ON SOFT MINIMAL OPEN SETS AND SOFT MAXIMAL OPEN SETS IN SOFT TOPOLOGICAL SPACES

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Abstract: In this paper a new class of sets called soft minimal open sets and soft maximal open sets in soft topological spaces are introduced and studied. A proper nonempty soft open subset (F, E) of (X, τ, E) is said to be a soft minimal open (resp. soft maximal open) set if and only if any soft open set which is contained (resp. contains) in (F, E) is eitherϕ (resp. X) or (F, E) itself. Also some of their properties have been investigated.

Keywords: Soft minimal open sets and soft maximal open sets.

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1. **Introduction And Preliminaries:** Topology is the branch of Mathematics the purpose of which is to elucidate and investigate ideas of continuity, within the frame work of Mathematics. The study of topological spaces, their continuous mappings and general properties makes up one branch of topology known as general topology.

Fuzzy set theory, which was firstly proposed by researcher L. A. Zadeh, in 1965, has become a very important tool to solve these kinds of problems and provides an appropriate framework for representing vague concepts by allowing partial membership. Fuzzy set theory has been studied by both mathematicians and computer scientists and many applications of fuzzy set theory have arisen over the years, such as fuzzy control systems, fuzzy automata, fuzzy logic, fuzzy topology etc. Beside this theory, there are also theory of probability, rough set theory which deal with finding solutions for these problems. Each of these theories has its inherent difficulties as pointed out in 1999 by Russian researcher Molodtsov, who introduced the concept of soft set theory which is a completely new approach for modeling vagueness and uncertainty. Applications of soft set theory in other disciplines and real-life problems are now catching momentum.

Molodtsov established the fundamental results of this new theory and successfully applied the soft set theory into several directions, such as smoothness of functions, operations research, Riemann integration, game theory, theory of probability and so on. Now, soft set theory and its applications are progressing rapidly in different fields.

In the years 2001 and 2003, F. Nakaoka and N. Oda, introduced and studied minimal open (resp. minimal closed) sets and maximal open (resp. maximal closed) sets, which are subclasses of open (resp. closed) sets. The complements of minimal open sets and maximal open sets are called maximal closed sets and minimal closed sets respectively. In the year 1999, Russian researcher Molodtsov [14], initiated the concept of soft sets as a new mathematical tool to deal with uncertainties while modeling problems in engineering physics, computer science, economics, social sciences and medical sciences. A soft set is a collection of approximate descriptions of an object. In 2002 and 2003, Maji, Biswas and Roy [12], gave some new definitions on soft sets and presented first practical application of soft sets in decision-making problems that is based on the reduction of parameters to keep the optimal choice objects. In 2003, Maji, Biswas and Roy [13], studied the theory of soft sets initiated by Molodtsov. They defined equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set and absolute soft set with examples. Soft binary operations like AND, OR and also the operations of union and intersection were also defined. In 2005, D. Chen [6], presented a new definition of soft set parametrization reduction and a comparison of it with attribute reduction in rough set theory. In 2005, D. Pie, D. Miao [20], discussed the relationship between soft sets and information systems. They showed that soft sets are a class of special information systems. In 2008, Z. Kong, L. Gao, L. Wong, S. Li [10], introduced the notion of normal parameter reduction of soft sets and its use to investigate the problem of sub-optimal choice and added a parameter set in soft sets.

In recent years, researchers have contributed a lot towards fuzzification of Soft Set Theory. In 2001, Maji P. K., Biswas R. and Roy A.R. [11], introduced the concept of Fuzzy Soft Set and some properties regarding fuzzy soft union, intersection, complement of a fuzzy soft set, De Morgan Law etc. In 2007, X. Yang, D. Yu, J. Yang, C. Wu [21], combined the interval-valued fuzzy set and soft set models and introduced the concept of interval-valued fuzzy soft set.

Topological structures of soft set and fuzzy soft set have been studied by some authors in recent years. In 2011, Muhammad Shabir and Munazza Naz and Naim

Cagman et al. initiated the study of soft topology and soft topological spaces independently. Muhammad Shabir and Munazza Naz [16], introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters and showed that a soft topological space gives a parameterized family of topological spaces. They introduced the definitions of soft open sets, soft closed sets, soft interior, soft closure and soft separation axioms. Also they obtained some interesting results for soft separation axioms, which are really valuable for research in this field. N. Cagman, S. Karatas and S. Enginoglu [5], defined the soft topology on a soft set, and presented its related properties and foundations of the theory of soft topological spaces. The notion of soft topology by Naim Cagman et al. is more general than that by Shabir and Naz.

At the same time, Abdulkadir Aygunoglu and Halis Aygun [4], introduced soft topological spaces and soft continuity of soft mappings. They also investigated initial soft topologies and soft compactness. In 2011, Sabir Hussain and Bashir Ahmad [2], investigated the properties of soft open (closed), soft neighborhood and soft closure. Also defined and discussed the properties of soft interior, soft exterior and soft boundary which are fundamental for further research on soft topology and foundations of the theory of soft topological spaces. In 2012, Bashir Ahmad and Sabir Hussain [1], defined soft exterior and studied its basic properties and establish several important results relating soft interior, soft exterior, soft closure, and soft boundary in soft topological spaces. Moreover, they characterized soft open sets, soft closed sets and soft clopen sets via soft boundary. In 2007, H.Hazra, P. Majumdar andS.K.Samanta [9], introduced the notions of topology on soft subsets and soft topology. Some basic properties of these topologies are studied.

We recall the following definitions, which are prerequisites for present study.

1.1 Definition [14]: Let U be an initial universe and E be a set of parameters. Let P(U) denote the power set of U and A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A $\rightarrow$P(U).

In other words, a soft set over U is a parametrized family of subsets of the universe U. For $\varepsilon \in A$, F(ε) may be considered as the set of ε-approximate elements of the soft set (F, A). Clearly, a soft set is not a set.

1.2 Example [12]: Let us consider a soft set (F, E) which describes the "attractiveness of houses" that Mr. X is considering to purchase. Suppose that there are six houses in the universe U = $\{h_1, h_2, h_3, h_4, h_5, h_6\}$ under consideration, and that E = $\{e_1, e_2, e_3, e_4, e_5\}$ is a set of decision parameters. The e_i(i = 1, 2, 3, 4, 5) denotes the parameters "expensive", "beautiful", "wooden", "cheap" and "in green surroundings" respectively. Consider the mapping F given by "houses (.)", where (.) is to be filled in by one of the parameters $e_i \in E$. For instance, $F(e_1)$ means "houses (expensive)", and its functional value is the set {h $\in$U : h is an expensive house}. Suppose that $F(e_1) = \{h_2, h_4\}$, $F(e_2) = \{h_1, h_3\}$, $F(e_3) = \phi$, $F(e_4) = \{h_1, h_3, h_5\}$ and $F(e_5) = \{h_1\}$. Then we can view the soft set (F, E) as consisting of the following collection of approximations:

(F, E) = {(expensive houses, $\{h_2, h_4\}$), (beautiful houses, $\{h_1, h_3\}$), (wooden houses,ϕ), (cheap houses, $\{h_1, h_3, h_5\}$), (in the green surroundings, $\{h_1\}$)}.

1.3. Definition [13]: For two soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a soft subset of (G, B) if

i) A $\subseteq$B and

ii) for all e $\in$A, F (e) and G(e) are identical approximations.

We write (F, A)$\tilde{\subset}$ (G, B). (F, A) is said to be a soft super set of (G, B), if (G, B) is a soft subset of (F, A). We denote it by (F, A) $\tilde{\supset}$ (G, B).

1.4. Definition [13]: Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

1.5. Definition [13]: Let E = $\{e_1, e_2, e_3 . . . , e_n\}$ be a set of parameters. The NOT set of $\neg$E denoted by $\neg$E is defined by $\neg E=\{e_1, e_2, e_3, \ldots e_n\}$, where $\neg e_i$ = not e_i for all i.

1.6. Definition [13]: The complement of a soft set (F, A) is denoted by $(F, A)^c=(F^c, \neg A)$ where, F^c: $\neg A \rightarrow P(U)$ is a mapping given by $F^c(\alpha) = U \setminus F(\neg\alpha)$, for all $\alpha \in \neg A$

Let us call F^c to be the soft complement function of F. Clearly $(F^c)^c$ is the same as F and $((F, A)^c)^c$= (F , A).

1.7. Definition [13]:A soft set (F, A) over U is said to be a NULL soft set denoted by "ϕ" if $\forall \varepsilon \in A$, F ($\varepsilon$) = ϕ, (null-set).

1.8. Definition [13]: If (F , A) and (G, B) are two soft sets then (F, A) AND (G, B) denoted by (F, A)Λ(G, B) is defined by (F, A) Λ(G, B) = (H, A × B), where H((α, β)) = F(α) $\cap$G(β), for all (α, β) $\cap$A × B.

1.9. Definition [13]: If (F, A) and (G, B) are two soft sets then (F, A) OR (G, B) denoted by (F, A) V (G, B) is defined by (F, A) V (G, B) = (O, A × B) where, O((α, β)) = F(α) $\cap$G(β) for all (α, β)$\cap$A×B.

1.10. Definition [13]: The union of two soft sets of (F, A) and (G, B) over the common universe U is the soft set (H, C), where C = A$\cup$B and for all e ε C,

$$H(e)=\begin{cases} F(e) & \text{if } e \in A - B \\ G(e) & \text{if } e \in B - A \\ F(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$

We write (F, A) $\cup$(G, B) = (H, C).

1.11. Definition [6]: The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe U, denoted (F, A) ∩(G, B), is defined as C=A∩B, and H(e) = F (e) ∩G(e) for all e ∈ C

1.12. Definition ([16]). Let τ be the collection of soft sets over X; then τ is called a soft topology on X if τ satisfies the following axioms:

i) Φ, $\widetilde{X}$ belong to τ.

ii) The union of any number of soft sets in τ belongs to τ.

iii) The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X. The members of τ are said to be soft open in X. A soft set (F, E) over X is said to be soft closed in X if its relative complement $(F, E)^c$ belongs to τ .

1.13. Definition [16]. Let (F, E) be a soft set over X and x ∈X. We say that x ∈(F, E) read as x belongs to the soft set (F, E), whenever x ∈F(α) for all α ∈E.

Note that for x ∈X, x ∉(F, E) if x ∉F(α) for some α ∈E.

1.14. Definition [16]: Let x ∈X; then (x, E) denotes the soft set over X for which x(α) = {x},∀α∈E.

1.15 Definition [2]: Let (X, τ, E)be a soft topological space over X, (G, E) be a soft set over X and x ∈X. Then (G, E) is said to be a soft neighborhood of x if there exists a soft open set (F, E) such that x ∈(F, E) $\widetilde{\subseteq}$ (G, E).

1.16 Definition [17]: Let (X, τ, E) be a soft topological space and (A, E) be a soft set over X.

i) The soft interior of (A, E) is the soft set sint(A, E) = ∪{(O, E): (O, E)is soft open and (O, E) $\widetilde{\subseteq}$ (A, E)}.

ii) The soft closure of (A, E)is the soft set scl(A, E) = ∩{(F, E) : (F, E)is soft closed and (A, E) $\widetilde{\subseteq}$ (F, E)}.

1.17 Definition [17]: A proper nonempty open subset U of a topological space X is said to be a minimal open set if any open set which is contained in U is ϕ or U.

1.18Definition [18]: A proper nonempty open subset U of a topological space X is said to be maximal open set if any open set which contains U is X or U.

1.19 Definition[19]: A proper nonempty closed subset F of a topological space X is said to be a minimal closed set if any closed set which is contained in F is ϕ or F.

1.20 Definition [19]: A proper nonempty closed subset F of a topological space X is said to be maximal closed set if any closed set which contains F is X or F.

Soft Minimal Open Sets

Let (X, τ, E) be a soft topological space over X.

Definition 2.1: A proper nonempty soft open subset (F, E) of (X, τ, E) is said to be a soft minimal open set if and only if any soft open set which is contained in (F, E) is eitherϕ or (F, E) itself.

Example 2.2: Let (X, τ, E) be a soft topological space and X be the universe set, E be the set of parameters.

$X = \{a, b, c\}, E = \{e_1, e_2\}, \tau$
$= \{X, \phi, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E)\}$

where$F_1(e_1) = \{b\}, F_1(e_2) = \{a\}$,
$F_2(e_1) = \{b, c\}, F_2(e_2) = \{a, b\}$,
$F_3(e_1) = \{a, b\}, F_3(e_2) = X$,
$F_4(e_1) = \{a, b\}, F_4(e_2) = \{a, c\}$,
$F_5(e_1) = \{b\}, F_5(e_2) = \{a, b\}$.

Therefore $(F_1, E), (F_2, E), (F_3, E), (F_4, E), \& \ (F_5, E),$ are soft open sets in soft topological spaces X. Therefore (F_1, E) is a soft minimal open set in soft topological space X.

Theorem 2.3:

i) Let (F, E) be a soft minimal open set and (G, E) a soft open set. Then $(F, E) \cap (G, E) = \phi or (F, E) \subset (G, E)$.

ii) Let (F, E) and (H, E) be soft minimal open sets. Then $(F, E) \cap (H, E) = \phi or \ (F, E) = (H, E)$.

Proof: i) Let (G, E) be the soft open set such that $(F, E) \cap (G, E) \neq \phi$. Since (F, E) is a soft minimal open set and$(F, E) \cap (G, E) \subset (F, E)$, we have $(F, E) \cap (G, E) = (F, E)$ .Therefore $(F, E) \subset (G, E)$.

ii) If$(F, E) \cap (H, E) = \phi$, then we see that $(F, E) \subset (H, E) \ and \ (H, E) \subset (F, E)$ by (i). Therefore (F, E)= (H, E).

Proposition 2.4: Let (F, E) be a soft minimal open set. If x is an element of(F, E), then $(F, E) \subset (G, E)$ for any soft open neighbourhood (G, E) of x.

Proof: Let (G, E) be a soft open neighborhood of x such that $(F, E) \not\subset (G, E)$, then $(F, E) \cap (G, E)$ is a soft open set such that $(F, E) \cap (G, E) \subsetneq (F, E)$ and $(F, E) \cap (G, E) \neq \phi$. This contradicts our assumption that(F, E) is a soft minimal open set.

Proposition 2.5: Let (F, E) be a soft minimal open set. Then

$(F, E) = \bigcap\{(G, E) : (G, E) \text{ is a soft open neighbourhood of } x\}$, for any element x of (F, E).

Proof: By Proposition 2.4 and the fact that (F, E) is a soft openneighbourhoodof$x$, we have

$(F, E) \subset \bigcap\{(G, E) : (G, E) \text{ is a soft open neighbourhood of } x\} \subset (F, E)$.

Therefore we have the result.

Theorem 2.6: Let (F, E) be a nonempty soft open set. Then the following three conditions are equivalent:

i) (F, E) is a soft minimal open set.

ii)$(F, E) \subset scl((G, E))$ for any nonempty soft subset (G, E) of (F, E).

iii) $scl((F,E)) = scl((G,E))$ for any nonempty soft subset (G,E) of (F,E).

Proof: (i)$\Longrightarrow (ii)$ Let (G,E) be any nonempty soft subset of (F,E). By Proposition 2.4, for any element x of (F,E) and any soft open neighbourhood (H,E) of x, we have

$(G,E) = (F,E) \cap (G,E) \subset (H,E) \cap (F,E)$. Then, we have $(H,E) \cap (G,E) \neq \phi$ and hence x is an element of scl $((G,E))$. It follows that $(F,E) \subset scl((G,E))$.

$(ii) \Longrightarrow (iii)$. For any nonempty soft subset (G,E) of (F,E), we have $scl((G,E)) = scl((F,E))$. On the other hand, by (ii),we see

$scl((F,E)) \subset scl\big(scl((G,E))\big) = scl((G,E))$.

Therefore we have $scl((F,E)) = scl((G,E))$ for any nonempty soft subset (G,E) of (F,E).

$(iii) \Longrightarrow (i)$. Suppose that (F,E) is not a soft minimal open set. Then there exists a nonempty soft open set (I,E) such that $(I,E) \subsetneq (F,E)$and hence there exists an element $x \in (F,E)$such that $x \notin scl(I,E)$.Then we have scl{x}$\subset$ scl$(I, E)^C$. It follows that scl{x}$\neq$ scl(F, E).

Example 2.7: From Example 2.2, $(F,E) = (F_1(e_1) = \{b\}, F_1(e_2) = \{a\})$ the soft minimal open set. Therefore

$scl\{(F_2,E)\} = X, scl\{(F_3,E)\} = X, scl\{(F_4,E)\} = X, scl\{(F_5,E)\} = X$**2.**

Soft Maximal Open Sets

Let (X,τ,E) be a soft topological space.

Definition 3.1: A proper nonempty soft open subset (F,E) of X is said to be a soft maximal open set if any soft open set which contains (F,E) is$X$ or (F,E) itself.

Example 3.2: Let (X,τ,E) be a soft topological spaces and X be the universe set, E be the set of parameters.

$X = \{a,b,c\}, E = \{e_1,e_2\}, \tau = \{X,\phi,(F_1,E),(F_2,E),(F_3,E),(F_4,E),(F_5,E)\} where F_1(e_1) = \{b\}, F_1(e_2) = \{a\}$; $F_2(e_1) = \{b,c\}, F_2(e_2) = \{a,b\}$ $F_3(e_1) = \{a,b\}, F_3(e_2) = X$; $F_4(e_1) = \{a,b\}, F_4(e_2) = \{a,c\} F_5(e_1) = \{b\}, F_5(e_2) = \{a,b\}$

Therefore

$(F_1, E), (F_2, E), (F_3, E), (F_4, E)$ & (F_5, E)

are soft open sets in soft topological spaces X. Therefore $(F_2, E), (F_3, E), and\ (F_4, E)$ are soft maximal open sets in soft topological space X.

Theorem 3.3:

i) Let (F,E) be a soft maximal open set and (H,E) be a soft open set. Then, $(F,E) \cup (H,E) = X$ or $(H,E) \subset (F,E)$.

ii) Let (F,E) and (G,E) be soft maximal open sets. Then,$(F,E) \cup (G,E) = X$ or $(F,E) = (G,E)$.

Proof: i) Let (H,E) be a soft open set such that $(F,E) \cup (H,E) \neq X$. Since (F,E) is a soft maximal open set and $(F,E) \subset (F,E) \cup (H,E)$, we have $(F,E) \cup (H,E) = (F,E)$

Therefore,$(H,E) \subset (F,E)$.

ii) If $(F,E) \cup (G,E) \neq X$ then $(F,E) \subset (G,E)$ and $(G,E) \subset (F,E)$ by (i). Therefore $(F,E) = (G,E)$.

Proposition 3.4: Let (F,E) be a soft maximal open set. If x is an element of (F,E), then for any soft open neighbourhood (H,E) of x,

$(H,E) \cup (F,E) = X$ or $(H,E) \subset (F,E)$.

Proof: By Theorem 3.3(i), we have the result.

Theorem 3.5: Let $(F_\propto,E),(F_\beta,E)$&$(F_\gamma,E)$ be soft maximal open sets such that

$(F_\propto,E) \neq (F_\beta,E)$. If $(F_\propto,E) \cap (F_\beta,E) \subset (F_\gamma,E)$, then $(F_\propto,E) = (F_\beta,E)$ or $(F_\beta,E) = (F_\gamma,E)$.

Proof: We have

$(F_\propto,E) \cap (F_\gamma,E) = (F_\propto,E) \cap \{(F_\gamma,E) \cap X\}$ Since $(F_\gamma,E) \cap X = (F_\gamma,E)$, therefore

$(F_\gamma,E) \subset X =$

$(F_\propto,E) \cap \{(F_\gamma,E) \cap [(F_\propto,E) U (F_\beta,E)]\}$ by theorem 3.3(ii)

$$= (F_\propto,E) \cap [\{(F_\gamma,E) \cap (F_\propto,E)\} \cup \{(F_\gamma,E) \cap (F_\beta,E)\}]$$

$$= (F_\propto,E) \cap \{(F_\gamma,E) \cap (F_\propto,E)\} \cup (F_\propto,E) \cap \{(F_\gamma,E) \cap (F_\beta,E)\}$$

$= [(F_\propto,E) \cap (F_\gamma,E)] \cup [(F_\gamma,E) \cap (F_\propto,E) \cap (F_\beta,E)]$=$(F_\propto,E) \cap (F_\gamma,E) \cup [(F_\propto,E) \cap (F_\beta,E)]$,

Since$(F_\propto,E) \cap (F_\beta,E) \subset (F_\gamma,E)$

=$(F_\propto,E) \cap [(F_\gamma,E) \cup (F_\beta,E)]$

Hence we have

$(F_\propto,E) \cap (F_\gamma,E) = (F_\propto,E) \cap [(F_\gamma,E) \cup (F_\beta,E)]$

If $(F_\gamma,E) \neq (F_\beta,E)$,then $(F_\gamma,E) \cup (F_\beta,E) = X$, and hence $(F_\propto,E) \cap (F_\gamma,E) = (F_\propto,E)$ which implies $(F_\propto,E) \subset (F_\gamma,E)$. Since $(F_\propto,E)$ and (F_γ,E) are soft maximal open sets, we have $(F_\propto,E)$=(F_γ,E).

Theorem 3.6: Let $(F_\propto,E),(F_\beta,E)$&$(F_\gamma,E)$ be soft maximal open sets which are different from each other, then

$(F_\propto,E) \cap (F_\beta,E) \not\subset (F_\propto,E) \cap (F_\gamma,E)$.

Proof: If $(F_\propto,E) \cap (F_\beta,E) \subset (F_\propto,E) \cap (F_\gamma,E)$. We have

$$[(F_\propto,E) \cap (F_\beta,E)] \cup [(F_\beta,E) \cap (F_\gamma,E)] \subset [(F_\propto,E) \cap (F_\gamma,E)] \cup [(F_\beta,E) \cap (F_\gamma,E)]$$

Hence

$$(F_\beta,E) \cap [(F_\propto,E) \cup (F_\gamma,E)] \subset [(F_\propto,E) \cup (F_\beta,E)] \cap (F_\gamma,E)$$

Since

$(F_{\propto}, E) \cup (F_{\gamma}, E) = X = (F_{\propto}, E) \cup (F_{\beta}, E)$, we have $(F_{\beta}, E) \subset (F_{\gamma}, E)$. It follows that $(F_{\beta}, E) = (F_{\gamma}, E)$ which is contradicts our assumption. Therefore $(F_{\beta}, E), (F_{\gamma}, E)$ are different soft maximal open sets.

Theorem 3.7: Let (F, E) be a soft maximal open set and x an element of (F, E). Then

$$(F, E) = \cup \left\{ \begin{matrix} (H, E): (H, E) \text{is a soft open neighborhood} \\ \text{of } x \text{ such that } (H, E) \cup (F, E) \neq X \end{matrix} \right\}$$

Proof: By Proposition 3.4 and the fact that (F, E) is a soft open neighborhood of x, we have $(F, E) \subset \bigcup\{(H, E): (H, E)$ is a soft open neighbourhood of x$\}$, such that $\{(F, E) \cup (H, E) \neq X\} \subset (F, E)$. Therefore we have the result.

4. Soft Closure, Soft Interior And Soft Maximal Open Sets:

Theorem 4.1: Let (F, E) be a soft maximal open set and x an element of $X - (F, E)$. Then $X - (F, E) \subset (G, E)$ for any soft open neighbourhood (G, E) of x.

Proof: Since $x \epsilon X - (F, E)$, we have $(G, E) \not\subset (F, E)$ for any soft open neighbourhood (G, E) of x. Then $(G, E) \cup (F, E) = X$, by Theorem 3.3(i). Therefore, $X - (F, E) \subset (G, E)$.

Theorem 4.2: Let (F, E) be a soft maximal open set, then either of the following (i) and (ii) holds.

(i) for each $x \epsilon X - (F, E)$ and each soft neighbourhood (G, E) of x, $(G, E) = X$.

(ii) there exists a soft open set (G, E) such that $X - (F, E) \subset (G, E)$ and $(G, E) \subsetneq X$.

Proof: If (i) does not hold, then there exists an element x of $X - (F, E)$ and a soft open neighbourhood (G, E) of x such that $(G, E) \subsetneq X$. By Theorem 4.1, we have

$$X - (F, E) \subset (G, E).$$

Theorem 4.3: Let (F, E) be a soft maximal open set. Then, either of the following (i) and (ii) holds.

$i)$ for each $x \epsilon X - (F, E)$ and each soft open neighbourhood of x, we have $X - (F, E) \subsetneq (G, E)$

$ii)$ there exists a soft open set (G, E) such that $X - (F, E) = (G, E) \neq X$.

Proof: Assume that (ii) does not hold. Then by Theorem 4.1, we have $X - (F, E) \subset (G, E)$, for each $x \epsilon X - (F, E)$ and each soft open neighbourhood (G, E) of x. Hence we have $X - (F, E) \subsetneq (G, E)$.

Theorem 4.4: Let (F, E) be a soft maximal open set. Then $scl(F, E) = X$ or $scl(F, E) = (F, E)$.

Proof: Since (F, E) is a soft maximal open set, only the following cases (i) and (ii) occur by Theorem 4.3.

(i) For each $x \epsilon X - (F, E)$ and each soft open neighbourhood (G, E) of x, we have $X - (F, E) \subsetneq (G, E)$. Let x be any element of $X - (F, E)$ and (G, E) any soft open neighbourhood of x. Since $X - (F, E) \neq (G, E)$. we have $(G, E) \cap (F, E) \neq \phi$ for any soft open neighbourhood (G, E) of x. Hence, $X - (F, E) \subset scl(F, E)$. Since $X = (F, E) \cup \{X - (F, E)\} \subset (F, E) \cup scl(F, E) = scl(F, E) \subset X$. We have $scl(F, E) = X$.

(ii) there exists a soft open set (G, E) such that $X - (F, E) = (G, E) \neq X$. Since $X - (F, E) = (G, E)$ is soft open set, (F, E) is soft closed set. Therefore $(F, E) = scl(F, E)$.

Theorem 4.5: Let (F, E) be a soft maximal open set. Then $sint\{X - (F, E)\} = X - (F, E)$ or $sint\{X - (F, E)\} = \phi$.

Proof: By Theorem 4.3, we have either, $sint\{X - (F, E)\} = \phi$ or

$$sint\{X - (F, E)\} = X - (F, E)$$

Theorem 4.6: Let (F, E) be a soft maximal open set and (H, E) a nonempty soft subset of $X - (F, E)$. Then $scl(H, E) = X - (F, E)$.

Proof: Since $\phi \neq (H, E) \subset X - (F, E)$, we have $(G, E) \cap (H, E) \neq \phi$ for any element x of $X - (F, E)$ and any soft open neighbourhood (G, E) of x by Theorem 4.1. Then, $X - (F, E) \subset scl(H, E)$. Since $X - (F, E)$ is a soft closed set and $(H, E) \subset X - (F, E)$, we see that $scl(H, E) \subset scl\{X - (F, E)\} = X - (F, E)$. Therefore $X - (F, E) = scl(H, E)$.

Theorem 4.7: Let (F, E) be a soft maximal open set and (I, E) a soft subset of X with $(F, E) \subsetneq (I, E)$. Then $scl(I, E) = X$.

Proof: Since $(F, E) \subseteq (I, E) \subset X$, there exists a nonempty soft subset (H, E) of $X - (F, E)$ such that $(I, E) = (F, E) \bigcup (H, E)$. Hence we have $scl(I, E) = scl\{(H, E) \bigcup (F, E)\} = scl(H, E) \cup scl(F, E) \supset (X - (F, E)) \cup (F, E) = X$. Therefore $scl(I, E) = X$.

Theorem 4.8: Let (F, E) be a soft maximal open set assume that the subset $X - (F, E)$ has two elements at least. Then $scl\{X - \{a\}\} = X$ for any element a of $X - (F, E)$.

Proof: Since $(F, E) \subsetneq X - \{a\}$ by our assumption, we have the result by Theorem 4.7.

Theorem 4.9: Let (F, E) be a soft maximal open set and (K, E) a proper soft subset of X with $(F, E) \subset (K, E)$. Then $sint(K, E) = (F, E)$.

Proof: If $(K, E) = (F, E)$, then $sint(K, E) = (F, E)$. Otherwise $(K, E) \neq (F, E)$, and we have $(F, E) \subsetneq (K, E)$. It follows that

$(F,E) \subset sint(K,E)$. Since (F,E) is a soft maximal open set, we have also
$sint(K,E) \subset (K,E)$.Therefore
$sint(K,E) = (F,E)$.

Theorem 4.10: Let (F,E) be a soft maximal open set and (H,E) a nonempty soft subset of $X - (F,E)$.Then
$X - scl(H,E) = sint\{X - (H,E)\} = (F,E)$.

Proof: Since $(F,E) \subset X - (H,E) \subsetneq X$ by our assumption, we have the result by Theorem 4.6 and 4.9

5. Soft Minimal Closed Sets And Soft Maximal Closed Sets:

Let (X,τ,E) be a soft topological space.

Definition 5.1: A proper nonempty soft closed subset (A,E) of X is said to be a soft minimal closed set if any soft closed set which is contained in (A,E) is ϕ or (A,E) itself.

Definition 5.2: A proper nonempty soft closed subset (A,E) of X is said to be a soft maximal closed set if any soft closed set which contains (A,E) is X or (A,E) itself.

Example 5.3: Let (X,τ,E) be a soft topological spaces and X be the universe set, E be the set of parameters.

$X = \{a,b,c\}, E = \{e_1,e_2\}, \tau = \{X,\phi,(F_1,E),(F_2,E),(F_3,E),(F_4,E),(F_5,E)\}$

$where F_1(e_1) = \{b\}, F_1(e_2) = \{a\}$

$\overline{F}_1(e_1) = \{a, c\}, \overline{F}_1(e_2) = \{b\ c\}$

$F_2(e_1) = \{b,c\}, F_2(e_2) = \{a,b\}$

$\overline{F}_2(e_1) = \{a\}, \overline{F}_2(e_2) = \{c\}$

$F_3(e_1) = \{a,b\}, F_3(e_2) = X$

$\overline{F}_3(e_1) = \{c\}, \overline{F}_3(e_2) = \phi$

$F_4(e_1) = \{a,b\}, F_4(e_2) = \{a,c\}$

$\overline{F}_4(e_1) = \{c\}, \overline{F}_4(e_2) = \{b\}$

$F_5(e_1) = \{b\}, F_5(e_2) = \{a,b\}$

$\overline{F}_5(e_1) = \{a, c\}, \overline{F}_5(e_2) = \{c\}$ Therefore

$(F_1, E), (F_2, E), (F_3, E), (F_4, E), \& \ (F_5, E)$, are soft open sets and

$(\overline{F}_1, E), (\overline{F}_2, E), (\overline{F}_3, E), (\overline{F}_4, E), and\ (\overline{F}_5, E)$ are soft closed sets in soft topological spaces X.

Therefore $(\overline{F}_2, E), (\overline{F}_3, E), and\ (\overline{F}_4, E)$ are soft minimal closed set and $(\overline{F}_1, E)$ is soft maximal closed set in soft topological space X.

Theorem 5.4: Let (A,E) be a soft subset of a soft topological space (X,τ,E).Then the following principle holds

i)(A,E) is a soft minimal closed set if and only if $(A,E)^c$ is a soft maximal open set.

$ii)(A,E)$ is a soft maximal closed set if and only if $(A,E)^c$ is a soft minimal open set.

Proof: Proof is obvious from definitions.

Theorem 5.5:

i) Let (A,E) be a soft minimal closed set and (C,E) a soft closed set. Then,
$(A,E) \cap (C,E) = \phi$ or $(A,E) \subset (C,E)$.

ii) Let (A,E) and (B,E) be soft minimal closed sets. Then, $(A,E) \cap (B,E) = \phi$ or
$(A,E) = (B,E)$.

Proof: i) Let (C,E) be a soft closed set such that $(A,E) \cap (C,E) \neq \phi$. Since (A,E) is a soft minimal closed set and $(A,E) \subset (A,E) \cap (C,E)$,we have $(A,E) \cap (C,E) \subset (F,E)$.
Therefore,$(A,E) \subset (C,E)$.

ii) If $(A,E) \cap (B,E) \neq \phi$ then $(A,E) \subset (B,E)$ and $(B,E) \subset (A,E)$ by (i). Therefore $(A,E) = (B,E)$.

Theorem 5.6: Let (A,E) and (A_λ, E) be soft minimal closed sets for any element $\lambda \in \Lambda$

i) If $(A,E) \subset (\cup_{\lambda \epsilon \Lambda} (A_\lambda, E))$, then there exists an element $\lambda\ of\ \Lambda$ such that $(A,E) = (A_\lambda, E)$

ii) If $(A,E) \neq (A_\lambda, E)$ for any element $\lambda\ of\ \Lambda$ then $\{(\cup_{\lambda \epsilon \Lambda} (A_\lambda, E))\} \cap (A,E) = \phi$.

Proof: i) Since$(A,E) \subset \{(\cup_{\lambda \epsilon \Lambda} (A_\lambda, E))\}$, we get $(A,E) = (A,E) \cap \cup_{\lambda \epsilon \Lambda} (A_\lambda, E) = \cup_{\lambda \epsilon \Lambda} \{(A,E) \cap (A_\lambda, E)\}$.
If $(A,E) \neq (A_\lambda, E)$for any element $\lambda\ of \Lambda$.
Then $(A,E) \cap (A_\lambda, E) = \phi$ for any element $\lambda\ of \Lambda$.
By Theorem 5.5(ii), hence we have
$\phi = \cup_{\lambda \epsilon \Lambda} \{(A,E) \cap (A_\lambda, E)\} = (A,E)$.
This is contradicts our assumption that (A,E) is a soft minimal closed set. Thus there exists an element $\lambda\ of \Lambda$ such that $(A,E) = (A_\lambda, E)$.

ii) If $\{(\cup_{\lambda \epsilon \Lambda} (A_\lambda, E))\} \cap (A,E) \neq \phi$ then there exists an element $\lambda\ of \Lambda$ such that $(A_\lambda, E) \cap (A,E) \neq \phi$. By Theorem 4.5(ii),we have $(A_\lambda, E) = (A,E)$ which is a contradiction. Therefore $(A_\lambda, E) \neq (A,E)$.

Theorem 5.7: Let (A_λ, E) and (A_γ, E) be soft minimal closed sets for any element $\lambda\ of \Lambda$ and $\gamma\ of\ \Gamma$. If there exists an element $\gamma\ of\ \Gamma$ such that $(A_\lambda, E) \neq (A_\gamma, E)$ for any element $\lambda\ of \Lambda$, then $\cup_{\gamma \epsilon \Gamma} (A_\gamma, E) \not\subset \cup_{\lambda \epsilon \Lambda} (A_\lambda, E)$.

Proof: Suppose that an element $\gamma'\ of\ \Gamma$ satisfies $(A_\lambda, E) \neq (A_\gamma, E)$ for any element $\lambda\ of \Lambda$. If $\cup_{\gamma \epsilon \Gamma} (A_\gamma, E) \subset \cup_{\lambda \epsilon \Lambda} (A_\lambda, E)$ then we get $(A_{\gamma'}, E) \subset \cup_{\lambda \epsilon \Lambda} (A_\lambda, E)$. Therefore 4.6(i), there exists an element $\lambda\ of \Lambda$ such that $(A_{\gamma'}, E) = (A_\lambda, E)$ which is contradiction. Therefore $(A_\lambda, E) \neq (A_\gamma, E)$.

Theorem 5.8: Let (A,E) be a soft minimal closed set, Then $sint(A,E) = (A,E)$ or $sint(A,E) = \phi$.

Proof: By Theorem 4.5, we put $(F,E) = X - (A,E)$, then we have the result.

Theorem 5.9: (i) Let (A,E) be a soft maximal closed set and (K,E) a soft closed set. Then $(A,E)\cup(K,E)=X$ or $(K,E)\subset(A,E)$.
(ii)Let (A,E) and (B,E) be soft maximal closed sets. Then $(A,E)\cup(B,E)=X$ or $(A,E)=(B,E)$.
Proof: The following Theorem 3.3, we have the result.

Theorem 5.10: Let (A,E) and (A_λ,E) be soft maximal closed sets for any element λ of Λ. If $(A,E)\neq(A_\lambda,E)$, for any element of Λ , then $\cap_{\lambda\in\Lambda}(A_\lambda,E)U(A,E)=X$.
Proof: By Theorem 3.3(ii)., we have the result.

Theorem 5.11: Let (A,E) and (A_λ,E) be soft maximal closed sets for any element λ of Λ. If $\cap_{\lambda\in\Lambda}(A_\lambda,E)\subset(A,E)$,then there exists an element λ of Λ such that $(A,E)=(A_\lambda,E)$.
Proof: Since $\cap_{\lambda\in\Lambda}(A_\lambda,E)\subset(A,E)$,we get $(A,E)=(A,E)\cup\{\cap_{\lambda\in\Lambda}(A_\lambda,E)\}=\cap_{\lambda\in\Lambda}\{(A,E)\cup(A_\lambda,E)\}$. If $(A,E)\cup(A_\lambda,E)=X$ for any element λ of Λ, then we have $X=\cap_{\lambda\in\Lambda}\{(A,E)\cup(A_\lambda,E)\}=(A,E)$. This contradicts our assumption that (A,E) is a soft maximal closed set. Then there exists an element λ of Λ such that $(A,E)\cup(A_\lambda,E)\neq X$. By Theorem 3.3(ii),we have the result.

Theorem 5.12: Let (A_λ,E) and (A_γ,E) be soft maximal closed sets for any elements $\lambda\in\Lambda$ and $\gamma\in\Gamma$. If $\cap_{\gamma\in\Gamma}(A_\gamma,E)\subset\cap_{\lambda\in\Lambda}(A_\lambda,E)$, then for any element λ of Λ, there exits an element γ of Γ such that $(A_\lambda,E)=(A_\gamma,E)$.
Proof: By Theorem 3.3(ii),we have the result.

Theorem 5.13: Let(A_α,E), (A_β,E) and (A_γ,E) be soft maximal closed sets which are different from each other, then $(A_\alpha,E)\cap(A_\beta,E)\not\subset(A_\alpha,E)\cap(A_\gamma,E)$. **Proof:** Following from Theorem 3.6,we have the result.

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