\leftarrow w_2)\subseteq L$ and $(w_1\rightarrow w_2)\subseteq L$.

Let $x,y\in M$ and $xy\in P$, Then $\phi^{-1}(xy)\in L$.

Let $w_1,w_2 \in A^*$ be such that $\phi(w_1)=x$ and $\phi(w_2)=y$.
Then $\phi(w_1w_2)=xy$ and so, $w_1w_2 \in L$.
Let $z \in P$ and let $\phi(w)=z$ for some $w \in A^*$. Then $w \in L$.
Since L is a subalgebra of $(A^*,\circ)$, we have $w_1w_2 \circ w \subseteq L$.
It follow that $w_1ww_2 \in L$.
So $\phi(w_1ww_2)=xzy \in P$.
Since $z \in P$ is arbitary, we have $xPy \subseteq P$.
Now assume that $x,y \in M$ and $xPy \subseteq P$.
Then for any $z \in P$, $xzy \in P$.
Let $w_1,w,w_2 \in A^*$ be such that $\phi(w_1)=x$, $\phi(w_2)=y$ and $\phi(w)=z$.
Then $\phi(w_1ww_2)=xzy \in P$.
So $w_1ww_2 \in L$. Also $w \in L$.
Since L is a subalgebra of $(A^*,\circ)$, $w_1ww_2 \circ w \subseteq L$.
So $w_1ww_2 \rightarrow w \subseteq L$.
Then $w_1w_2 \in L$.
So $\phi(w_1w_2)=xy \in P$.
Thus $xy \in P \Leftrightarrow xPy \subseteq P$ holds.
Conversely assume that $xy \in P \Leftrightarrow xPy \subseteq P$.
We have to prove that L is a subalgebra of $(A^*,\circ)$.
Let $w_1,w_2 \in L$ where $\phi(w_1)=a$ and $\phi(w_2)=b$. Then $ab \in P$.
Now $w_1 \circ w_2= (w_1 \leftarrow w_2) \cup (w_1 \rightarrow w_2)$.
Let $w_1=u_1v_1$ for $u_1,v_1 \in A^*$.
Let $\phi(u_1)=x_1$ and $\phi(v_1)=y_1$.
Then $\phi(w_1)=\phi(u_1v_1)=x_1y_1 \in P$.
Now by the given conditions, $x_1Py_1 \subseteq P$.
Since $b \in P$, we have $x_1by_1 \in P$.
Since $\phi(u_1w_2v_1)=x_1by_1$, we have $u_1w_2v_1 \in L$ implies $w_1 \leftarrow w_2 \subseteq L$.
Now suppose that $w_1=u_2w_2v_2$ for some $u_2,v_2 \in A^*$.
Let $\phi(u_2)=x_2$ and $\phi(v_2)=y_2$,
Then $a=\phi(w_1)=\phi(u_2w_2v_2)=x_2by_2 \in P$.
So by condition on P, we have $x_2y_2 \in P$. Since $\phi(u_2v_2)=x_2y_2 \in P$, we get $u_2v_2 \in L$.
So $(w_1 \rightarrow w_2) \subseteq L$.
Thus $w_1 \circ w_2 \subseteq L$.
Thus $L \circ L \subseteq L$.
Hence L is a subalgebra of hyperalgebra $(A^*,\circ)$.

Definition 2.11: A Language L is C-Simple or Congruence Simple if L is a class of its syntactic congruence. That is L is C-Simple iff $u_1Lu_2 \cap L \neq \phi \Rightarrow u_1Lu_2 \subseteq L$.[5]

Theorem 2.3: Let $L=P\phi^{-1}$ where $\phi: A* \rightarrow M$ is a morphism and let $1 \in L$. Then L is a subalgebra of $(A^*,\circ)$ iff L is C-Simple

Proof: Assume that L is a subalgebra.
For proving L is C-Simple, we have to prove that $u_1Lu_2 \cap L \neq \phi \Rightarrow u_1Lu_2 \subseteq L$.
Let $u_1Lu_2 \cap L \neq \phi$ for some $u_1,u_2 \in A^*$.
Let $u_1wu_2 \in L$ for $w \in L$ with $\phi(u_1)=x$,$\phi(w)=a$ and $\phi(u_2)=y$.
Then $xay \in P$ and $a \in P$.
By theorem 2.2, we get $xy \in P$.
Since $\phi(u_1u_2)=xy$, we have $u_1u_2 \in L$.
Since L is a subalgebra, we have $u_1u_2 \circ w \subseteq L$for all$w \in L$.
That is $u_1Lu_2 \subseteq L$.
Therefore L is C-Simple.
Conversely assume that L is C-Simple.
Let $w_1,w_2 \in L$.
We have to prove that $(w_1 \leftarrow w_2) \subseteq L$ and $(w_1 \rightarrow w_2) \subseteq L$.
Suppose that $w_1=u_1u_2$ for $u_1,u_2 \in A^*$.
Since $1 \in L$, we have $u_1Lu_2 \cap L \neq \phi$.
Since L is C-Simple, we have $u_1Lu_2 \subseteq L$.
So $u_1w_2u_2 \in L$.
Thus $(w_1 \leftarrow w_2) \subseteq L$.
Similarly $(w_1 \rightarrow w_2) \subseteq L$.
That is $w_1 \circ w_2 \subseteq L$ implies $L \circ L \subseteq L$.
It follows that L is a subalgebra of hyperalgebra $(A^*,\circ)$

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A R Rajan/ Emeritus Professor in Mathematics/,Leelamma K. V/Associate Professor/ Mar Ivanios College TVM.