ANALYSIS OF AN M[X]/G/1 FEEDBACK RETRIAL QUEUE WITH SECOND MULTI-OPTIONAL SERVICE AND MODIFIED BERNOULLI VACATIONS

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Abstract: This paper deals with the steady state behaviour of a batch arrival feedback retrial queueing system with second multi optional services, modified Bernoulli vacation for impatient customers, where the server is subject to breakdowns and delaying repair. Any arriving batch may balk (or renege) the system at some particular times. All the customers demand the first 'essential' service, whereas only some of them demand the second multiple 'optional' service. We construct the mathematical model and derive the probability generating functions of number of customers in the system/orbit by using the supplementary variable method. Some important system performances are obtained.

Keywords: retrial queue; optional service, modified Bernoulli vacations; impatient customer

Introduction: The theory of retrial queues have been extensively applied in the study of communication and computer networks. The special characteristic of retrial queues is that, a customer who finds a busy server does not leave the system or joins a queue. He joins an orbit (retrial group) from where he makes repeated attempts to obtain service. Several survey articles and monographs have been published on retrial queues; see Artalejo [1]. Krishnakumar and Arivudainambi [2] have investigated a single server retrial queue with Bernoulli schedule and general retrial times. Queueing systems with vacations and server breakdowns have been found to be useful in modeling the systems in which the server has additional tasks. Single server queueing systems with server breakdowns and vacation have been studied by many researchers including Choudhury and Deka [3]. The suggested model has potential application in the transfer model of an email system. Simple mail transfer protocol (SMTP) is used to deliver the messages between the mail servers. In this paper, we study a model M[X]/G/1 feedback retrial queue with second multi optional service, modified Bernoulli vacations and delaying repair.

Model Description: In this section, we develop a model for batch arrival feedback retrial queue with second multi optional service, modified Bernoulli vacation for impatient customers, where the server is subject to breakdowns and delaying repair. The detailed description of the model is given as follows:

Arrival process: Customers arrive in batches according to a compound Poisson process with rate λ. Let X_u denote the number of customers belonging to the u^{th} arrival batch, where X_u , u =1,2,3,...are with a common distribution $\Pr[X_u = n] = \chi_n$, n=1,2,3...and $X(z)$ denotes the probability generating function of X.

Retrial process: We assume that there is no waiting space and therefore if an arriving batch of customers finds the server free, the arrival beings his service one from the batch and rest of them join into pool of blocked customers called an orbit. If an arriving batch finds the server being busy, vacation or breakdown, the arrivals either leave the service area with probability b and join into an orbit, or balk the system with probability $1-b$. Measured from the moment when the server becomes idle, the customer at the head of the retrial queue competes with potential primary customers to decide which customer will enter service next. If a primary customer arrives first, the retrial customer may cancel its attempt for service and either returns to its position in the retrial queue with probability r or quits the system with probability $1-r$. Inter-retrial times have an arbitrary distribution $R(t)$ with corresponding Laplace-Stieltijes transform (LST) $R^*(\vartheta)$

Service process: The First Essential Service (FES) is needed to all arriving customers and the service time has a general distribution. The service time of the first essential service is denoted by the random variable S_o with distribution function $S_0(t)$ and LST $S_0^*(\vartheta)$. As soon as first essential service completed, with probability r_k ($1 \le k \le m$), the customer may chose for a certain Second Multi-Optional Service (SMOS) from m ($m \ge 1$) kinds of different services, or else with probability $r_0 = 1 - \sum_{k=1}^{m} r_k$ he leaves the system. The service times follows a general random variable S_k with d.f $S_k(t)$ and LST $S_k^*(\vartheta)$.

Feedback process: After service complet-ion epoch the served unsatisfied custo-mers may join into the orbit with prob-ability θ or may leaves the system with complementary probability $1-\theta$.

Vacation process: After completion of service to each customer, the server may take a vacation of random length V with probability p, and with probability q = $1-p$ it waits for the next customer to serve. If orbit is empty, the server always takes a vacation. At the end of a vacation, the server remains idle for the customer from the orbit or new arrival customers. The vacation time of the server is of random length V with distribution function $V(t)$ and LST $V^*(\vartheta)$.

Breakdown process: While the server is working with any phase of service, it may breakdown at any time and the service channel will fail for a short interval of time i.e. server is down for a short interval of time. The breakdowns i.e. server's life times are generated by exogenous Poisson processes with rates α_o for FES and α_k for SMOS, which we may call some sort of disaster during FES and SMOS periods respectively.

Delay process: As soon as breakdown occurs the server is sent for repair, during that time it stops providing service to the arriving batch of and waits for repair to start, which we may refer to as waiting period of the server. We define the waiting time as delay time. The delay time for FES D_o of the server follows with d.f. $D_o(y)$ and LST $D_0^*(\vartheta)$. The delay time for SMOS D_k of the server follows with d.f. $D_k(y)$ and LST $D_k^*(\vartheta)$.

Repair process: The customer who was just being served before server breakdown waits for the remaining service to complete. The repair time (denoted by G_o for FES and G_k for SMOS) distributions of the server for both the phases of service are assumed to be arbitrarily distributed with d.f. $G_o(y)$ for FES and $G_k(y)$ for SMOS. It is LST $G_0^*(\vartheta)$ and $G_k^*(\vartheta)$.

Various stochastic processes involved in the system are assumed to be independent of each other. In the steady state, we assume that $R(0)=0R(\infty)=1, S_o(0)=0, S_o(\infty)=1, V(0)=0, V(\infty)=0, S_k(0)=0, S_k(\infty)=1$ are continuous at $x=0$ and $D_o(0)=0$, $D_k(\infty)=1$, $G_o(0)=0$, $G_k(\infty)=1$ are conti-nuous at $y = 0$ ($1\leq k \leq m$). The state of system at time t can be described by the bivariate Markov process $\{C(t), N(t); t\geq 0\}$ where $C(t)$ denotes the server state (0,1,2,3,4) and $R^0(t), S_0^0(t), S_k^0(t), V^0(t), D_0^0(t), D_k^0(t), G_0^0(t)$ and $G_k^0(t)$ depending, if the server is idle, busy on FES or SMOS, vacation, delaying repair on FES or SMOS and repair on FES or SMOS respectively. $N(t)$ corresponding to the number of customers in orbit at time t.

So that the functions $a(x), \mu_k(x), \gamma(x), \eta_k(y)$ and $\xi_k(y)$ are the conditional completion rates for repeated attempts, service, vacation, delay time and repair time on respective phases respectively ($0\leq k \leq m$).

$$a(x)dx = \frac{dR(x)}{1-R(x)}, \mu_k(x)dx = \frac{dS_k(x)}{1-S_k(x)}, \gamma(x)dx = \frac{dV(x)}{1-V(x)},$$

$$\eta_k(y)dy = \frac{dD_k(y)}{1-D_k(y)}, \xi_k(y)dy = \frac{dG_k(y)}{1-G_k(y)}.$$

Then the embedded Markov chain $\{Z_n; n\in N\}$ is ergodic if and only if, $\rho<1$. where $\rho = (E(X)+r-1)\left[1-R^*(\lambda)\right]+\varpi b$.

$$\varpi = \lambda E(X)\begin{pmatrix} E(S_0)[1+\alpha_0(E(D_0)+E(G_0)] \\ +\sum_{k=1}^{m} r_k E(S_k)[1+\alpha_k(E(D_k)+E(G_k)]+pE(V) \end{pmatrix}$$

Steady state distribution: In this section, we first develop the steady state difference-differential equations for the retrial system. Then we derive the probability generating functions for the server state and the number of customers in the system/orbit. For the process $\{N(t), t\geq 0\}$, we define the probabilities $P_0(t) = P\{C(t)=0, N(t)=0\}$ and probability densities, for $t\geq 0, x\geq 0, n\geq 0$ and ($0\leq k\leq m$)

$$P_n(x,t)dx = P\left\{C(t)=0, N(t)=n, x\leq R^0(t)<x+dx\right\},$$

$$\Pi_{k,n}(x,t)dx = P\left\{C(t)=1, N(t)=n, x\leq S_k^0(t)<x+dx\right\},$$

$$\Omega_n(x,t)dx = P\left\{C(t)=2, N(t)=n, x\leq V^0(t)<x+dx\right\},$$

$$Q_{k,n}(x,y,t)dy = P\begin{Bmatrix} C(t)=3, N(t)=n, \\ y\leq D_k^0(t)<y+dy \;/ S_k^0(t)=x \end{Bmatrix},$$

$$R_{k,n}(x,y,t)dy = P\begin{Bmatrix} C(t)=4, N(t)=n, \\ y\leq G_k^0(t)<y+dy / S_k^0(t)=x \end{Bmatrix}.$$

By the method of supplementary variable technique, we obtain the following system of governing equations,

$$\lambda bP_0 = \int_0^\infty \Omega_0(x)\gamma(x)dx \tag{1}$$

$$\frac{dP_n(x)}{dx} + [\lambda + a(x)]P_n(x) = 0, \; n\geq 1 \tag{2}$$

$$\begin{aligned}\frac{d\Pi_{k,n}(x)}{dx} + [\lambda+\alpha_k+\mu_k(x)]\Pi_{k,n}(x) = \lambda b\sum_{u=1}^{n}\chi_u\Pi_{k,n-u}(x) \\ +\lambda(1-b)\Pi_{k,n}(x) + \int_0^\infty \xi_k(y)R_{k,n}(x,y)dy, n\geq 1, (0\leq k\leq m)\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{d\Omega_n(x)}{dx} + [\lambda+\gamma(x)]\Omega_n(x) = \lambda b\sum_{u=1}^{n}\chi_u\Omega_{n-u}(x) \\ +\lambda(1-b)\Omega_n(x), n\geq 1\end{aligned} \tag{4}$$

$$\begin{aligned}\frac{dQ_{k,n}(x,y)}{dy} + [\lambda+\eta_k(y)]Q_{k,n}(x,y) = \lambda(1-b)Q_{k,n}(x,y) \\ +\lambda b\sum_{u=1}^{n}\chi_u Q_{k,n-u}(x,y), n\geq 1, (0\leq k\leq m)\end{aligned} \tag{5}$$

$$\begin{aligned}\frac{dR_{k,n}(x,y)}{dy} + [\lambda+\xi_k(y)]R_{k,n}(x,y) = \lambda(1-b)R_{k,n}(x,y) \\ +\lambda b\sum_{u=1}^{n}\chi_u R_{k,n-u}(x,y), n\geq 1, (0\leq k\leq m)\end{aligned} \tag{6}$$

The steady state boundary conditions are

$$\begin{aligned}P_n(0) = (1-\theta)qr_0\int_0^\infty \Pi_{0,n}(x)\mu_0(x)dx + \int_0^\infty \Omega_n(x)\gamma(x)dx \\ +(1-\theta)q\sum_{k=1}^{m}\int_0^\infty \Pi_{k,n}(x)\mu_k(x)dx + \theta qr_0\int_0^\infty \Pi_{0,n-1}(x)\mu_0(x)dx \\ +\theta q\sum_{k=1}^{m}\int_0^\infty \Pi_{k,n-1}(x)\mu_k(x)dx, \; n\geq 1, (1\leq k\leq m)\end{aligned} \tag{7}$$

$$\Pi_{0,n}(0)=\int_0^\infty P_{n+1}(x)a(x)dx+\lambda r\sum_{u=1}^{n}\chi_u\int_0^\infty P_{n-u+1}(x)dx$$
$$+\lambda(1-r)\sum_{u=1}^{n}\chi_u\int_0^\infty P_{n-u+2}(x)dx+\lambda b\chi_{n+1}P_0,\ n\geq 1 \quad (8)$$

$$\Pi_{k,n}(0)=r_k\int_0^\infty \Pi_{0,n}(x)\mu_0(x)dx,\ n\geq 1,\ (1\leq k\leq m) \quad (9)$$

$$\Omega_0(0)=(1-\theta)\left\{\begin{array}{l} r_0\int_0^\infty \Pi_{0,0}(x)\mu_0(x)dx \\ +\sum_{k=1}^{m}\int_0^\infty \Pi_{k,0}(x)\mu_k(x)dx \end{array}\right\},\ n=0,\ (1\leq k\leq m) \quad (10)$$

$$\Omega_n(0)=p(1-\theta)\left\{r_0\int_0^\infty \Pi_{0,n}(x)\mu_0(x)dx+\sum_{k=1}^{m}\int_0^\infty \Pi_{k,n}(x)\mu_k(x)dx\right\} \quad (11)$$
$$+p\theta\left\{r_0\int_0^\infty \Pi_{0,n-1}(x)\mu_0(x)dx+\sum_{k=1}^{m}\int_0^\infty \Pi_{k,n-1}(x)\mu_k(x)dx\right\},n\geq 1,\ (1\leq k\leq m)$$

$$Q_{k,n}(x,0)=\alpha_k P_{k,n}(x),\ n\geq 0,\ \text{for}(0\leq k\leq m) \quad (12)$$

$$R_{k,n}(x,0)=\int_0^\infty Q_{k,n}(x,y)\eta_k(y)dy,\ n\geq 0\ (0\leq k\leq m) \quad (13)$$

The normalizing condition is

$$P_0+\sum_{n=1}^\infty\int_0^\infty P_n(x)dx+\sum_{n=0}^\infty\left(\sum_{k=0}^{m}\int_0^\infty \Pi_{k,n}(x)dx+\int_0^\infty \Omega_n(x)dx\right)$$
$$+\sum_{n=0}^\infty\left(\sum_{k=0}^{m}\left[\int_0^\infty\int_0^\infty D_{k,n}(x,y)dxdy++\int_0^\infty\int_0^\infty R_{k,n}(x,y)dxdy\right]\right)=1 \quad (14)$$

To solve the above equations, then we define the generating functions for $|z|\leq 1$, for $(0\leq k\leq m)$

$$P(x,z)=\sum_{n=1}^\infty P_n(x)z^n;\ \Pi_k(x,z)=\sum_{n=0}^\infty \Pi_{k,n}(x)z^n;\ \Omega(x,z)=\sum_{n=1}^\infty \Omega_n(x)z^n;$$

$$Q_k(x,y,z)=\sum_{n=0}^\infty Q_{k,n}(x,y)z^n;R_k(x,y,z)=\sum_{n=0}^\infty R_{k,n}(x,y)z^n;\ X(z)=\sum_{n=1}^\infty \chi_n z^n$$

Now multiplying the steady state equation and steady state boundary condition (2)-(13) by z^n and summing over$n$, then solving the partial differential that equations, it follows that for $(0\leq k\leq m)$

$$P(x,z)=P(0,z)[1-R(x)]e^{-\lambda x} \quad (15)$$
$$\Pi_k(x,z)=\Pi_k(0,z)[1-S_k(x)]e^{-A_k(z)x} \quad (16)$$
$$\Omega(x,z)=\Omega(0,z)[1-V(x)]e^{-b(z)x} \quad (17)$$
$$Q_k(x,y,z)=Q_k(x,0,z)[1-D_k(y)]e^{-b(z)y} \quad (18)$$
$$R_k(x,y,z)=R_k(x,0,z)[1-G_k(y)]e^{-b(z)y} \quad (19)$$

where

$$A_k(z)=b(z)+\alpha_k[1-D_k^*(b(z))G_k^*(b(z))]\ \text{and}\ b(z)=\lambda b\left(1-X(z)\right)$$

Solving the above equations (2)-(13), then integrating the equations (15)-(19) with respect to x and y define the partial probability generating functions as,

$$P(z)=\int_0^\infty P(x,z)dx,\ \Pi_k(z)=\int_0^\infty \Pi_k(x,z)dx,\ \Omega(z)=\int_0^\infty \Omega(x,z)dx,$$

$$Q_k(z)=\int_0^\infty Q_k(x,z)dx,\ R_k(z)=\int_0^\infty R_k(x,z)dx,\ \text{for}\ (0\leq k\leq m)$$

$$P(z)=\frac{bP_0[1-R^*(\lambda)]}{V^*(\lambda b)}\times\frac{Nr(z)}{Dr(z)} \quad (20)$$

$$Nr(z)=V^*(\lambda b)\left\{X(z)(1-\theta+\theta z)S_0^*[A_0(z)]\left(q+pV^*\left[b(z)\right]\right)\right.$$
$$\left.\left[r_0+\sum_{k=1}^{m}r_kS_k^*[A_k(z)]\right]-z\right\}+zq\left(V^*\left[b(z)\right]-1\right)$$

$$Dr(z)=z-(1-\theta+\theta z)S_0^*[A_0(z)]\left[r_0+\sum_{k=1}^{m}r_kS_k^*[A_k(z)]\right]$$
$$R(z)\left(q+pV^*\left[b(z)\right]\right)$$

where $R(z)=\left[R^*(\lambda)+\frac{X(z)}{z}[1-r+rz](1-R^*(\lambda))\right]$

$$M(z)=\left\{R(z)\left(q\left(V^*\left[b(z)\right]-1\right)-V^*(\lambda b)\right)+V^*(\lambda b)X(z)\right\}$$

$$\Pi_k(z)=\frac{r_k\lambda bP_0S_0^*[A_0(z)]\left(1-S_k^*(A_k(z))\right)M(z)}{A_k(z)V^*(\lambda b)Dr(z)} \quad (21)$$

$$\Omega(z)=\frac{\lambda bP_0\left(1-V^*[b(z)]\right)M(z)}{b(z)V^*(\lambda b)Dr(z)}\times\left\{\begin{array}{l}\left((1-\theta+\theta z)S_0^*[A_0(z)]\right. \\ \left.\left[r_0+\sum_{k=1}^{m}r_kS_k^*[A_k(z)]\right]\right)\end{array}\right\} \quad (22)$$

$$Q_k(z)=\frac{\alpha_kr_k\lambda bP_0S_0^*[A_0(z)]\left(1-S_k^*(A_k(z))\right)\left(1-D_k^*(b(z))\right)M(z)}{A_k(z)b(z)V^*(\lambda b)Dr(z)}$$
(23)

$$R_k(z)=\frac{\left\{\begin{array}{l}r_k\alpha_k\lambda bP_0S_0^*[A_0(z)]D_k^*[b(z)] \\ \times\left(1-S_k^*(A_k(z))\right)\left(1-G_k^*(b(z))\right)M(z)\end{array}\right\}}{A_k(z)b(z)V^*(\lambda b)Dr(z)} \quad (24)$$

The probability generating function of number of customers in the system is

$$K(z)=P_0+P(z)+\Omega(z)+z\sum_{k=0}^{m}\left(\Pi_k(z)+Q_k(z)+R_k(z)\right)$$

$$K(z)=\frac{Nr1(z)}{\left(1-X(z)\right)Dr(z)} \quad (25)$$

$$Nr1(z)=P_0\left\{z\left\{1-S_0^*\left(A_0(z)\right)\left(r_0+\sum_{k=1}^{m}r_kS_k^*(A_k(z))\right)\right\}M(z)\right.$$
$$+\left(1-V^*(b(z))\right)\left\{\begin{array}{l}qz-(1-\theta+\theta z)\left\{\left(q+pV^*(\lambda b)\right)R(z)-pV^*(\lambda b)X(z)\right\} \\ \left(S_0^*[A_0(z)]\left[r_0+\sum_{k=1}^{m}r_kS_k^*[A_k(z)]\right]\right)\end{array}\right\}$$
$$\left.+\left[1-X(z)\right]\left\{\begin{array}{l}V^*(b(z))(1-\theta+\theta z)\left(q+pV^*(\lambda b)\right)\left(r_0+\sum_{k=1}^{m}r_kS_k^*(A_k(z))\right) \\ S_0^*\left(A_0(z)\right)\left(X(z)b(1-R^*(\lambda))-R(z)\right)+zV^*(\lambda b) \\ \left[1-b(1-R^*(\lambda))\right]+zqb(1-R^*(\lambda))\left[V^*(b(z))-1\right]\end{array}\right\}\right\}$$

The probability generating function of number of customers in the orbit is

$$H(z)=P_0+P(z)+\Omega(z)+\sum_{k=0}^{m}\left(\Pi_k(z)+Q_k(z)+R_k(z)\right) \quad (26)$$

The probability that the server is idle (P_o)

$$P_0 + P(1) + Q(1) + \sum_{k=0}^{m} \left(\Pi_k(1) + Q_k(1) + R_k(1) \right) = 1.$$

$$P_0 = \frac{Nr}{Dr};\ Nr = V^*(\lambda b)\Big(1-\theta-(E(X)+r-1)(1-R^*(\lambda))-\varpi b\Big)$$

$$Dr = \left\{ \Big(q\lambda bE(V)+V^*(\lambda b)\Big)\begin{pmatrix}(1-\theta)-(1-R^*(\lambda)) \\ \big[(E(X)+r-1)-bE(X)\big]\end{pmatrix} \right. \qquad (27)$$

$$-bV^*(\lambda b)(1-R^*(\lambda))(1-\theta)+\lambda bV^*(\lambda b)$$

$$\left\{1-r+(1-R^*(\lambda))(b-1)E(X)\right\}\begin{pmatrix} E(S_0)[1+\alpha_0(E(D_0)+E(G_0)]+ \\ \sum_{k=1}^{m} r_k E(S_k)[1+\alpha_k(E(D_k)+E(G_k)] \end{pmatrix}$$

Note that, $P(z)$ $\Omega(z)$, $\Pi_k(z)$, $Q_k(z)$ and $R_k(z)$ are the PGF of number of customers in the system when server being idle, (vacation, busy on phases, delaying repair on phases and under repair on phases) respectively (for 0≤ $k \le m$).

Performance measures: In this section, from the above results, we derive the steady state probability that the server is idle during the retrial time (P), busy on FES or SMOS ($\Pi_o$or Π_k), on vacation (Ω), delaying repair on FES or SMOS ($Q_o$or Q_k), repair on FES or SMOS ($R_o$or R_k) and The mean number of ordinary customers in the orbit and system, (L_q, L_s) respectively.

$$P = P(1) = \frac{b\left(1-R^*(\lambda)\right)}{Dr}\left\{q\lambda bE(X)E(V)+V^*(\lambda b)\left(\varpi b+\theta+E(X)-1\right)\right\}$$

$$\Pi_k = \sum_{k=0}^{m} \Pi_k(1) = \Gamma \sum_{k=1}^{m} r_k \lambda bE(S_k) \Big/ Dr$$

$$\Omega = \Omega(1) = \frac{\lambda bE(V)}{Dr}\left\{\begin{matrix} q\omega\left(1-\theta-\lambda bE(X)\right)+pV^*(\lambda b)E(X) \\ -\left(q+pV^*(\lambda b)\right)\left(1-R^*(\lambda)\right)\left(E(X)+r-1\right)\end{matrix}\right\}$$

$$Q_k = \sum_{k=0}^{m} Q_k(1) = \Gamma \sum_{k=1}^{m} \alpha_k \lambda bE(S_k)E(D_k) \Big/ Dr$$

$$R_k = \sum_{k=0}^{m} R_k(1) = \Gamma \sum_{k=1}^{m} \alpha_k \lambda bE(S_k)E(G_k) \Big/ Dr$$

$$L_q = \lim_{z\to 1}\frac{d}{dz}H(z) = H'(1) = P_0 + P'(1) + \Omega'(1) + \sum_{k=0}^{m}\left(\Pi'_k(1)+Q'_k(1)+R'_k(1)\right)$$

$$L_s = \lim_{z\to 1}\frac{d}{dz}K(z) = K'(1) = L_q + \sum_{k=0}^{m}\left(\Pi_k(1)+Q_k(1)+R_k(1)\right)$$

$$W_s = \frac{L_s}{\lambda E(X)} \text{ and } W_q = \frac{L_q}{\lambda E(X)}$$

$$\textit{where } \Gamma = \left\{V^*(\lambda b)\left(E(X)-\left(1-R^*(\lambda)\right)\left(E(X)+r-1\right)\right)+q\lambda bE(X)E(V)\right\}$$

Conclusion: In this paper, we have studied a batch arrival feedback retrial queue with second multi optional service, modified Bernoulli vacations for impati-ent customers and subject to server break-downs and delaying repair. The prob-ability generating functions of the number of customers in the system and orbit are found by using the supplementary variable technique. The performance measures like, the mean number of customers in the system/orbit, the average waiting time of customer in the system/orbit are also obtained.

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