APPROXIMATE SOLUTION FOR TIME-DELAY FRACTIONAL FISHER'S EQUATION BY SHIFTED JACOBI - GAUSS COLLOCATION METHOD

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Abstract: In this paper, the fractional Fisher's equation with finite time delay is considered for the purpose to develop approximate scheme for its numerical solutions. The shifted Jacobi-Gauss collocation scheme is used in combination with the operational matrix of fractional derivatives and the delay Jacobi operational matrix for the numerical solution of the defined system. The fractional derivatives are described in Caputo sense. We provide complete theoretical treatment to approximate the solutions under this approach; it reduces to such problems by solving a system of algebraic equations. The applicability of this technique is presented in suitable example.

Keywords: KPP- Fisher delayed reaction–diffusion equation; Fractional differential equations; Jacobi-Gauss collocation; Newton's iterative method.

Introduction: Non-linear Phenomena are of fundamental importance in various fields of science and engineering. Non-linear partial differential equations play important role in non-linear physics (See [5] and references there in). In recent years, considerable interest in fractional partial differential equations has been motivated because of their growing applications in many fields. Several analytical algorithms have been investigated for treating these equations analytically to obtain closed form solutions such as variational iteration method, Fourier transform method, etc.,. However, there are only few types of these equations in which the analytical solutions are available. Therefore, numerical schemes have to be used in general. The Fishers equation is first introduced by Fisher as a deterministic model of the wave propagation of favored gene in population [6]. Also the Fisher's equation arises in many fields such as physical, biological etc., which are described by the interaction of diffusion and reaction process [5].

Recently, continuous orthogonal polynomials have been used instead of numerical approximate solution of Fractional Differential Equations (FDEs) [1]. In [3], the authors introduced a new Jacobi operational matrix for fractional derivative in the Caputo sense, and solve the FDEs. In [4], Jacobi delay operational matrix for the integer order differential equations with the time delay in any interval is studied. In [6], the authors was studied the monotone travelling wave fronts of the KPP-Fisher delay equation. The analytical solutions to the fractional Fisher equation were studied by applying the fractional sub-equation in [7].

In this paper is organized as follows. First, in Section 2, the authors review the basic properties of shifted Jacobi polynomials and some definition of fractional calculus. In Section 3, gives operational matrices to any interval for fractional derivative in Caputo sense of shifted Jacobi polynomials derived in the previous section. Also in this section, we completely developed delay Jacobi coefficient matrix for the shifted Jacobi polynomials. In Section 4, provided the theoretical procedure of numerical approximation for the time-delay fractional Fisher's equation solved by the way of constructing the shifted Jacobi Gauss collocation method. The proposed method is applied to numerical example is given finally.

2. Preliminaries: In this section, some necessary concepts, definitions and basic results for Jacobi polynomial, fractional calculus are summarized [1], [4].

2.1. Shifted Jacobi polynomial: The Jacobi polynomial $P_i^{(a,b)}(z)$ is defined in the interval$1 \leq z \leq 1$. The shifted Jacobi polynomial in the form as:

$$J_i^{(a,b)}(t) = \sum_{k=0}^{i} \frac{(-1)^{i-k}\Gamma(i+b+1)}{\Gamma(k+b+1)\Gamma(i+a+b+1)} \times \frac{\Gamma(i+k+a+b+1)}{\Gamma(i-k+1)\Gamma(k+1)}\left[\frac{t-t_0}{t_f-t_0}\right]^k$$

Where $t_0 \leq t \leq t_f$, (1)

The recurrence relation of the shifted Jacobi polynomial as follows

$$2i(i+a+b)(2i+a+b-2)J_i^{(a,b)}(t) = (2i+a+b-1)[(2i+a+b)(2i+a+b-2)\left(\frac{2t-t_0-t_f}{t_f-t_0}\right)+a^2-b^2]J_{i-1}^{(a,b)}(t). \quad (2)$$

The orthogonality property of the shifted Jacobi polynomial is,

$$\int_{t_0}^{t_f} W^{(a,b)}(t)\, J_i^{(a,b)}(t) J_j^{(a,b)}(t)dt = \begin{cases} 0, & i \neq j \\ h_{ij}^{(a,b)} & i = j \end{cases},$$

Where $W^{(a,b)}(t) = (t-t_0)^b(t_f-t)^a$ is the weighted function, and

$$u_N(x) = \sum_{i=0}^{N} c_j J_{L,j}^{(a,b)}(x) = C^T \Phi_{L,N}^{(a,b)}(x),$$

If we approximate $u(x)$ by the first $(N+1)-$ terms, then we write
where $C^T = [C_0, C_1, \dots, C_N]$,
$\Phi_{L,N}^{(a,b)}(x)$
$= \left[J_{L,0}^{(a,b)}(x),\ J_{L,1}^{(a,b)}(x)\ ,\dots,\ J_{L,N}^{(a,b)}(x)\right]^T$
Similarly, let $u(x,t)$ be an infinitely differentiable function defined on $0 < x \le L$ and $t_0 < t \le t_f$. Then it is possible to express as

$$u(t,x) = \sum_{i=0}^{M}\sum_{j=0}^{N} u_{ij} J_{t_f,i}^{(a,b)}(t) J_{t_f,j}^{(a,b)}(x) = \Phi_{t_f,M}^T(t) U \Phi_{L,N}^{(a,b)}(x) \quad (3)$$

as $M, N \to \infty$, the approximation becomes equal to the exact function, where U is the coefficient matrix and $\Phi^{(a,b)}$ is the Jacobi vectror. These are defined as:

$$\Phi_{L,N}^{(a,b)}(x) = \left[J_{L,0}^{(a,b)}(x),\ \dots, J_{L,N}^{(a,b)}(x)\right]^T$$

$$U = \begin{pmatrix} u_{00} & \dots & u_{0,N} \\ \vdots & \vdots & \vdots \\ u_{M0} & \dots & u_{M,N} \end{pmatrix},$$

where the coefficients u_{ij} are given by,

$$u_{ij} = \int_{t_0}^{t_f}\int_0^L u(t,x) J_{t_f,i}^{(a,b)}(t) J_{t_f,j}^{(a,b)}(x) \mathrm{W}_{\mathrm{t_f}}^{(\mathrm{a,b})}(\mathrm{t}) \times d\mathrm{W}_{\mathrm{L}}^{(\mathrm{a,b})}(\mathrm{x}) dt dx, \quad i = 0,1,2,\dots,M, \quad j = 0,1,2,\dots,N.$$

2.2. Basic Definitions:

Definition.2.1 TheRiemann-Liouville fractional integral operator of order $\alpha > 0$ is defined as:

$$I_{a^+}^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)}\int_{a^+}^{t} \frac{f(s)}{(t-s)^{1-\alpha}} ds, \ t > 0,$$

Provided that the integral on right hand side exists, where $\Gamma(\cdot)$ is the gamma function.

Definition.2.2: The Caputo derivative of order α with the lower limit a for a function f can be written as

$$^{c}D_{0^+}^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)}\int_{a^+}^{t} \frac{f^n(s)}{(t-s)^{\alpha+1-n}} ds = I^{n-\alpha} f^n(t), \ t > 0, \quad 0 \le n-1 < \alpha < n,$$

provided that the right hand side is point-wise defined on $[a, \infty)$ where $n = [\alpha] + 1$.

3. O**perational matrix of fractional Caputo derivatives:**

In this section we will drive the operational matrix for the fractional derivative in the Caputo sense [3].

Theorem 3.1: Let $\Phi(t)$ be shifted Jacobi vector defined in equation and let also$\upsilon > 0$. Then

$$D^{\upsilon}\Phi(t) = D^{(\upsilon)}\Phi(t), \quad (4)$$

$$D^{(\upsilon)} = \begin{pmatrix} 0 & 0 & \dots & 0 \\ & \vdots\vdots\vdots\vdots & & \\ \Omega_\upsilon(\lceil\upsilon\rceil,0) & \Omega_\upsilon(\lceil\upsilon\rceil,1) & \dots & \Omega_\upsilon(\lceil\upsilon\rceil,M) \\ \vdots & \vdots & \vdots & \vdots \\ \Omega_\upsilon(M,0) & \Omega_\upsilon(M,1) & \dots & \Omega_\upsilon(M,M) \end{pmatrix}$$

Where $\Omega_\upsilon(i,j) = \sum_{k=\lceil\upsilon\rceil}^{I} \delta_{ijk}and\delta_{ijk}$is given by

$$\delta_{ijk} = \frac{(-1)^{i-k}\Gamma(j+b+1)\Gamma(i+b+1)}{h_{jk}\Gamma(j+a+b+1)\Gamma(k+b+1)} \times \frac{\Gamma(i+k+a+b+1)(t_f-t_0)^{a+b-\upsilon+1}}{\Gamma(i+a+b+1)\Gamma(i-k+1)\Gamma(k-\upsilon+1)} \times \sum_{f=0}^{j} \frac{(-1)^{j-f}\Gamma(j+f+a+b+1)}{\Gamma(f+b+1)\Gamma(j-f+1)} \times \frac{\Gamma(a+1)\Gamma(f+k-\upsilon+b+1)}{\Gamma(f+1)\Gamma(f+k+a+b-\upsilon+2)}.$$

Here $D^{(\upsilon)}$, the first $\lceil\upsilon\rceil$ rows are all zeros.

Proof: Using Theorem 3.4 in [3] and applying equations (1) and (3), the theorem can be proved.

3.2. Delay Jacobi polynomial: For an arbitrary delay $\tau > 0$, the continuous real valued function $u(t-\tau, x)$ can be approximated as delay Jacobi vector $\Phi^{(a,b)}(t-\tau)$. Then this equation is written as Jacobi vector with a coefficient delay matrix H. To do this $u(t-\tau, x)$ is assumed to be smooth continuous function and the approximation is derived as follows:

$$u(t-\tau,x) = \sum_{i=0}^{M}\sum_{j=0}^{N} u_{ij} J_{t_f,i}^{(a,b)}(t-\tau) J_{L,j}^{(a,b)}(x) = \Phi_{t_f,M}^T(t-\tau) U \Phi_{L,N}(x),$$

where $\Phi_{t_f,M}^{(a,b)}(t-\tau)$ is delay Jacobi vector. First we find this delay Jacobi vector, using the approximation is derived as follows:

$$u(t-\tau,x) = \sum_{i=0}^{M}\sum_{j=0}^{N} u_{ij} J_{t_f,i}^{(a,b)}(t-\tau) J_{L,j}^{(a,b)}(x) = \Phi_{t_f,M}^T(t-\tau) U \Phi_{L,N}(x),$$

Where $\Phi_{t_f,M}^{(a,b)}(t-\tau)$ is delay Jacobi vector. First we find this delay Jacobi vector, using the shifted Jacobi polynomial (1) by replacing t into $-\tau$,

$$J_0^{(a,b)}(t-\tau) = 1 = J_0^{(a,b)}(t)$$

$$J_1^{(a,b)}(t-\tau) = -\left[\frac{(a+1)t_0 + (b+1)t_f}{(t_f - t_0)}\right] + \left[\frac{(a+b+2)(t-\tau)}{t_f - t_0}\right] = J_0^{(a,b)}(t) - \frac{\tau(a+b+2)}{(t_f - t_0)} J_1^{(a,b)}(t).$$

The delay Jacobi function can be approximated in the following Jacobi series,

$$\Phi_{t_f,M}^{(a,b)}(t-\tau) = \sum_{j=0}^{M} h_{i,j} J_{t_f,j}^{(a,b)}(t) = H\Phi_{t_f,M}^{(a,b)}(t).$$

Here H is the coefficient delay matrix and it is a lower-triangular matrix. That is, Hcan be written as,

$$H=\begin{bmatrix} h_{0,0} & 0 & 0 & 0 \\ h_{1,0} & h_{1,1} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ m_{0,0} & h_{m,1} & h_{m,2} & h_{m,m} \end{bmatrix}_{(m+1)\times(m+1)}$$

Its elements $h_{i,j}$ can be obtained as follows: Using shifted Jacobi recurrence relation (2) by replacing t into $t-\tau$, we obtain the following relations,

$$h_{0,0}=1;\ h_{1,0}=-\frac{\tau(a+b+2)}{t_f-t_0};$$

$$h_{1,1}=1;\ h_{i,j}=0\ for j>i$$

$$\begin{aligned}2\mathrm{i}(\mathrm{i}+\mathrm{a}+\mathrm{b})(2\mathrm{i}+\mathrm{a}+\mathrm{b}-2)\mathrm{h}_{\mathrm{i},0} &= (2\mathrm{i}+\mathrm{a}+\mathrm{b})(2\mathrm{i}+\mathrm{a}+\mathrm{b}-1)(2\mathrm{i}+\mathrm{a}+\mathrm{b} \\ &-2)\times[\frac{2(\mathrm{a}+1)(\mathrm{b}+1)}{(3+\mathrm{a}+\mathrm{b})(2+\mathrm{a}+\mathrm{b})}h_{i-1,1} \\ &-\frac{\mathrm{a}-\mathrm{b}}{\mathrm{a}+\mathrm{b}+2}h_{i-1,0}]+(2i+a+b-1) \\ &\times[\mathrm{a}^2-\mathrm{b}^2-\frac{2\tau}{t_f-t_0}(2\mathrm{i}+\mathrm{a}+\mathrm{b})(2\mathrm{i}+\mathrm{a} \\ &+\mathrm{b}-2)\ h_{i-1,0}-2(\mathrm{i}+\mathrm{a}-1)(\mathrm{i}+\mathrm{b}-1) \\ &\times(2\mathrm{i}+\mathrm{a}+\mathrm{b})\mathrm{h}_{\mathrm{i}-2,0},\end{aligned}$$

$$\begin{aligned}2\mathrm{i}(\mathrm{i}+\mathrm{a}+\mathrm{b})(2\mathrm{i}+\mathrm{a}+\mathrm{b}-2)\mathrm{h}_{\mathrm{i},\mathrm{j}} &= (2i+a+b)(2i+a+b-1)(2i+a+b \\ &-2)\times[\frac{2j(j+a+b)}{(2j+a+b)(2j+a+b-1)} \\ &\times h_{i-1,j-1}-\frac{a^2-b^2}{(2j+a+b)(2j+a+b+2)} \\ &\times h_{i-1,j}-+\frac{2(j+a+1)(j+b+1)}{(2j+a+b)(2j+a+b+2)} \\ &\times h_{i-1,j-1}+(2i+a+b-1)[\mathrm{a}^2-\mathrm{b}^2 \\ &-\frac{2\tau}{t_f-t_0}(2i+a+b)(2i+a+b-2)h_{i-1,j} \\ &-2(i+a-1)(i+b-1)(2i+a+b)h_{i-2,j},\end{aligned}$$

for$j\geq 1$.Therefore, the above recursive equations have the elements of the delay Jacobi matrix H.

4. **Shifted Jacobi-Gauss collocation method:** Let us first introduce some basic notation will be used in the sequel. We set ([1])

$$S_M(t_0,t_f)=span\left\{J_{t_f,0}^{(a,b)},J_{t_f,1}^{(a,b)},\dots,J_{t_f,M}^{(a,b)}\right\}.$$

We recall the Jacobi-Gauss-interpolation. For any positive integer $M, S_M(t_0,t_f)$ stands for the set of all algebraic polynomial of degrees at most M. If we denote by

$$t_{M,i}^{a,b}\left(t_{t_f,M,i}^{(a,b)}\right),0\leq i\leq M$$

and $\quad \varpi_{M,i}^{(a,b)}\left(\varpi_{t_f,M,i}^{(a,b)}\right),0\leq i\leq M,$

to the nodes (zeros) and Christoffel numbers of the shifted Jacobi-Gauss quadrature on the interval $(-1,1)$, (t_0,t_f) respectively, we can easily show that,

$$\mathrm{t}_{\mathrm{t_f},\mathrm{M},\mathrm{i}}^{(\mathrm{a},\mathrm{b})}=\left(\frac{t_f-t_0}{2}\right)t_{M,i}^{a,b}+\left(\frac{t_f+t_0}{2}\right),$$

$$\varpi_{t_f,M,i}^{(a,b)}=\left(\frac{t_f-t_0}{2}\right)^{a+b+1}\varpi_{M,i}^{(a,b)}.$$

4.1., Fractional Fisher's time delay equation: Consider nonlinear time-delay fractional type Fisher's equation is of the form [5]-[7]:

$$\frac{\partial^{\alpha}\mathrm{u}(\mathrm{x},\mathrm{t})}{\partial \mathrm{t}^{\alpha}}=\epsilon\frac{\partial^{2\alpha}u(x,t)}{\partial x^{2\alpha}}+u(x,t)\times[1-u(x,t-\tau)],\qquad (5)$$

$$0\leq x\leq L,\qquad t\geq 0.$$

where $0<\alpha\leq 1$, the initial condition

$$u(x,t)=g(x,t),-\tau\leq t\leq 0,\qquad (6)$$

and, there are two continuous real valued functions$r_1(x),r_2(x)$ and ϵ is a real constant, the boundary condition is:

$$u(0,t)=r_1(x),u(L,t)=r_2(x),t\geq 0.$$

Fist, Convert the given initial condition (6) to the zero constraints and combined to the given system. We can write the equation (5) as:

$$\frac{\partial^{\alpha}\mathrm{u}(\mathrm{x},\mathrm{t})}{\partial \mathrm{t}^{\alpha}}-\epsilon\frac{\partial^{2\alpha}u(x,t)}{\partial x^{2\alpha}}+u(x,t)\times[u(x,t-\tau)-1]-u(x,0)-g(x)=0$$

Now,we approximate $\mathrm{u}(x,t), u(x,0)$, $u(x,t-\tau)$ and $g(x,t)$ by the shifted Jacobi polynomials are:

$$u_{M,N}(x,t)=\Phi_{t_f,M}^T(t)U\Phi_{L,N}(x);\ \mathrm{u_{M,N}}(\mathrm{x},0)=\Phi_{\mathrm{t_f},\mathrm{M}}^{\mathrm{T}}(0)\mathrm{U}\Phi_{\mathrm{L},\mathrm{N}}(\mathrm{x});$$

$$u_{M,N}(x,t-\tau)=\begin{cases}\Phi_{t_f,M}^T(t)G\Phi_{L,N}(x),0\leq t\leq\tau,\\ \Phi_{t_f,M}^T(t)H^TU\Phi_{L,N}(x),\tau\leq t\leq 2\tau,\\ \Phi_{t_f,M}^T(t)H^TU\Phi_{L,N}(x),(n-1)\tau\leq t\leq n\tau.\\ n>0.\end{cases}$$

$$g_{M,N(x,t)}=\Phi_{t_f,M}^T(t)G\Phi_{L,N}(x)$$

Where U is an unknown coefficients of $(M+1)\times(N+1)$ matrix while G is known matrix that can be written as:

$$G=\begin{bmatrix} g_{0,0} & g_{0,1} & \cdots & g_{0,N} \\ g_{1,0} & g_{1,1} & \cdots & g_{1,N} \\ \vdots & \vdots & \ddots & \vdots \\ g_{M,0} & g_{M,1} & \cdots & g_{M,N} \end{bmatrix}_{(M+1)\times(N+1)}$$

where the coefficients g_{ij}, $i=0,1,\cdots,M$, $j=0,1,\cdots N$ can be evaluated by

$$g_{ij=}\frac{1}{h_{t_f,i}^{(a,b)}h_{t_f,j}^{(a,b)}}\int_{t_0}^{t_f}\int_0^L g(t,x)J_{t_f,i}^{(a,b)}(t)J_{t_f,j}^{(a,b)}(x)\times\mathrm{W}_{\mathrm{t_f}}^{(\mathrm{a},\mathrm{b})}(\mathrm{t})\mathrm{W}_{\mathrm{L}}^{(\mathrm{a},\mathrm{b})}(\mathrm{x})dxdt.$$

$$\mathrm{i}=0,1,2,\dots,\mathrm{M},\ \ \mathrm{j}=0,1,2,\dots,\mathrm{N}.$$

For general function $g(x,t)$, it is more difficult to compute the previous integral exactly. Using the Jacobi-Gauss quadrature formula we can approximate the coefficients g_{ij} as:

$$g_{ij}=\frac{1}{h_{t_f,i}^{(a,b)}h_{t_f,j}^{(a,b)}}\sum_{\delta=0}^{M}\sum_{\epsilon=0}^{N}g\left(x_{L,N,\epsilon}^{(a,b)},t_{t_f,M,\delta}^{(a,b)}\right)\times J_{t_f,i}^{(a,b)}\left(t_{t_f,M,\delta}^{(a,b)}\right)J_{t_f,j}^{(a,b)}\left(x_{L,N,\epsilon}^{(a,b)}\right)\varpi_{L,N,\epsilon}^{(a,b)}\varpi_{t_f,M,\delta}^{(a,b)},$$

$\mathrm{i}=0,1,2,\dots,\mathrm{M},\ \ \mathrm{j}=0,1,2,\dots,\mathrm{N}.$

where $x_{L,N,\epsilon}^{(a,b)}$, $0 \le \epsilon \le N$ are the zeros of Jacobi-Gauss quadrature in the interval $(0, L)$ with$\varpi_{L,N,\epsilon}^{(a,b)}$, $0 \le \epsilon \le N$ are the corresponding Christoffel numbers and $t_{t_f,M,\delta}^{(a,b)}$, $0 \le \delta \le M$ are the zeros of Jacobi-Gauss quadrature in the interval $(0, t_f)$ with $\varpi_{t_f,M,\delta}^{(a,b)}$,$0 \le \delta \le M$ are the corresponding Christoffel numbers. Using Theorem 3.1 and equations (4), one can write:

$$\frac{\partial^\alpha u(x,t)}{\partial x^\alpha} \simeq \Phi_{t_f,M}^T(t) D_\alpha^T U \Phi_{L,N}(x); \quad \frac{\partial^{2\alpha} u(x,t)}{\partial x^{2\alpha}} \simeq \Phi_{t_f,M}^T(t) U D_{2\alpha} \Phi_{L,N}(x).$$

By substituting this above approximation in given system (5) by using step method. Take, $0 < x < L$, $0 < t < \tau$, we get

$$\Phi_{t_f,M}^T(t) D_\alpha^T U \Phi_{L,N}(x) - \epsilon \Phi_{t_f,M}^T(t) U D_{2\alpha} \times \Phi_{L,N}(x) + \Phi_{t_f,M}^T(t) U \Phi_{L,N}(x) \times \left(\Phi_{t_f,M}^T(t) G \Phi_{L,N}(x) - 1\right) + \Phi_{t_f,M}^T(0\ ) \times U\Phi_{L,N}(x) g(x) = 0,$$

we collocate at $(M+1)(N-1)$ points as:

$$\Phi_{t_f,M}^T(t_i) D_\alpha^T U \Phi_{L,N}(x_j) - \epsilon \Phi_{t_f,M}^T(t_i) U D_{2\alpha} \Phi_{L,N}(x_j) + \Phi_{t_f,M}^T(t_i) \times U \Phi_{L,N}(x_j) \left(\Phi_{t_f,M}^T(t_i) G \Phi_{L,N}(x_j) - 1\right) + \Phi_{t_f,M}^T(0) U \Phi_{L,N}(x_j) g(x_j) = 0,$$

Where $t_i, i = 0, 1, \ldots, M$ are the roots of $P_{t_f,M+1}^{(a,b)}(t)$, while $x_j, j = 0, 1, \ldots, N-2$ are the roots of $P_{L,N-1}^{(a,b)}(x)$; this is generates $(M+1)(N-1)$nonlinearalgebraic equations in theunknown expansion coefficients, $u_{ij}, i = 0, 1, \ldots M$, $j = 0, 1, \ldots, N-2$, and the rest of this system is obtained from the boundary conditions as:

$$\Phi_{t_f,M}^T(t_i) U \Phi_{L,N}(0) = r_1(t_i);$$
$$\Phi_{t_f,M}^T(t_i) U \Phi_{L,N}(L) = r_2(t_i), \quad i = 0, 1, 2, \ldots M.$$

The above determined $(M+1)(N-1)$ system of equations may be combined with the above $2(M+1)$ equations to be written as $(M+1)(N+1)$ system of nonlinear algebraic equations in the unknown expansion coefficientsu_{ij}, that may be solved by using Newton's iterative method.

Consequently, $u_{M,N}(x,t)$ given in equation can be calculated as follows,

$$u_{M,N}(x,t) = \Phi_{t_f,M}^T(t) U \Phi_{L,N}(x), \quad 0 < x \le L, \quad 0 < t \le \tau.$$

Set $u_{M,N}(x,\tau) = u(x,\tau) = s_1(x)$. Similarly, we apply above technique for the interval $0 < x < L$, $\tau \le t \le (2\tau)$

With the boundary condition, then the equation (5) as:

$$\Phi_{t_f,M}^T(t_i) D_\alpha^T U \Phi_{L,N}(x_j) - \epsilon \Phi_{t_f,M}^T(t_i) U \times D_{2\alpha} \Phi_{L,N}(x_j) + \Phi_{t_f,M}^T(t_i) U \Phi_{L,N}(x_j) \times \left(\Phi_{t_f,M}^T(t_i) H^T U \Phi_{L,N}(x_j) - 1\right) + \Phi_{t_f,M}^T(0) U \Phi_{L,N}(x_j) - s_1(x_j) = 0.$$

It follows that, $u_{M,N}(x,t)$ given in equation can be calculated as,

$$u_{M,N}(x,t) = \Phi_{t_f,M}^T(t) U \Phi_{L,N}(x), \quad 0 < x \le L, \quad \tau < t \le 2\tau.$$

Set$u_{M,N}(x, 2\tau) = u(x, 2\tau) = s_2(x)$. Continue applying this above process in finite terms, we get the approximate solution $u_{(M,N)}(x,t)$in the interval $x \in [0, L]$, $t \in [(n-1)\tau, n\tau]$, $n = 1, 2, \cdots$.

Example 4.2: Consider the time-delay fractional Fisher's equation ([5], [7]):

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = 6 \frac{\partial^{2\alpha} u(x,t)}{\partial x^{2\alpha}} + u(x,t) \times [1 - u(x, t-1)], \quad (7)$$
$$0 \le x \le 1, \quad 0 < t \le 2,$$

Subject to initial condition

$$u(x,t) = \frac{1}{4}\left(-1 + \tanh \frac{x}{2\sqrt{6}}\right)^2, \quad 0 \le x \le 1, \quad -1 \le t \le 0,$$

and boundary conditions

$$u(0,t) = \frac{1}{4}\left(1 + \tanh \frac{5t}{12}\right)^2, t > 0,$$
$$u(1,t) = \frac{1}{4}\left(-1 + \tanh \frac{1}{12}(\sqrt{6} - 5t)\right)^2.$$

Figure 1

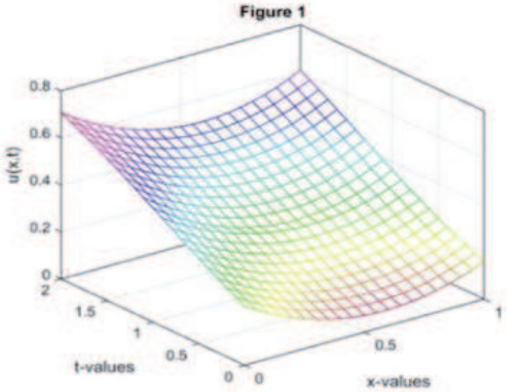
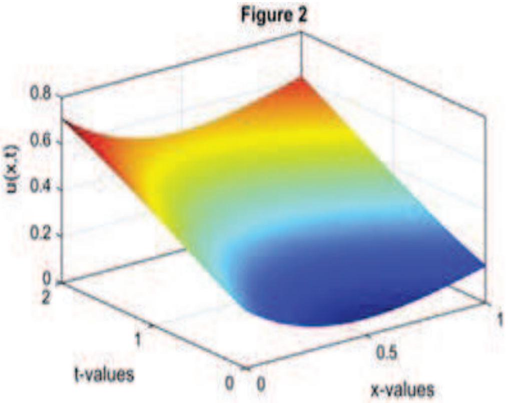


Figure 2



We solve (7) for different values of α. By using the theoretical technique given in section 4.1, one can obtain approximate solution for the given system, also applying the Legendre basis ($a = 0, b = 0$) with $M = N = 3$. In Figures 1 and 2 presents the approximate solutions of $\alpha = 1$, 0.75 respectively.

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