THE TRIPLE Γ^3 OF TENSOR PRODUCTS IN ORLICZ SEQUENCE SPACES

T.V.G. SHRI PRAKASH, M. CHANDRAMOULEESWARAN, N. SUBRAMANIAN

Abstract: Let X be a Banach lattice and Γ_M^3 be an triple entire Orlicz sequence space associated to an Orlicz function with the Δ_2-condition. In this paper discuss the some general topological properties of tensor products.

Keywords: analytic sequence, triple sequences, Γ^3 space, Musielak-Orlicz function, p-metric space, Banach metric lattice, positive tensor product.

2000 Mathematics subject classification: 46B42, 46B28.

Introduction: Throughout w, Γ and Λ denote the classes of all, entire and analytic scalar valued single sequences, respectively. We write w^3 for the set of all complex sequences (x_{mnk}), where m, n, k $\in \mathbb{N}$, the set of positive integers. Then, w^3 is a linear space under the coordinate wise addition and scalar multiplication.

We can represent triple sequences by matrix. In case of double sequences we write in the form of a square. In the case of a triple sequence it will be in the form of a box in three dimensional case.

Some initial work on double sequence spaces is found in N. Subramanian et al. (2008, 2009), Hardy (1917) and many others. Later on investigated by some initial work on triple sequence spaces is found in Esi et al. (2015, 2014), Sahiner et al. (2007) and many others. The initial work on modulus function or Orlicz functions and some other sequence spaces is found in Y. Altin et al. (2003, 2006, 2009).

Let (x_{mnk}) be a triple sequence of real or complex numbers. Then the series $\sum_{m,n,k=1}^{\infty} x_{mnk}$ is called a triple series. The triple series $\sum_{m,n,k=1}^{\infty} x_{mnk}$ is said to be convergent if and only if the triple sequence (S_{mnk}) is convergent, where

$S_{mnk} = \sum_{i,j,q=1}^{m,n,k} x_{ijq} \ (m,n,k = 1,2,3,\ldots)$.

A sequence x = (x_{mnk}) is said to be triple analytic if

$\sup_{m,n,k} |x_{mnk}|^{\frac{1}{m+n+k}} < \infty$.

The vector space of all triple analytic sequences are usually denoted by Λ^3. A sequence x = (x_{mnk}) is called triple entire sequence if

$|x_{mnk}|^{\frac{1}{m+n+k}} \to 0$ as m, n, k $\to\infty$.

The vector space of all triple entire sequences are usually denoted by Γ^3. The space Λ^3 and Γ^3 is a metric space with the metric

$$d(x,y) = \sup_{m,n,k}\left\{|x_{mnk} - y_{mnk}|^{\frac{1}{m+n+k}} : m,n,k : 1,2,3,\ldots\right\} \ldots (1)$$

for all x = $\{x_{mnk}\}$ and y = $\{y_{mnk}\}$ in Γ^3. Let ϕ = {finite sequences}.

Consider a double sequence x = (x_{mnk}). The $(m, n, k)^{th}$ section $x^{[m,n,k]}$ of the sequence is defined by

$x^{[m,n,k]} = \sum_{i,j,q=0}^{m,n,k} x_{ijq}\delta_{ijq}$ for all m, n, k $\in$ N,

$$\delta_{mnk} = \begin{bmatrix} 0 & 0 & \ldots 0 & 0 & \ldots \\ 0 & 0 & \ldots 0 & 0 & \ldots \\ \vdots & & & & \\ 0 & 0 & \ldots 1 & 0 & \ldots \\ 0 & 0 & \ldots 0 & 0 & \ldots \end{bmatrix}$$

with 1 in the $(m, n, k)^{th}$ position and zero other wise.

Consider a triple sequence x = (x_{mnk}). The $(m, n, k)^{th}$ section $x^{[m,n,k]}$ of the sequence is defined by

$x^{[m,n,k]} = \sum_{i,j,q=0}^{m,n,k} x_{ijq}\mathfrak{I}_{ijq}$ for all m, n, k $\in$ N ; where $\mathfrak{I}_{ijq}$ denotes the triple sequence whose only non zero term in the $(i, j, k)^{th}$ place for each i, j, k $\in$ N.

Let M and Φ be mutually complementary Orlicz functions. Then, we have

(i) For all u, y ≥ 0,
uy ≤ M(u) + Φ(y), (Young's inequality) [See [Kampthan et al., (1981)] … (2)

(ii) For all u ≥ 0,
$u\eta(u) = M(u) + \Phi(\eta(u))$ … (3)

(iii) For all u ≥ 0, and 0 <λ< 1,
$M(\lambda u) \le \lambda M(u)$ … (4)

A sequence M = (M_{mnk}) of Orlicz function is called a Musielak- Orlicz function. A sequence g = (g_{mnk}) defined by

$g_{mnk}(v) = \sup\{|v|\, u - M_{mnk}(u) : u \ge 0\}$, m, n, k = 1, 2, …

is called the complementary function of a Musielak-Orlicz function M. For a given Musielak-Orlicz function M, the Musielak-Orlicz sequence space t_M is defined by

$M_f = \left\{x \in w^3 : I_M(|x_{mnk}|)^{1/m+n+k} \to 0 \text{ as } m,n,k \to \infty\right\}$,

where I_M is a convex modular defined by

$I_M(x) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty} M_{mnk}(|x_{mnk}|)^{1/m+n+k}, x = (x_{mnk}) \in t_M$

We consider t_M equipped with the Luxemburg metric

$$d(x,y)=\sup_{mnk}\left\{\inf\left(\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}M_{mnk}\left(\frac{|x_{mnk}|^{1/m+n+k}}{mnk}\right)\right)\le 1\right\}.$$

The positivity perspective, it is known that the projective tensor and the injective tensor product of two Banach lattices are, in general not Banach lattices.

Notations: For a vector space X, a vector $\bar{x}=(x_{ijk})_{ijk}$ $\in X^{N\times N\times N}$ and $n\in\mathbb{N}$, we write $\bar{x}(\le n)$ is a three dimensional matrix from first term to n^{th} term and remaining term zero and $\bar{x}(>n)$ is a three dimensional matrix from first term to n^{th} term zero and start with $(n+1)^{th}$ term.

If X is an ordered set, the usual order on $X^{N\times N\times N}$ is defined by $\bar{x} = (x_{ijk})_{ijk}\ge 0 \Leftrightarrow (x_{ijk}) \ge 0$ for each i, j, k $\in\mathbb{N}$ for Banach lattice X, X^* denotes its dual space, B_X denotes its closed unit ball, and X^+ denotes its positive cone.

Definitions and Preliminaries: A sequence $x = (x_{mnk})$ is said to be triple analytic if $\sup_{mnk}|x_{mnk}|^{\frac{1}{m+n+k}}<\infty$. The vector space of all triple analytic sequences is usually denoted by Λ^3. A sequence $x = (x_{mnk})$ is called triple entire sequence if $|x_{mnk}|^{\frac{1}{m+n+k}}\to 0$ as m,n, k→∞. The vector space of triple entire sequence is usually denoted by Γ^3. The space Γ^3 is a metric space with the metric

$$d(x,y)=\sup_{m,n,k}\left\{\left(|x_{mnk}-y_{mnk}|\right)^{\frac{1}{m+n+k}}: m,n,k:1,2,3,...\right\} \quad ...(5)$$

for all $x = \{x_{mnk}\}$ and $y = \{y_{mnk}\}$ in Γ^3.

Let $n\in\mathbb{N}$ and X be a real vector space of dimension w, where $n \le m$. A real valued function $d_p(x_1, ..., x_n) = ||(d_1(x_1, 0), ..., d_n(x_n, 0))||_p$ on X satisfying the following four conditions:

(i) $||(d_1(x_1, 0), ..., d_n(x_n, 0))||_p = 0$ if and only if $d_1(x_1, 0), ..., d_n(x_n, 0)$ are linearly dependent,

(ii) $||(d_1(x_1, 0), ..., d_n(x_n, 0))||_p$ is invariant under permutation,

(iii) $||(\alpha d_1(x_1, 0), ..., d_n(x_n, 0))||_p = |\alpha|\ ||(d_1(x_1, 0), ..., d_n(x_n, 0))||_p$, $\alpha\in\mathbb{R}$.

(iv) $d_p((x_1, y_1), (x_2, y_2), ..., (x_n, y_n)) := \sup\{d_X(x_1, x_2, ..., x_n), d_Y(y_1, y_2, ..., y_n)\}$, for $x_1, x_2, ..., x_n\in X$, $y_1, y_2, ..., y_n\in Y$ is called the p product metric of the Cartesian product of n metric spaces is the p norm of the n-vector of the norms of the n subspaces.

A trivial example of p product metric of n metric space is the p norm space is $X = \mathbb{R}$ equipped with the following Euclidean metric in the product space is the p norm:

$||(d_1(x_1, 0), ..., d_n(x_n, 0))||_E = \sup(|\det(d_{mn}(x_{mn}, 0))|) =$

$$\sup\left[\begin{vmatrix} d_{11}(x_{11},0) & d_{12}(x_{12},0) & \dots & d_{1n}(x_{1n},0)\\ d_{21}(x_{21},0) & d_{22}(x_{22},0) & \dots & d_{2n}(x_{1n},0)\\ \vdots & & & \\ d_{n1}(x_{n1},0) & d_{n2},(x_{n2},0) & \dots & d_{nn}(x_{nn},0)\end{vmatrix}\right]$$

where $x_i = (x_{i1}, ..., x_{in}) \in R^n$ for each i = 1, 2, ..., n.

If every Cauchy sequence in X converges to some L ∈ X, then X is said to be complete with respect to the p-metric. Any complete p-metric space is said to be p-Banach metric space.

3.1. Definition

Positive tensor products: For Banach lattices X, Y and Z, let X ⊗ Y ⊗ Z denote the algebraic tensor product of X, Y and Z. For each $u=\sum_{m=1}^{r}\sum_{n=1}^{s}\sum_{k=1}^{t}x_{mnk}\otimes y_{mnk}\otimes z_{mnk}\in X\otimes Y\otimes Z$, define $T_u : X^*\to Y, Z$ by $T_u(x^*) = \sum_{m=1}^{r}\sum_{n=1}^{s}\sum_{k=1}^{t}x^*(x_{mnk})y_{mnk}z_{mnk}$ for each $x^*\in X^+$.

Then injective cone on X ⊗ Y ⊗ Z is defined to be $C_i = \{u \in X\otimes Y\otimes Z: T_u(x^*) \in Y^*\forall x^* \in X^{*+}\}$.

3.2. Definition: Let X $\bar{\otimes}_i$ Y $\bar{\otimes}_i$ Z denote the completion of X ⊗ Y ⊗ Z with respect d(., .). Then X $\bar{\otimes}_i$ Y $\bar{\otimes}_i$ Z with C_i as its positive cone is a Banach lattice called the positive injective tensor product of X, Y and Z.

The positive cone on X ⊗ Y ⊗ Z is defined to be

$$C_p=\left\{\sum_{m=1}^{r}\sum_{n=1}^{s}\sum_{k=1}^{t}x_{mnk}\otimes y_{mnk}\otimes y_{mnk}: r,s,t\in N, x_{mnk}\in X^+, y_{mnk}\in Y^+\right\}.$$

We define the following spaces:

For a Banach metric lattice X, let

$$\Gamma_M^3(X)=\left\{\bar{x}=(x_{ijk})_{ijk}\in X^{N\times N\times N}:\left(x^*|x_{ijk}|^{1/i+j+k}\right)_{ijk}\in\Gamma^3,\forall x^*\in X^{*+}\right\}$$

The metric defined to be

$$d(x,y)=\sup\left\{\left\|\left(x^*\left(\left|x_{ijk}-y_{ijk}\right|\right)^{1/i+j+k}\right):x^*\in B_{X^{*+}}\right\|\right\},$$

$x=(x_{ijk})_{ijk}\in\Gamma_M^3(X)$

Let

$$\Gamma_M^3(X)=\left\{\bar{x}\in\Gamma_M^3(X):\lim_{i,j,k}\left\|\left(\left|\bar{x}_{ijk}(>n)\right|\right)^{1/i+j+k}\right\|\to 0 \text{ as } i,j,k\to\infty\right\},$$

with the metric

$$d(x,y)=\sup\left\{\left\|\left(\left|\left(\bar{x}_{ijk}-\bar{y}_{ijk}\right)(>n)\right|\right)^{1/i+j+k}\right\|\forall\left(\bar{x}_{ijk}\right)\in\Gamma_M^3\left(X^{*+}\right)\right\}$$

4. Some New Orlicz Sequence Spaces of Tensor Product: The main aim of this article is to introduce the following sequence spaces and examine the topological and algebraic properties of the resulting sequence spaces. Let $M = (M_{mnk})$ be a sequence Orlicz function,

$\left(\bar{X},\left\|\left(d(\bar{x}_1,0),d(\bar{x}_2,0),...,d(\bar{x}_{n-1},0)\right)\right\|_p\right)$ be a p-metric space, and consider

$\mu_{mnk}(\bar{x}) = \left\|\left(\left|\bar{x}_{ijk}(>n)\right|\right)^{1/i+j+k}\right\|$.

We define the following sequence spaces as follows:

$\left[\Gamma^3_M, \left\|\left(d(\bar{x}_1,0), d(\bar{x}_2,0), \ldots, d(\bar{x}_{n-1},0)\right)\right\|_p\right] =$

$\lim_{mnk\to\infty}\left\{\sum_m\sum_n\sum_k\left[M_{mnk}\left(\left\|\mu_{mnk}(\bar{x}), \left(d(\bar{x}_1,0), d(\bar{x}_2,0), \ldots, d(\bar{x}_{n-1},0)\right)\right\|_p\right)\right] = 0\right\}$,

and

$\left[\Lambda^3_M, \left\|\left(d(\bar{x}_1,0), d(\bar{x}_2,0), \ldots, d(\bar{x}_{n-1},0)\right)\right\|_p\right] =$

$\sup\left\{\sum_m\sum_n\sum_k\left[M_{mnk}\left(\left\|\mu_{mnk}(\bar{x}), \left(d(\bar{x}_1,0), d(\bar{x}_2,0), \ldots, d(\bar{x}_{n-1},0)\right)\right\|_p\right)\right] < \infty\right\}$.

5. Main Results

5.1. Theorem: Let M = (M_{mnk}) be a Musielak-Orlicz function, the tensor product of Orlicz sequence spaces $\left[\Gamma^3_{M\mu}, \left\|\left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right]$ and $\left[\Lambda^3_{M\mu}, \left\|\left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right]$ arelinear spaces.

Proof: It is routine verification. Therefore the proof is omitted.

5.2. Theorem: Let M = (M_{mnk}) be a Musielak-Orlicz function, the tensor product of Orlicz sequence space $\left[\Gamma^3_{M\mu}, \left\|\left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right]$ is a paranormed space with respect to the paranorm defined by

g(x)= $\left\{\left[M_{mnk}\left(\left\|\mu_{mnk}(x), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right)\right]\right\}$.

Proof: Clearly g(x) ≥ 0 for x = (x_{mnk}) ∈ $\left[\Gamma^3_{M\mu}, \left\|\left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right]$

Since $M_{mnk}(0) = 0$, we get g(0) = 0.

Conversely, suppose that g(x) = 0, then

$\left\{\left[M_{mnk}\left(\left\|\mu_{mnk}(x), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right)\right]\right\}$

Suppose that $\mu_{mnk}(x) \neq 0$ for each m, n, k $\in N \times N \times N$. Then

$\left\|\mu_{mnk}(x), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p \to \infty$.

It follows that

$\left(\left[M_{mnk}\left(\left\|\mu_{mnk}(x), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right)\right]\right) \to \infty$

which is a contradiction. Therefore $\mu_{mnk}(x) = 0$. Let $\left(\left[M_{mnk}\left(\left\|\mu_{mnk}(x), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right)\right]\right)$ and

$\left(\left[M_{mnk}\left(\left\|\mu_{mnk}(y), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right)\right]\right)$

Then by using Minkowski's inequality, we have

$\left(\left[M_{mnk}\left(\left\|\mu_{mnk}(x+y+z), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right)\right]\right) \leq$

$\left(\left[M_{mnk}\left(\left\|\mu_{mnk}(x), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right)\right]\right)$ +

$\left(\left[M_{mnk}\left(\left\|\mu_{mnk}(y), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right)\right]\right)$ +

$\left(\left[M_{mnk}\left(\left\|\mu_{mnk}(z), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right)\right]\right)$.

So we have

g(x+y+z)=

$\left\{\left[M_{mnk}\left(\left\|\mu_{mnk}(x+y+z), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right)\right]\right\} \leq$

$\left\{\left[M_{mnk}\left(\left\|\mu_{mnk}(x), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right)\right]\right\}$ +

$\left\{\left[M_{mnk}\left(\left\|\mu_{mnk}(y), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right)\right]\right\}$ +

$\left\{\left[M_{mnk}\left(\left\|\mu_{mnk}(z), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right)\right]\right\}$.

Therefore,

g(x + y + z) ≤ g(x) + g(y) + g(z).

Finally, to prove that the scalar multiplication is continuous. Let λ be any complex number. Therefore paranormed by,

g(λx)= $\left\{\left[M_{mnk}\left(\left\|\mu_{mnk}(\lambda x), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right)\right]\right\}$.

Hence it is continuous. This completes the proof.

5.3. Theorem: (i) If the tensor product of Orlicz sequence spaces (M_{mnk}) and (N_{mnk}) are satisfies Δ_2-condition, then

$\left[\Gamma^3_{M\mu}, \left\|\mu_{mnk}(x), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right] =$

$\left[\Gamma^3_{N\mu}, \left\|\mu_{uvw}(x), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right]$.

(ii) If the tensor product of Orlicz sequence spaces (N_{mnk}) and (M_{mnk}) are satisfies Δ_2-condition, then

$\left[\Gamma^3_{N\mu}, \left\|\mu_{mnk}(x), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right] =$

$\left[\Gamma^3_{M\mu}, \left\|\mu_{mnk}(x), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right]$.

Proof: Let the tensor product of Orlicz sequence spaces (M_{mnk}) and (N_{mnk}) are satisfiesΔ_2-condition, we get

$\left[\Gamma^3_{N\mu}, \left\|\mu_{mnk}(x), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right] \subset$

$\left[\Gamma^3_{N\mu}, \left\|\mu_{mnk}(x), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right] \ldots \quad (6)$

To prove the inclusion

$\left[\Gamma^3_{M\mu}, \left\|\mu_{mnk}(x), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right] \subset$

$\left[\Gamma^3_{N\mu}, \left\|\mu_{mnk}(x), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right]$,

let a $\in \left[\Gamma^3_{M\mu}, \left\|\mu_{mnk}(x), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right]$.

Then for all {x_{mnk}} with

(x_{mnk}) $\in \left[\Gamma^3_{M\mu}, \left\|\mu_{mnk}(x), \left(d(x_1), d(x_2), \ldots, d(x_{n-1})\right)\right\|_p\right]$ we have

$$\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}|x_{mnk}a_{mnk}|<\infty \qquad \dots (7)$$

Since the tensor product of Orlicz sequence (M_{mnk}) satisfies Δ_2-condition, then $(y_{mnk}) \in \left[\Gamma^3_{M\mu}, \|\mu_{mnk}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p\right]$,

we get $\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty}|y_{mnk}a_{mnk}|<\infty$ by (7). Thus $(a_{mnk}) \in \left[\Gamma^3_{M\mu}, \|\mu_{mnk}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p\right]$ = $\left[\Gamma^3_{N\mu}, \|\mu_{mnk}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p\right]$ and hence $(a_{mnk}) \in \left[\Gamma^3_{N\mu}, \|\mu_{mnk}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p\right]$. This gives that

$$\left[\Gamma^3_{M\mu}, \|\mu_{mnk}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p\right] \subset \left[\Gamma^3_{N\mu}, \|\mu_{mnk}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p\right] \dots (8)$$

we are granted with (7) and (8)

$$\left[\Gamma^3_{M\mu}, \|\mu_{mnk}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p\right] = \left[\Gamma^3_{N\mu}, \|\mu_{mnk}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p\right]$$

(ii) Similarly, one can prove that $\left[\Gamma^3_{N\mu}, \|\mu_{mnk}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p\right] \subset \left[\Gamma^3_{M\mu}, \|\mu_{mnk}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p\right]$ if the sequence (N_{mnk}) satisfies Δ_2-condition.

5.4. Proposition: The tensor product of Orlicz sequence space $\left[\Gamma^3_{M\mu}, \|\mu_{mnk}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p\right]$ is not solid.

Proof: The result follows from the following example.

Example: Consider x = (x_{mnk}) =

$$\begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ \vdots & & & \\ 1 & 1 & \dots & 1 \end{bmatrix} \in \left[\Gamma^3_{M\mu}, \|\mu_{mnk}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p\right].$$

Let

$$\alpha_{mnk} = \begin{bmatrix} (-1)^{m+n+k} & (-1)^{m+n+k} & \dots & (-1)^{m+n+k} \\ (-1)^{m+n+k} & (-1)^{m+n+k} & \dots & (-1)^{m+n+k} \\ \vdots & & & \\ (-1)^{m+n+k} & (-1)^{m+n+k} & \dots & (-1)^{m+n+k} \end{bmatrix}$$

for all m, n, $k \in N \times N \times N$.

Then $\alpha_{mnk}x_{mnk} \notin \left[\Gamma^3_{M\mu}, \|\mu_{mnk}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p\right]$.

Hence $\left[\Gamma^3_{M\mu}, \|\mu_{mnk}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p\right]$ is not solid.

5.5. Proposition: The tensor product of Orliczsequence space $\left[\Gamma^3_{M\mu}, \|\mu_{mnk}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p\right]$ is not monotone.

Proof: The proof follows from Proposition 5.4.

5.6. Theorem: The tensor product of Orlicz sequence spaces $\left[\Gamma^3_{\mu}, \|(d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p\right]$ and $\left[\Lambda^3_{\mu}, \|(d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p\right]$ are not convergence free.

Example: Consider,

$$\left[M_{mnk}\left(\|\mu_{mnk}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|\right)_p\right] = \left[M\left(\|\mu_{mnk}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|\right)_p\right] = \left[\left(\|\mu_{mnk}(x), (d(x_1), d(x_2), \dots, d(x_{n-1}))\|\right)_p\right]$$

for all m, n, $k \in N \times N \times N$, for m, n, k are odd. If m, n, k even, consider the sequence $(x_{mnk}) = (mnk)^{-(m+n+k)}$ for all m, n, $k \in N \times N \times N$ belongs to each of $\left[\Gamma^3_{\mu}, \|(d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p\right]$ and $\left[\Lambda^3_{\mu}, \|(d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p\right]$.Considerthe sequence (y_{mnk}) defined by $(y_{mnk})^{1/m+n+k} = m^2n^2k^2$, for all m, n, $k \in N \times N \times N$. Then (y_{mnk}) neither belongs to $\left[\Gamma^3_{\mu}, \|(d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p\right]$ nor $\left[\Lambda^3_{\mu}, \|(d(x_1), d(x_2), \dots, d(x_{n-1}))\|_p\right]$. Hence the tensor product Orlicz sequence spaces are not convergence free.

References:

1. Apostol, T., 1978. Mathematical Analysis, Addison-Wesley, London.
2. Altin, Y., 2009. Properties of some sets of sequences defined by a modulus function, Acta Mathematica Scientia. Series B. (English Edition), No. 2, 427-434.
3. B. Shanthi Gowri, Gnanambal Ilango, Region Growing Segmentation of Magnetic Resonance; Mathematical Sciences international Research

Journal ISSN 2278 – 8697 Vol 3 Issue 2 (2014), Pg 792-795

4. Bektas, Ç.A. and Altin, Y., 2003. The Sequence Spaces on Seminormed Spaces, Indian Journal of Pure and Applied Mathematics, 34(4), 529-534.
5. Basarir, M. and Solancan, O., On some double sequence spaces, J. Indian Acad. Math., 21(2) (1999), 193-200.
6. Bromwich, T.J.I'A., An introduction to the theory of infinite series Macmillan and Co. Ltd., New York (1965).
7. Esi, A. and Savas, E., 2015. On lacunary statistically convergent triple sequences in probabilistic normed space, Applied Mathematics and Information Sciences, 9 (No. 5), 2529-2534.
8. Esi, A., 2014. On some triple almost lacunary sequence spaces defined by Orlicz functions, Research and Reviews: Discrete Mathematical Structures, 1(2), 16-25.
9. Esi, A., and Necdet Catalbas, M., 2014. Almost convergence of triple sequences, Global Journal of Mathematical Analysis, 2(1), 6-10.
10. Et, M., Altin, Y., Choudhary, Y. and Tripathy, B.C. 2006. On some classes of sequences defined by sequences of Orlicz functions, Mathematical Inequalities and Applications, 9 (No. 2), 335-342.
11. Hardy, G.H., On the convergence of certain multiple series, Proc. Camb. Phil. Soc., 19 (1917), 86-95.
12. *K V Vidyasagar,* An Alternate Proof of the De Moivre's theorem. ; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 2 Issue 1 (2013), Pg 1-5
13. Kamthan, P.K. and Gupta, M. Sequence spaces and series, Lecture notes, Pure and Applied Mathematics, 65 Marcel Dekker, In c., New York, 1981.
14. Krasonselskii, M.A. and Rutickii, Y.B., Convex functions and Orlicz spaces, Gorningen, Netherlands, 1961.
15. Lindenstrauss, J. and Tzafriri, L. On Orlicz sequence spaces, Israel J. Math., 10 (1971), 379-390.
16. Nakano, H., Concave modulars, J. Math. Soc. Japan, 5 (1953), 29-49.
17. Qingying Bu, Donghai Ji and Yongjin Li, Copies of ℓ_1 in positive tensor products of Orlicz sequence spaces, Quaestiones Mathematicae, 34, (2011), 407-415.
18. D. Anandha Selvam, M. Davamani Christober, A Study on Just Excellent and Very Excellent Weakly; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 3 Issue 2 (2014), Pg 800-802
19. Subramanian, N. and Misra, U.K. Characterization of gai sequences via double Orlicz space, Southeast Asian Bulletin of Mathematics, 35 (2011), 687-697.
20. Subramanian, N., Tripathy, B.C. and Murugesan, C., The double sequence space of Γ^2, Fasciculi Math., 40 (2008) 91-103.
21. Subramanian, N., Tripathy, B.C. and Murugesan, C. The Cesàro of double entire sequences, International Mathematical Forum, 4, No. 2 (2009), 49-59.
22. Şahiner, A., Gürdal, M. and Düden, F.K. 2007. Triple sequences and their statistical convergence, Selçuk Journal of Applied Mathematics, 8(2), 49-55.

***

T.V.G. Shri Prakash/ N.Subramanian/
Department of Mathematics/ SASTRA University/Thanjavur-613 401/India.
M.Chandramouleeswaran/Department of Mathematics/
Saiva Bhanu shathriya College/Arupukottai – 626 101/ India.