PRODUCTION INVENTORY MODEL FOR NON-DETERIORATIVE ITEMS WITH LINEAR DEMAND

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Abstract: Demand considered in most of the classical inventory models is constant and the assumption of a constant demand rate may not be always appropriate for such consumer goods type of inventory (e.g. milk, vegetables, radioactive materials, volatile liquids, etc.), while in most of the practical cases the demand changes with time. Also, most of the works on inventory models do not take the optimal solution in higher order equation. In this paper, a production inventory model for non-deteriorative items with linear demand is considered and the optimum solution is derived in higher order equation. The develop optimum objective of this research is to cycle time and optimum production quantity with minimize the total cost in each model. Numerical example is provided to illustrate the theoretical results and made the sensitivity analysis of parameters on the optimal solutions. The validation of result, this model was coded in Microsoft Visual Basic 6.0

Key words: Production, Demand, Cycle time, Optimality, linear demand.

Introduction: The Economic Production quantity (EPQ) model is widely used by practitioners as a decision-making tool for the control of inventory. The classical EPQ model considers the ideal case whether the value of inventory items are unaffected by time and replenishment is done instantaneously. In real life cases, however, the ideal case is not quite as applicable and most of the practical cases the demand changes with time. The aim of the paper is to develop an EPQ model for a single-item inventory having a time-varying linear demand. The linear time-dependence of demand of the form $D(t) = a + bt, a \geq 0, b \neq 0.$ This type of demand has a better representation of time-varying market. If we compare the other two types of time-dependents like linear and exponential, it is seen that linear time-dependence demand leads to uniform change in the real market. At the same instant, exponential time-dependence demand also seems to be unrealistic because an exponential rate of change is very high and it is in doubt that the market demand of any product may undergo a high rate of change like exponential function. Thus the alternative and probably more realistic approach is to consider the quadratic time-dependence of demand which may represent all types of time-dependence depending on the signs of the parameters of the time-quadratic demand function. The first classical Economic Order Quantity (EOQ) formula, which is also known square-root-formula, was developed by F.W Harris (1913). In this formula, demand of items was assumed to be constant over time. But in the real life situations, demand rate of any item is always in a dynamic state. Siver and Meal (1973) developed an approximate solution procedure, known as Silver-meal-Heuristic, for the case of general deterministic time-varying demand pattern. Then the economic production quantity (EPQ) inventory model was proposed by Taft (1918).Dave, U and Patel L.K. (1981) considered an EOQ model in which the demand rate is changing linearly with time and the deteriorating is assumed to be a constant fraction of the onhand inventory. Kun-Jen Chund and Sui-Fu Tsai (1997) considered the inventory replenishment policy over a fixed planning period for a deteriorating item having a deterministic demand pattern with a linear trend and shortages. Tinn-Tsair Teng et al. (1999) assumed that the demand function is positive and fluctuating with time and the models are developed with deteriorating items and shortages. Ghosh and Chaudhuri (2004) developed an inventory model for a deteriorating item, a quadratic time-vary demand and shortages in inventory. A two-parameter weibull distribution is taken to represent the time of deterioration. Sana, S et al. (2004) developed a production inventory model for a deteriorating item over a finite planning horizon with a linear time-varying demand, finite production rate and shortages. Ghosh, S.K. and Chaudhuri, K.S. (2006) developed an EOQ model over a finite time-horizon for a deteriorating item with a quadratic, time-dependent demand, allowing shortages in inventory and the rate of deterioration is taken to be time-proportional and it is assumed that shortage occur in every cycle. Gour Chandra Mahata (2011) constructed an inventory level for deteriorating items with instantaneous replenishment, exponential decay rate and a time varying linear demand without shortages under permissible delay in payments. Mingbao Cheng et al. (2011) considered an inventory model for time-dependent deteriorating items with trapezoidal type demand rate and partial backlogging. Ghosh et al. (2012) considered an optimal inventory replenishment policy for a deteriorating items time-quadratic demand and time-dependent partial backlogging which depends on the length of the waiting time for the next replenishment over a finite time horizon and variable replenishment cycle. Vinod Kumar Mishra et al.(2013) considered a deterministic inventory model with time-dependent demand and time-varying holding cost where deteriorating is time proportional and the model considered here allows shortages and the demand is partially backlogged. In this paper, a production inventory model for non-deteriorative items with linear demand is considered and the optimum solution is derived in higher order equation. The objective of this research is to develop optimum cycle time and optimum production quantity with minimize the total cost in each model. Numerical example is provided to illustrate the theoretical results and made the sensitivity analysis of parameters on the optimal solutions. The validation of result, this model was coded in Microsoft Visual Basic 6.0. This paper is organized as follows. Section 2 is concerned with assumptions and notations, Section 3 presents mathematical model for finding the optimal solutions and numerical example. Finally, the paper summarizes and concludes in section 4.

Assumptions and Notations: The following assumptions are used to formulate the problem.

1) Initial inventory level is zero and planning horizon is infinite.
2) The demand rate is a linear function where D – $a + bt, a \geq 0, b \neq 0$. Here 'a' and 'b' are constants and "a" stands for the initital demand and "b" is position trend in demand.
3) Shortages are not allowed.
4) Holding cost per unit per year is known.
5) Items are produced/ purchased and added to the inventory.
6) The item is a single product; it does not interact with any other inventory items.
7) The production rate is always greater than or equal to the sum of the demand rate.

The following notations are used in our analysis: P – Production rate in units per unit time, Q*-Optimal size of production run, C_p – Production Cost per unit, C_h -Holding cost per unit/year, C_0 – Setup cost / ordering cost, T – Cycle time, T_1 - The time during which the stock is building up at a constant rate of P-D units per unit time that is Production time.

Mathematical Model: Figure 1 represents the EPQ model with linear demand. The inventory on-hand increases with the rate P-D, which is the production rate minus consumption rate, until time T_1 when the production process stops and the inventory on hand reaches its maximum level, Q_1. After that point, the inventory level decreases with the consumption rate D, until it becomes zero at the end of the cycle T, when the production process is resumed again. Here, $D = a + bt$

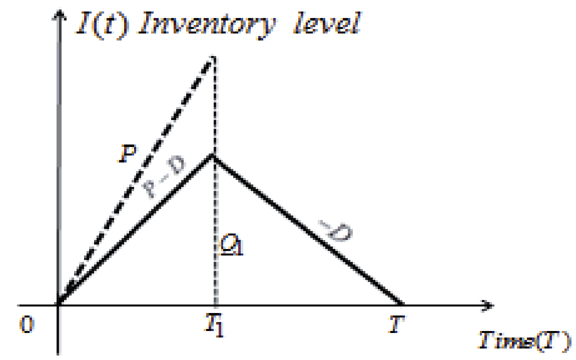

Figure 1 : Production inventory model with linear demand

The differential equations are

$$\frac{d}{dt} I(t) = P - (a + bt); 0 \leq t \leq T_1 \quad (1)$$

$$\frac{d}{dt} I(t) = -(a + bt); T_1 \leq t \leq T \quad (2)$$

with the boundary conditions $I(0) = 0; I(T) = 0; I(T_1) = Q_1$ (3)

The solutions of above differential equations are

From (1), $I(t) = \int_0^t (P - (a + bt))dt = Pt - at - \frac{b}{2}t^2$ (4)

From(2); $I(t) = \int_t^T (a+bt)dt = a(T-t) + \frac{b}{2}\left(T^2 - t^2\right)$ (5)

From the equations (1) and (2);

$PT_1 - aT_1 - \frac{b}{2}T_1^2 = a(T-T_1) + \frac{b}{2}\left(T^2 - T_1^2\right)$, Therefore, $T_1 = \frac{a}{P}T + \frac{b}{2P}T^2$ (6)

From the equations (1) and (3),

$I(T_1) = Q_1 \Rightarrow PT_1 - aT_1 - \frac{b}{2}T_1^2$, therefore, $Q_1 = (P-a)T_1 - \frac{b}{2}T_1^2$ (7)

Total Cost: The total cost comprise of the sum of the Production cost, Ordering cost, holding cost, Deteriorating cost. They are grouped together after evaluating the above cost individually.

(i) Ordering Cost / unit time = $\frac{C_0}{T}$ (8)

(ii) Production Cost /unit time = DC_P (9)

(iii) Holding Cost / unit time : Holding cost is applicable to both stages of the production cycle, as described by

HC = $\frac{C_h}{T}\left[\int_0^{T_1} I(t)dt + \int_{T_1}^{T} I(t)dt\right]$

$$= \frac{C_h}{T}\left[\begin{array}{l}\int_0^{T_1}\left((P-a)t - \frac{bt^2}{2}\right)dt \\ +\int_{T_1}^{T}\left(a(T-t) + \frac{b}{2}\left(T^2 - t^2\right)\right)dt\end{array}\right]$$

$$= \frac{C_h}{T}\left[\frac{(P-a)}{2}T_1^2 + \frac{a}{2}(T-T_1)^2 + \frac{b}{3}T^3 - \frac{b}{2}T^2T_1\right]$$

$$= C_h\left[\frac{(P-a)}{2T}T_1^2 + \frac{a}{2T}(T-T_1)^2 + \frac{b}{3}T^2 - \frac{b}{2}TT_1\right] \quad (10)$$

Therefore, Total Cost (TC)

= Purchase Cost + Ordering Cost + Holding Cost $TC = \frac{C_0}{T} + DC_P + C_h\left[\begin{array}{l}\frac{(P-a)}{2T}T_1^2 + \frac{a}{2T}(T-T_1)^2 \\ +\frac{b}{3}T^2 - \frac{b}{2}TT_1\end{array}\right]$

(11)

Differentiating the Total Cost (11) with respect to T_1,

$$\frac{\partial}{\partial T_1}(TC) = \frac{C_h}{T}\left[(P-a)T_1 - a(T-T_1) - \frac{b}{2}T^2\right] = 0$$

$(P-a)T_1 - a(T-T_1) - \frac{b}{2}T^2 = 0$ Therefore, $T_1 = \frac{a}{p}t + \frac{b}{2p}T^2$ (12)

Differentiating the total cost (11) with respect to T,

$$\frac{\partial}{\partial T}(TC) = \frac{-C_0}{T^2} + C_h\left[\begin{array}{l}\frac{-(P-a)T_1^2}{2T^2} + \frac{a(T^2 - T^2_1)}{2T^2} + \\ \frac{2}{3}bT - \frac{b}{2}T_1\end{array}\right] = 0$$

$$-(P-a)T_1^2 + a(T^2 - T_1^2) + \frac{4}{3}bT^3 - 2bT^2T_1 = \frac{2C_0}{C_h}$$

$$-PT_1^2 + aT^2 - 2bT^4 + \frac{4}{3}bT^3 - bT_1T^2 - \frac{2C_0}{C_h} = 0$$

Substitute the value of T_1, eliminate T^4 and higher power of T and simplify,

$$P\left[\frac{aT}{P} + \frac{bT^2}{2P}\right]^2 - aT^2 - \frac{4bT^3}{3} + \frac{abT^3}{P} + \frac{b^2T^4}{2P} = -\frac{2C_0}{C_h}$$

$$\frac{-3b^2}{4P}T^4 + \frac{4Pb - 6ab}{3P}T^3 + \frac{(aP - a^2)T^2}{P} = \frac{2C_0}{C_h}$$

Therefore, $2(3ab - 2Pb)T^3 - 3a(P-a)T^2 + \frac{6PC_0}{C_h} = 0$ (13)

Which is the optimum solution equation for T.

Note : When b =0 and D=a, then $T = \sqrt{\frac{2PC_0}{DC_h(P-D)}}$ which is the standard inventory model.

For example, Let us consider,

P=5000, D = 4500, C_0 =100, C_h =10, C_d =100, a=4100, b =2220

Optimum Solution:

T = 0.1803, $T_1\, 0.1551$, Q = 811.38, Q_1 = 112.87,Setup cost = 554.64,

Production cost = 450,000, Holding Cost = 602.75, Total cost = 451157.39

Sensitivity Analysis

Table -1: Production Inventory model for non-deteriorative items with linear demand

Parameters		Optimum values				
		T	T_1	Q	Q_1	Total Cost
C_0	80	0.1594	0.1364	717.48	102.09	451067.70
	90	0.1701	0.1459	757.55	107.68	446495.44
	100	0.1803	0.1550	811.34	112.86	451157.39
	110	0.1901	0.1639	851.34	117.71	448969.30
	120	0.1996	0.1726	902.86	122.25	453469.97
C_h	8	0.2043	0.1768	930.34	124.42	456384.29
	9	0.1912	0.1649	870.67	118.23	456451.19
	10	0.1803	0.1550	811.34	112.86	451157.39
	11	0.1710	0.1467	773.83	108.16	453665.34
	12	0.1630	0.1396	730.36	104.00	449243.62
C_p	80	0.1803	0.1550	811.34	112.86	361178.08
	90	0.1803	0.1550	811.34	112.86	406180.67
	100	0.1803	0.1550	811.34	112.86	451157.39
	110	0.1803	0.1550	811.34	112.86	496185.84
	120	0.1803	0.1550	811.34	112.86	541188.43
a	3900	0.1604	0.1308	682.71	124.91	426884.63
	4000	0.1690	0.1416	739.59	119.33	438746.25
	4100	0.1803	0.1550	811.34	112.86	451157.39
	4200	0.1958	0.1730	907.87	105.20	464571.13
	4300	0.2200	0.2000	1053.58	95.59	479845.20
b	2020	0.1785	0.1528	796.42	113.96	447229.46
	2120	0.1794	0.1539	803.81	113.42	449195.22
	2220	0.1803	0.1550	811.34	112.86	451157.39
	2320	0.1812	0.1562	819.14	112.28	453194.54
	2420	0.1821	0.1574	827.09	111.68	455230.09

Observation: It is observed from the table 1

1. With the increase in setup cost per unit (C_0), optimum quantity (Q*), maximum inventory (Q_1), Production time (T_1), cycle time (T) and total cost increases.
2. With the increase in holding cost per unit (C_h), optimum quantity (Q*), maximum inventory (Q_1), production time (T_1) and cycle time (T) decreases but total cost increases.
3. Similarly, other parameters, production cost per unit (C_p), rate of demand "a" and "b"can also be observed from the table-1.

Conclusion: In this paper, a production inventory model for non-deteriorative items with linear demand is considered and the optimum solution is derived in higher order equation. The objective of this research is to develop optimum cycle time and optimum production quantity with minimize the total cost in each model. Numerical example is given to illustrate the theoretical results and made the sensitivity analysis of parameters on the optimal solutions. The validation of result, this model was coded in Microsoft Visual Basic 6.0

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