HARMONIC MEAN LABELING ON SOME MORE SPECIAL TYPE OF GRAPHS

C. DAVID RAJ, C. JAYASEKARAN

Abstract: A graph G = (V, E) with p vertices and q edges is called a Harmonic mean graph if it is possible to label the vertices x∈V with distinct labels f(x) from 1, 2, ..., q + 1 in such a way that when each edgee = xy is labeled with $f(e = xy) = \left\lceil \frac{2f(x)f(y)}{f(x)+f(y)} \right\rceil$ or $\left\lfloor \frac{2f(x)f(y)}{f(x)+f(y)} \right\rfloor$, then the edge labels are distinct. In this case f is called Harmonic mean labeling of G. In this paper, we apply Harmonic mean labeling technique on some special type of graphs.

Key words: Graph, Harmonic mean labeling, Harmonic mean graph.

Introduction: Let G(p, q) be a graph with p = |V(G)| vertices and q = |E(G)| edges, where V(G) and E(G) respectively denote the vertex set and edge set of the graph G. In this paper, we consider the graphs which are simple, finite and undirected.

The concept of graph labeling was introduced by Rosa[5] in 1967. Harmonic mean labeling of graphs was introduced by S. Somasundaram, R. Ponraj and S.S.Sandhya in [6] and they investigated several Harmonic mean graphs in [7, 8]. Some of the Harmonic mean graphs are investigated by C. Jayasekaran, C. David Raj and S.S Sandhya in [3, 4, 9]. A detailed survey of graph labeling is available in Gallian[1]. For graph theoretic terminology and notations we refer to Harary[2]. In this paper, we investigate Harmonic mean labeling of some special type of graphs. The following definitions are useful for the present investigation.

Definition 1.1: A graph G = (V, E) with p vertices and q edges is called a Harmonic mean graph if it is possible to label the vertices x∈V with distinct labels f(x) from 1, 2, ..., q + 1 in such a way that when each edge e = xy is labeled with $f(e = xy) = \left\lceil \frac{2f(x)f(y)}{f(x)+f(y)} \right\rceil$ or $\left\lfloor \frac{2f(x)f(y)}{f(x)+f(y)} \right\rfloor$, then the edge labels are distinct. In this case f is called Harmonic mean labeling of G.

Definition 1.2: Consider two copies of $C_m \odot K_1$. Connect two vertices of degree one from each copy by an edge, the new graph obtained is called joint sum of $C_m \odot K_1$.

Definition 1.3: The corona of two graphs G_1 and G_2 is the graph $G = G_1 \odot G_2$ formed by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.

Main Results:

Theorem 2.1: A graph obtained by joining an end vertex of a path with an end vertex of a crown by an edge is a Harmonic mean graph.

Proof: Let $u_1u_2u_3 \ldots u_mu_1$ be the cycle C_m. Let v_i be the vertex which is adjacent to u_i, $1 \le i \le m$. The resultant graph is the crown $C_m \odot K_1$. Let $w_1w_2w_3 \ldots w_n$ be the path P_n. Join v_m and w_1 by an edge v_mw_1. The resultant graph is the required graph G whose edge set is E = $\{u_iu_{i+1}, u_mu_1 / 1 \le i \le m-1\} \cup \{u_iv_i / 1 \le i \le m\} \cup \{w_iw_{i+1} / 1 \le i \le n-1\}$.

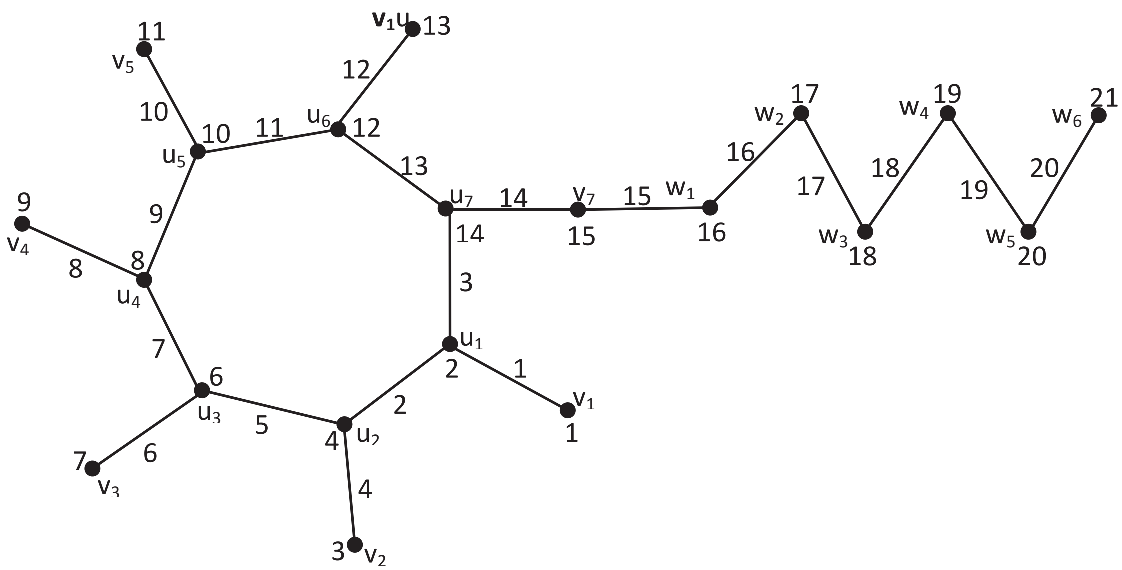

Figure 2.1.

Define a function f: V(G) → {1, 2, …,q + 1} by $f(u_i) = 2i$, $1 \le i \le m$;
$f(v_i) = 2i - 1$, $1 \le i \le 2$;
$f(v_i) = 2i + 1$, $3 \le i \le m$;
$f(w_i) = 2m + i + 1$, $1 \le i \le n$;
Then the edges get the labels
$f(u_1v_1) = 1$; $f(u_iv_i) = 2i$, $2 \le i \le m$;
$f(u_1u_2) = 2$;
$f(u_iu_{i+1}) = 2i + 1$, $2 \le i \le m - 1$; $f(u_mu_1) = 3$;
$f(v_mw_1) = 2m + 1$;
$f(w_iw_{i+1}) = 2m + i + 1$, $1 \le i \le n - 1$;
In the view of above labeling pattern f provides a Harmonic mean labeling for G
Example 2.2: Harmonic mean labeling of the graph obtained by joining an end vertex of the path P_6 with an end vertex of the crown $C_6ʘK_1$ by an edge is given in figure 2.1.
Theorem 2.3: The joint sum of two copies of $C_nʘK_1$ is a Harmonic mean graph.
Proof: Let G be the joint sum of two copies of $C_nʘK_1$. Let $u_1, u_2, u_3, \dots, u_n, v_1, v_2, v_3, \dots, v_n$ be the vertices of first copy of $C_nʘK_1$ and let $s_1, s_2, s_3, \dots, s_n, t_1, t_2, t_3, \dots, t_n$ be the vertices of second copy of $C_nʘk_1$. Join u_1 and s_1 by an edge u_1s_1. The resultant graph is G. Define a function f: V(G) → {1, 2, …,q + 1} by
$f(u_1) = 1$; $f(u_i) = 4i - 1$, $2 \le i \le n$;
$f(v_1) = 3$; $f(v_i) = 4i$, $2 \le i \le n$;
$f(s_1) = 2$; $f(s_i) = 4i + 2$, $2 \le i \le n$;

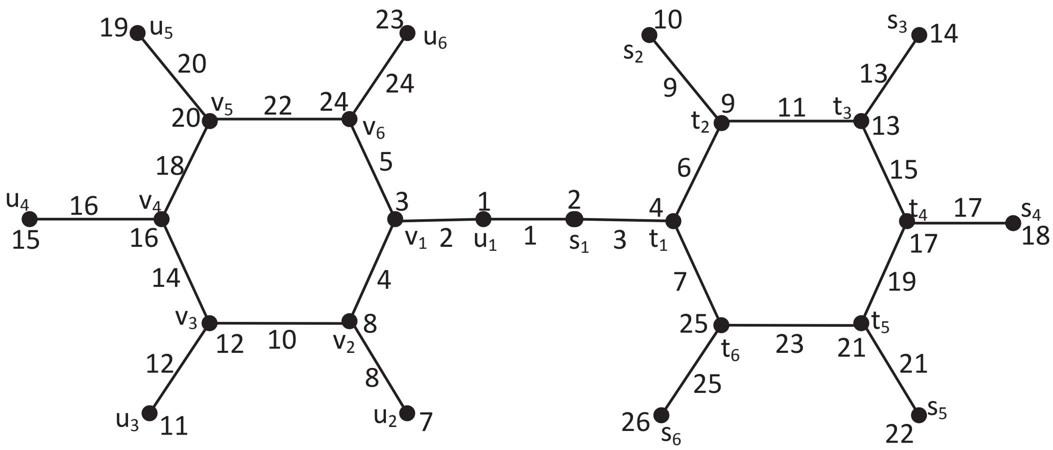

Figure 2.2
Theorem 2.5.A graph obtained by joining an end vertex of a crown with a vertex of a circle by an edge is a Harmonic mean graph.
Proof: Let $v_1v_2v_3 \dots v_mv_1$ be the cycle C_m. Let $u_i$be the vertex which is adjacent to v_i, $1 \le i \le m$. The resultant graph is the crown $C_mʘK_1$. Let $w_1w_2w_3 \dots w_nw_1$ be the cycle C_n. Join u_1 and w_1 by an edge u_1w_1.The resultant graph is the required graph G
$f(t_1) = 4$; $f(t_i) = 4i + 1$, $2 \le i \le n$;
Then the edges get the labels
$f(u_1s_1) = 1$; $f(u_1v_1) = 2$; $f(u_iv_i) = 4i$, $2 \le i \le n$;
$f(v_1v_2) = 4$; $f(v_iv_{i+1}) = 4i + 2$, $2 \le i \le n - 1$; $f(v_nv_1) = 5$;
$f(s_1t_1) = 3$; $f(s_it_i) = 4i + 1$, $2 \le i \le n$;
$f(t_1t_2) = 6$; $f(t_it_{i+1}) = 4\,i + 3$, $2 \le i \le n - 1$; $f(t_nt_1) = 7$;
Thus f provides a Harmonic mean labeling for G. Hence G is a Harmonic mean Graph.
Example 2.4: Harmonic mean labeling of joint sum of two copies of $C_6ʘK_1$is given in figure 2.2.
whose edge set is $E = \{u_iu_{i+1}, u_mu_1/1 \le i \le m - 1\} \cup \{u_iv_i\ /\ 1 \le i \le m\} \cup \{\ w_iw_{i+1}, w_nw_1\ /\ 1 \le i \le n - 1\}$. Define a function f: V(G) → {1, 2, …, q + 1} by
$f(u_1) = 1$; $f(u_i) = 2i + 2$, $2 \le i \le n$;
$f(v_1) = 2$; $f(v_i) = 2i + 3$, $2 \le i \le n$;
$f(w_1) = 3$; $f(w_i) = 2n + i + 2$, $2 \le i \le m$;
Then the edges get the labels
$f(u_1w_1) = 2$; $f(u_1v_1) = 1$;

$f(u_iv_i) = 2i + 3, 2 \le i \le n;$
$f(v_1v_2) = 3;$
$f(v_iv_{i+1}) = 2i + 4, 2 \le i \le n - 1;$
$f(v_nv_1) = 4;$
$f(w_1w_2) = 5;$
$f(w_iw_{i+1}) = 2n + i + 2, 2 \le i \le m - 1;$

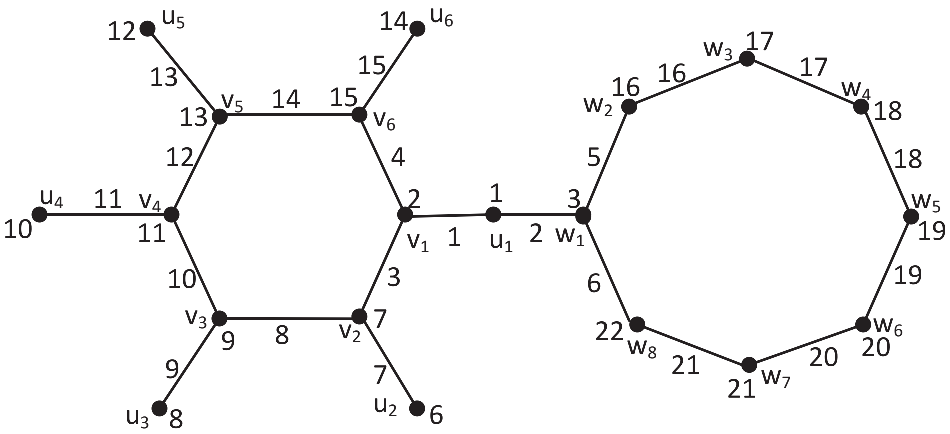

Figure 2.3.

Example 2.6. A Harmonic mean labeling of the graph obtained by joining a vertex of cycle C_8 with an end vertex of the crown $C_6 \odot K_1$ by an edge is given in figure 2.3

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C. David Raj/Department of Mathematics/ Malankara Catholic College,Mariagiri/ Kaliyakkavilai/Kanyakumari - 629 153/ Tamil Nadu.
C. Jayasekaran/Department of Mathematics/ Pioneer Kumaraswamy College/ Nagercoil/Kanyakumari/ Pin - 629 003/Tamil Nadu.