A DISPLAYED INVENTORY MODEL WITH QUALITY CONSTRAINT USING PENTAGONAL FUZZY NUMBER

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Abstract: This paper explores a Fuzzy Multi-item displayed inventory model with alternative power supply cost (Power generator). The cost parameters and the constraints are represented by the pentagonal fuzzy number. The model is solved by fuzzy geometric programming method. The optimal order quantity, number of display quantity and percentage of quality have been determined. A numerical example is given illustrate the model.

Keywords: Display inventory, Economic Order Quantity (EOQ), Fuzzy geometric programming, Pentagonal fuzzy number, Nearest interval approximation.

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1. **Introduction:** For several years, classical economic order quantity (EOQ) problems with different variations were solved by many researchers and had been reported in the reference books and survey papers (e.g. Churchman, Ackoff and Arnoff 1957 [3], Hardley and Whitin 1958 [6], Silver and Peterson 1985 [12], etc,).

Merchandisers and manufacturers must manage the inventory as part of the ongoing operations of business. A manufacturer must deal with raw materials inventory, work-in-progress inventory and finished goods inventory. Merchandisers and retailers confront the possibility that products in inventory, if retained too long, may no longer be in demand. Quality in inventory management systems is vitally important to the prosperity and long-term stability of a company, as how a business manages inventory can have a direct effect on overall profits, both in the short-term and long-term.

Defective products in the market can lead to the loss of reputation and customer loyalty. One dissatisfied customer will tell 100 others, which means the loss of both present and future customers. It will also affect the brand image, leading to loss of good will and customer loyalty. If the trend is not corrected, and the quality is not restored, the company will have to close down. So quality of an item is important to inventory management.

While modelling an inventory problem, generally the demand depends on some parameter. Many researchers discussed the following two types of demand, (1) Time-dependent demand and (2) Stock-dependent demand. But in practical situation demand rate of an item may be related to quality of the product. If the quality of an item is high, then the demand rate is increases.

But all these inventory problems are solved with the assumption that the co-efficient or cost parameters are specified in a precise way. In real life, there are many diverse situations due to uncertainty. Here the inventory costs are imprecise, that is fuzzy in nature.

Early works in using fuzzy concept in decision making were done by Zadeh (1965) [15] and Bellman (1970) [2] through introducing fuzzy goals, costs and constraints. Later, the fuzzy linear programming models was formulated and an approach for solving linear programming model with fuzzy numbers has been presented by Zimmermann (1978) [16].

The non-linear optimization problems have been solved by various non-linear optimization techniques. Among those techniques, geometric programming (GP) is an efficient and effective method to solve a particular type of nonlinear problems. After the first introduction by Zener, Duffin et al [1967][4] developed the GP method. Kotchenberger was the first to use it on inventory problems. Later on Worrall and Hall (1982) [14] analyzed a multi-item inventory model with several constraints using posynomial GP method. Later, the Geometric programming techniques were discussed by Abou-el-Ata, M. O.,Fergany, H. A., and El-Wakeel, M.F. (2003) [1]. Recently N.K.Mandal and T.K.Roy (2006) [9] presented a displayed inventory model with triangular fuzzy number. In their paper demand rate is dependent on the displayed inventory level and there is a limitation on total display space.

Lately, power scarcity is affecting the small scale industries such as textiles, pickle manufacturing company, mushrooms growing farm and milk productions companies etc. To solve this problem, generators are being installed, it incurs a cost. This paper introduces / refers the cost as “Alternative power supply cost”. Also the pentagonal fuzzy number is defined. So the display inventory model by using pentagonal fuzzy number with alternative power supply cost has been considered. In this paper, a multi item displayed inventory model under shelf-space constraint and quality cost constraint in fuzzy environment is formulated. Also power generator has been used in both back room storage area and display area.

The parameters involved in this paper are assumed to be imprecise in nature and are represented by pentagonal fuzzy number with different types of left and right membership functions. The model is then reduced to multi-objective decision- making inventory problem and is solved by fuzzy geometric method. Finally a numerical example is given to illustrate the model.

2. Assumptions and Notations:

A multi-item displayed inventory model with generator cost is formulated under the following assumptions and notations.

Assumptions:

1. The unit cost of the item is independent of the order quantity Q.
2. The display cost does not depend on the length of cycle time T.
3. The outstanding order is atmost one per cycle.
4. Lead time is zero.
5. Shortages are not allowed.
6. Demand rate depends on percentage of quality (y_i) and purchasing cost (C_i) for i th item $D_i = C_i y_i^{a_i}$ ($0 < a_i < 1,\ 0 < C_i < k$), where k is a finite number.
7. Full-shelf merchandising policy has been adopted, where the display area is always kept fully stocked, so the inventory is replenished as soon as the back room inventory reaches zero. The displayed inventory will always be at its maximum. The inventory level decreases at a constant rate.
8. Alternative power supply (power generator) cost is allowed.

Notations:

The following are for i th item (i =1,2,3.......n)

n –number of items

S_i – number of display quantity (decision variable)

Q_i –number of order quantity (decision variable)

y_i – percentage of quality (decision variable)

D_i – demand rate

w_i - shelf space per unit

T_i – cycle time $= \dfrac{Q_i}{C_i y_i^{a_i}}$

$\tilde{p}_i$ – fuzzy selling price per unit per cycle

$\tilde{C}_i$ – fuzzy purchasing cost per unit per cycle

$\tilde{C}_{1i}$ – fuzzy holding cost per unit per unit time

$\tilde{C}_{2i}$ – fuzzy display shelf cost per unit per unit time

$\tilde{C}_{3i}$ – fuzzy set up cost per cycle

$\tilde{P}_i$ – fuzzy production rate

$\tilde{g}_i$ – fuzzy alternative power supply cost (power generator) per unit per unit time

PF – fuzzy profit function

$\tilde{W}$ – fuzzy total display-shelf space

$\tilde{M}$ – fuzzy Miscellaneous amount

θ_i – instantaneous inventory level of the entire system including both the back room storage and the display inventory.

3.Mathematical Model in Crisp Environment:

The inventory model is formulated to maximum the average net profit, which includes the gross revenues, unit purchasing cost, set up cost, holding cost and the display cost under the limited display space – constant and quality cost constraint.

Average profit = Gross revenues per unit – purchasing price per unit – set up cost per unit time – holding cost per unit time – generator cost per unit time – display shelf space cost per unit time.

Hence, the average profit function is

$$PF(S,Q,y) = \sum_{i=1}^{n} \begin{bmatrix} D_i\, p_i - D_i\, C_i - \\ \dfrac{C_{3i}\, D_i}{Q_i} - \left(1 - \dfrac{D_i}{P_i}\right)\left(\dfrac{C_{1i} + g_i}{2}\right)\theta_i \\ -C_{2i}\, S_i \end{bmatrix} \qquad \text{------- (1)}$$

Where the average inventory is $\dfrac{\theta_i}{2} = \dfrac{Q_i}{2} + S_i$

Average profit function is reduced to

$$PF(S,Q,y) = \sum_{i=1}^{n} \begin{bmatrix} C_i\, y_i^{a_i}\left(p_i - C_i\right) - \dfrac{C_{3i}\, C_i\, y_i^{a_i}}{Q_i} - \\ \dfrac{(C_{1i} + g_i)\, Q_i}{2} - (C_{1i} + g_i + C_{2i}) S_i \\ + \dfrac{(C_{1i} + g_i)\, C_i\, y_i^{a_i} Q_i}{2\, P_i} + \\ \dfrac{(C_{1i} + g_i)\, C_i\, y_i^{a_i} S_i}{P_i} \end{bmatrix}$$

Subject to:

$$\sum_{i=1}^{n} w_i\, S_i \le W$$

$$\sum_{i=1}^{n} C_i\, y_i\, Q_i \le \frac{M}{2}$$

The problem is then stated as

$$\max\ PF(S,Q,y) = \sum_{i=1}^{n} \begin{bmatrix} K_{1i} y_i^{a_i} - \dfrac{K_{2i} y_i^{a_i}}{Q_i} - K_{3i}\, Q_i \\ -K_{4i} S_i + K_{5i} y_i^{a_i}\, Q_i \\ +K_{6i} y_i^{a_i}\, S_i \end{bmatrix} \qquad \text{-- (2)}$$

Subject to:

$$\sum_{i=1}^{n} K_{7i}\, S_i \le 1$$

$$\sum_{i=1}^{n} K_{8i}\, y_i\, Q_i \le 1$$

Where

$$K_{1i} = C_i(P_i - C_i)\; K_{2i} = C_{3i} C_i\; K_{3i} = \frac{(C_{1i} + g_i)}{2}$$

$$K_{4i} = (C_{1i} + g_i + C_{2i})$$

$$K_{5i} = \frac{(C_{1i} + g_i)C_i}{2P_i}\; K_{6i} = \frac{(C_{1i} + g_i)C_i}{P_i}$$

$$K_{7i} = \frac{w_i}{W}\; K_{8i} = \frac{2C_i}{M}$$

The standard geometric programming is

$$\max\; PF(S,Q,y) = \sum_{i=1}^{n} \begin{bmatrix} -K_{1i} y_i^{a_i} + \frac{K_{2i} y_i^{a_i}}{Q_i} + \\ K_{3i} Q_i + K_{4i} S_i - \\ K_{5i} y_i^{a_i} Q_i - K_{6i} y_i^{a_i} S_i \end{bmatrix} \quad \text{-------- (3)}$$

Subject to:

$$\sum_{i=1}^{n} K_{7i}\, S_i \le 1$$

$$\sum_{i=1}^{n} K_{8i}\, y_i\, Q_i \le 1$$

This primal problem (3) is a constrained signomial problem with 5n-1 degree of difficulty. The corresponding dual problem is

$$\max\; D_L = -\prod_{i=1}^{n} \begin{bmatrix} \left(\frac{K_{1i}}{w_{1i}}\right)^{-w_{1i}} \left(\frac{K_{2i}}{w_{2i}}\right)^{w_{2i}} \left(\frac{K_{3i}}{w_{3i}}\right)^{w_{3i}} \\ \left(\frac{K_{4i}}{w_{4i}}\right)^{w_{4i}} \left(\frac{K_{5i}}{w_{5i}}\right)^{-w_{5i}} \\ \left(\frac{K_{6i}}{w_{6i}}\right)^{-w_{6i}} \left(\frac{K_{7i}}{w_{7i}}\right)^{w_{7i}} \left(\frac{K_{8i}}{w_{8i}}\right)^{w_{8i}} \\ \left(\sum_{i=1}^{n} w_{7i}\right)^{\sum_{i=1}^{n} w_{7i}} \left(\sum_{i=1}^{n} w_{8i}\right)^{\sum_{i=1}^{n} w_{8i}} \end{bmatrix}^{-1} \quad \text{---------- (4)}$$

Subject to:

$$-w_{1i} + w_{2i} + w_{3i} + w_{4i} - w_{5i} - w_{6i} = -1$$

$$-w_{2i} + w_{3i} - w_{5i} + w_{8i} = 0$$

$$w_{4i} - w_{6i} + w_{7i} = 0$$

$$-a_i w_{1i} + a_i w_{2i} - a_i w_{5i} - a_i w_{6i} + w_{8i} = 0$$

By using geometric programming theorem [4], the analytical expression for the decision variables Q_i,y_i and S_i are obtained.

$$Q_i^* = \sqrt{\frac{K_{2i}\, w_{5i}}{K_{5i}\, w_{2i}}}$$

$$S_i^* = \frac{K_{5i}\, w_{6i}\, Q_i^*}{K_{6i}\, w_{5i}} \quad \text{---------- (5)}$$

$$y_i^* = \frac{\left(K_{7i}\, w_{8i}\, S_i^*\right)}{\left(K_{8i}\, Q_i^* w_{7i}\right)}$$

4. Pentagonal Fuzzy Number and its Nearest interval Approximation:

Definition 4.1: A pentagonal fuzzy number $\widetilde{A}$ described as a normalized convex fuzzy subset on the real line R whose membership function $\mu_{\widetilde{A}}(x)$ is defined as follows

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & , \quad x \le a \\ \omega_A\left(\frac{x-a}{b-a}\right) & , \quad a \le x \le b \\ \omega_A + (1-\omega_A)\left(\frac{x-b}{c-b}\right) & , \quad b \le x \le c \\ \omega_A + (1-\omega_A)\left(\frac{d-x}{d-c}\right) & , \quad c \le x \le d \\ \omega_A\left(\frac{e-x}{e-d}\right) & , \quad d \le x \le e \\ 0 & , \quad x \ge e \end{cases}$$

Where $0.6 \le \omega_A < 1$ and a, b, c, d and e are real numbers.

This type of fuzzy number be denoted as $\widetilde{A} = (a,b,c,d,e;\omega_A)_{PFN}$ $\mu_{\widetilde{A}}(x)$ satisfies the following conditions:

1. $\mu_{\widetilde{A}}$ is a continuous mapping from R to the closed interval [0,1].
2. $\mu_{\widetilde{A}}$ is a convex function.
3. $\mu_{\widetilde{A}} = 0, -\infty < x \le a.$
4. $\mu_{\widetilde{A}} = L(x)$ is strictly increasing on (a,c).
5. $\mu_{\widetilde{A}} = 1,\; x = c.$
6. $\mu_{\widetilde{A}} = R(x)$ is strictly decreasing on (c,e).
7. $\mu_{\widetilde{A}} = 0,\; e \le x < \infty.$

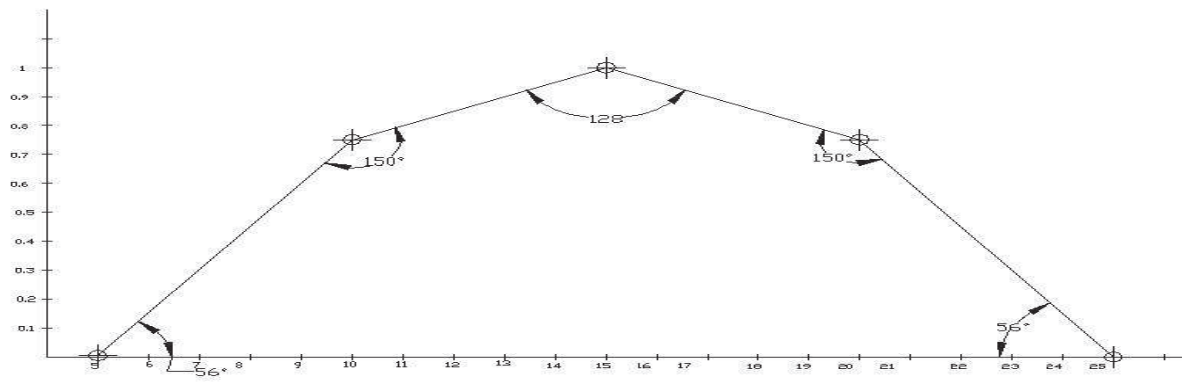

Figure 1: Graphical representation of Pentagonal fuzzy number

Remarks:

1. If $x = c$ then $\mu_{\tilde{A}} = 1$, therefore it is a uni-model fuzzy number.
2. If $0 \le \omega_A < 0.6$ then $\widetilde{A}$ becomes a triangular fuzzy number.
3. If $\omega_A = 1$ then $\widetilde{A}$ becomes a trapezoidal fuzzy number.

Nearest Interval Approximation: Suppose $\widetilde{A}$ and $\widetilde{B}$ are two fuzzy numbers with α-cuts are $[A_L(\alpha), A_R(\alpha)]$ and $[B_L(\alpha), B_R(\alpha)]$ respectively. Then the distance between $\widetilde{A}$ and $\widetilde{B}$ is

$$d(\tilde{A},\tilde{B}) = \sqrt{\begin{array}{l}\int_0^1 ((A_L(\alpha) - B_L(\alpha))^2 \, d\alpha) + \\ \int_0^1 (A_R(\alpha) - B_R(\alpha))^2 \, d\alpha\end{array}}$$

Given $\widetilde{A}$ is a pentagonal fuzzy number. We have to find a closed interval $C_d(\widetilde{A})$, which is the nearest to $\widetilde{A}$ with respect to metric d. We can do it since each interval is also a fuzzy number with constant α –cut for all $\alpha \in [0,1]$ Hence $\left(C_d(\widetilde{A})\right)\alpha = [C_L, C_R]$. Now we have to minimize

$$d(\tilde{A}, C_d\tilde{A}) = \sqrt{\begin{array}{l}\int_0^1 (A_L(\alpha) - C_L)^2 \, d\alpha + \\ \int_0^1 (A_R(\alpha) - C_R)^2 \, d\alpha\end{array}}$$

In order to minimize $d(\widetilde{A}, C_d\widetilde{A})$ it is sufficient to minimize the function $D(C_L, C_R) = d^2(\widetilde{A}, C_d\widetilde{A})$. The first partial derivatives are

$$\frac{\partial D(C_L, C_R)}{\partial C_L} = -2\int_0^1 A_L(\alpha)\, d\alpha + 2C_L \quad \text{and}$$

$$\frac{\partial D(C_L, C_R)}{\partial C_R} = -2\int_0^1 A_R(\alpha)\, d\alpha + 2C_R$$

Solving $\dfrac{\partial D(C_L, C_R)}{\partial C_L} = 0$ and $\dfrac{\partial D(C_L, C_R)}{\partial C_R} = -2\int_0^1 A_R(\alpha)\, d\alpha + 2C_R = 0$ the following expressions are derived

$$C_L = \int_0^1 A_L(\alpha) d\alpha \quad and \quad C_R = \int_0^1 A_R(\alpha) d\alpha$$

Again since $\dfrac{\partial D^2(C_L, C_R)}{\partial C_L{}^2} = 2 > 0$ and $\dfrac{\partial D^2(C_L, C_R)}{\partial C_R{}^2} = 2 > 0$

$$H(C_L, C_R) = \frac{\partial D^2(C_L, C_R)}{\partial C_L{}^2} \cdot \frac{\partial D^2(C_L, C_R)}{\partial C_R{}^2} - \frac{\partial D^2(C_L, C_R)}{\partial C_R{}^2}$$

So $D(C_L, C_R)$, i.e. $d(\widetilde{A}, C_d\widetilde{A})$ is global minimum. Therefore, the interval

$$A_L(\alpha) = \begin{cases} a + \dfrac{\alpha(b-a)}{\omega_A} & if \quad a \le x \le b \\ b + \dfrac{(\alpha - \omega_A)(c-b)}{1-\omega_A} & if \quad b \le x \le c \end{cases}$$

.

$$A_R(\alpha)=\begin{cases} d-\dfrac{(\alpha-\omega_A)\,(d-c)}{1-\omega_A} & if \quad c\le x\le d \\ e-\dfrac{\alpha\,(e-d)}{\omega_A} & if \quad d\le x\le e \end{cases}$$

By the nearest interval approximation method, the lower and limit of the interval are

$$C_L(\alpha)=\frac{1}{2(1-\omega_A)}\begin{bmatrix}(b+c)+\omega_A(a-b-2c)\\ -\omega_A{}^2(a-c)\end{bmatrix}$$

$$C_R(\alpha)=\frac{1}{2(1-\omega_A)}\begin{bmatrix}(d+c)+\omega_A(e-d-2c)\\ -\omega_A{}^2(e-c)\end{bmatrix}$$

Similarly, when $\tilde{A}$ is a parabolic fuzzy number

$$C_L(\alpha)=\frac{1}{3}\left[(b+2c)+(a+b-2c)\omega_A\right]$$

$$C_R(\alpha)=\frac{1}{3}\left[(d+2c)+(e+d-2c)\omega_A\right]$$

Similarly, when $\tilde{A}$ is a hyperbolic numberLet $\tilde{A}=(a,b,c,d,e)$ be a pentagonal fuzzy number. The α–level interval of $\tilde{A}$ is defined as $A_\alpha=[A_L(\alpha),A_R(\alpha)]$.

When $\tilde{A}$ is a linear fuzzy number (LFN), the left and right α cuts are

$C_d(\tilde{A})=\left[\int_0^1 A_L(\alpha)\,d\alpha,\ \int_0^1 A_R(\alpha)\,d\alpha\right]$ is the nearest interval approximation of fuzzy number $\tilde{A}$ with respect to the metric d.

$$C_L{}^*=\int_0^1 A_L(\alpha)\,d\alpha \quad and$$

$$C_R{}^*=\int_0^1 A_R(\alpha)\,d\alpha$$

$$C_L(\alpha)=\begin{bmatrix}\dfrac{\sqrt{b^2-a^2}\,\omega_A}{2}\begin{bmatrix}\dfrac{b}{\sqrt{b^2-a^2}}+\\ \dfrac{a^2}{b^2-a^2}\\ \log\left(\dfrac{\sqrt{b^2-a^2}+b}{a}\right)\end{bmatrix}+\\ \dfrac{\sqrt{c^2-b^2}\,(1-\omega_A)}{2}\begin{bmatrix}\dfrac{c}{\sqrt{c^2-b^2}}+\\ \dfrac{b^2}{c^2-b^2}\\ \log\left(\dfrac{\sqrt{c^2-b^2}+c}{b}\right)\end{bmatrix}\end{bmatrix}$$

$$C_R(\alpha)=\begin{bmatrix}\dfrac{\sqrt{d^2-c^2}\,(1-\omega_A)}{2}\\ \left[\dfrac{c}{\sqrt{d^2-c^2}}+\dfrac{d^2}{d^2-c^2}\tan^{-1}\left(\dfrac{\sqrt{d^2-c^2}}{c}\right)\right]+\\ \dfrac{\sqrt{e^2-d^2}\,(\omega_A)}{2}\\ \left[\dfrac{d}{\sqrt{e^2-d^2}}+\dfrac{e^2}{e^2-d^2}\tan^{-1}\left(\dfrac{\sqrt{e^2-d^2}}{d}\right)\right]\end{bmatrix}$$

5. The Proposed Inventory Model In Fuzzy Environment:

If the cost parameters and total display shelf space parameters are fuzzy numbers, then the problem (2) is transformed to

$$\max\ P\tilde{F}(S,Q,y)=\sum_{i=1}^{n}\begin{bmatrix}\tilde{C}_i\,y_i^{a_i}\left(\tilde{p}_i-\tilde{C}_i\right)-\dfrac{\tilde{C}_{3i}\,\tilde{C}_i\,y_i^{a_i}}{Q_i}-\\ \dfrac{(\tilde{C}_{1i}+\tilde{g}_i)Q_i}{2}-(\tilde{C}_{1i}+\tilde{g}_i+\tilde{C}_{2i})S_i\\ +\dfrac{(\tilde{C}_{1i}+\tilde{g}_i)\tilde{C}_i\,y_i^{a_i}Q_i}{2\tilde{P}_i}+\\ \dfrac{(\tilde{C}_{1i}+\tilde{g}_i)\tilde{C}_i\,y_i^{a_i}S_i}{\tilde{P}_i}\end{bmatrix}$$

Subject to:

$$\sum_{i=1}^{n} w_i\,S_i\le\tilde{W}$$

$$\sum_{i=1}^{n} C_i\,y_i\,Q_i\le\frac{\tilde{M}}{2}$$

Where ~ represents the fuzzification of the parameters. In our proposed model, the cost parameters g_i, $p_i, C_i, C_{1i}, C_{2i}, C_{3i}, P_i, W$ and M are considered as pentagonal fuzzy numbers.

$\tilde{p}_i=(p_{i_1}, p_{i_2}, p_{i_3}, p_{i_4}, p_{i_5})$

$\tilde{C}_i=(C_{i_1}, C_{i_2}, C_{i_3}, C_{i_4}, C_{i_5})$

$\tilde{P}_i=(P_{i_1}, P_{i_2}, P_{i_3}, P_{i_4}, P_{i_5})$

$\tilde{C}_{1i}=(C_{1i_1}, C_{1i_2}, C_{1i_3}, C_{1i_4}, C_{1i_5})$

$\tilde{C}_{2i}=(C_{2i_1}, C_{2i_2}, C_{2i_3}, C_{2i_4}, C_{2i_5})$

$\tilde{C}_{3i}=(C_{3i_1}, C_{3i_2}, C_{3i_3}, C_{3i_4}, C_{3i_5})$

$\tilde{g}_i=(g_{i_1}, g_{i_2}, g_{i_3}, g_{i_4}, g_{i_5})$

$\tilde{W}=(W_1,W_2,W_3,W_4,W_5)$

$\tilde{M}=(M_1,M_2,M_3,M_4,M_5)$

our proposed model is reduced to,

$$\max\ PF(S,Q,y)=$$
$$\sum_{i=1}^{n}\left[\begin{array}{l}[C_{i_L},C_{i_R}]\,y_i^{a_i}\left([p_{i_L},p_{i_R}]-[C_{i_L},C_{i_R}]\right)\\ -\dfrac{[C_{3i_L},C_{3i_R}][C_{i_L},C_{i_R}]\,y_i^{a_i}}{Q_i}\\ -\dfrac{\left([C_{1i_L},C_{1i_R}]+[g_{i_L},g_{i_R}]\right)Q_i}{2}\\ -\left([C_{1i_L},C_{1i_R}]+[g_{i_L},g_{i_R}]+[C_{2i_L},C_{2i_R}]\right)S_i\\ +\dfrac{\left([C_{1i_L},C_{1i_R}]+[g_{i_L},g_{i_R}]\right)[C_{i_L},C_{i_R}]\,y_i^{a_i}Q_i}{2[P_{i_L},P_{i_R}]}\\ +\dfrac{\left(C_{1i}+[g_{i_L},g_{i_R}]\right)[C_{i_L},C_{i_R}]\,y_i^{a_i}S_i}{[P_{i_L},P_{i_R}]}\end{array}\right]$$
$$=[PF_L(S,Q,y),\ PF_R(S,Q,y)]$$

Subject to:

$$\sum_{i=1}^{n} w_i\,S_i\le W$$
$$\sum_{i=1}^{n}[C_{i_L},C_{i_R}]\,y_i\,Q_i\le\frac{M}{2}$$

Where

$$PF_L(S,Q,y)=\sum_{i=1}^{n}\left[\begin{array}{l}K_{1i_L}y_i^{a_i}-\dfrac{K_{2i_L}y_i^{a_i}}{Q_i}-K_{3i_L}Q_i-\\ K_{4i_L}S_i+K_{5i_L}y_i^{a_i}Q_i+K_{6i_L}y_i^{a_i}S_i\end{array}\right]$$

Where

$$K_{1i_L}=C_{i_L}(P_{i_L}-C_{i_R})\quad K_{2i_L}=C_{3i_R}C_{i_R}$$
$$K_{3i_L}=\frac{(C_{1i_R}+g_{i_R})}{2}\quad K_{4i_L}=(C_{1i_R}+g_{i_R}+C_{2i_R})$$
$$K_{5i_L}=\frac{(C_{1i_L}+g_{i_L})C_{i_L}}{2P_{i_R}}\quad K_{6i_L}=\frac{(C_{1i_L}+g_{i_L})C_{i_L}}{P_{i_R}}$$

$$PF_R(S,Q,y)=\sum_{i=1}^{n}\left[\begin{array}{l}K_{1i_R}y_i^{a_i}-\dfrac{K_{2i_R}y_i^{a_i}}{Q_i}-\\ K_{3i_R}Q_i-K_{4i_R}S_i+\\ K_{5i_R}y_i^{a_i}Q_i+K_{6i_R}y_i^{a_i}S_i\end{array}\right]$$

Where

$$K_{1i_R}=C_{i_R}(P_{i_R}-C_{i_L})\quad K_{2i_R}=C_{3i_L}C_{i_L}$$
$$K_{3i_R}=\frac{(C_{1i_L}+g_{i_L})}{2}$$
$$K_{4i_R}=(C_{1i_L}+g_{i_L}+C_{2i_L})$$
$$K_{5i_R}=\frac{(C_{1i_R}+g_{i_R})C_{i_R}}{2P_{i_L}}$$
$$K_{6i_R}=\frac{(C_{1i_R}+g_{i_R})C_{i_R}}{P_{i_L}}$$
$$PF_C(S,Q,y)=\left[\frac{PF_L+PF_R}{2}\right]$$

6. Cases of proposed Inventory Model With Pentagonal Fuzzy Number:

Case 1: All the cost parameters are fuzzified and the total display shelf-space parameter and miscellaneous amount are deterministic.

$$\max\ P\tilde{F}(S,Q,y)=\sum_{i=1}^{n}\left[\begin{array}{l}\tilde{C}_i\,y_i^{a_i}\left(\tilde{p}_i-\tilde{C}_i\right)-\\ \dfrac{\tilde{C}_{3i}\,\tilde{C}_i\,y_i^{a_i}}{Q_i}\\ -\dfrac{(\tilde{C}_{1i}+\tilde{g}_i)Q_i}{2}-\\ (\tilde{C}_{1i}+\tilde{g}_i+\tilde{C}_{2i})S_i\\ +\dfrac{(\tilde{C}_{1i}+\tilde{g}_i)\tilde{C}_i\,y_i^{a_i}Q_i}{2\tilde{P}_i}+\\ \dfrac{(\tilde{C}_{1i}+\tilde{g}_i)\tilde{C}_i\,y_i^{a_i}S_i}{\tilde{P}_i}\end{array}\right]\quad ----(6)$$

Subject to:

$$\sum_{i=1}^{n} w_i\,S_i\le W$$
$$\sum_{i=1}^{n} C_i\,y_i\,Q_i\le\frac{M}{2}$$

Using the nearest interval approximation, the above model is defuzzified as follows

$$\max\ PF(S,Q,y)=[PF_L,PF_R]$$

Subject to:

$$\sum_{i=1}^{n} w_i\,S_i\le W$$
$$\sum_{i=1}^{n} C_i\,y_i\,Q_i\le\frac{M}{2}$$

The model is converted into a multi-objective non-linear programming problem

$$\max\ PF_L(S,Q,y)=\sum_{i=1}^{n}\left[\begin{array}{l}K_{1i_L}y_i^{a_i}-\dfrac{K_{2i_L}y_i^{a_i}}{Q_i}-K_{3i_L}Q_i\\ -K_{4i_L}S_i+K_{5i_L}y_i^{a_i}Q_i+K_{6i_L}y_i^{a_i}S_i\end{array}\right]$$

----(7)

$$\max\ PF_C(S,Q,y)=\sum_{i=1}^{n}\left[\begin{array}{l}K_{1i_C}y_i^{a_i}-\dfrac{K_{2i_C}y_i^{a_i}}{Q_i}-K_{3i_C}Q_i-\\ K_{4i_C}S_i+K_{5i_C}y_i^{a_i}Q_i+K_{6i_C}y_i^{a_i}S_i\end{array}\right]$$

------(8)

Subject to:

$$\sum_{i=1}^{n} w_i\, S_i \le W$$

$$\sum_{i=1}^{n} C_i\, y_i\, Q_i \le \frac{M}{2}$$

The multi-objective inventory problem (7) is solved by the geometric programming technique and a pay-off matrix of order 2×2 is formed.

The standard geometric programming problem is

$$\max\ PF_L(S,Q,y) = \sum_{i=1}^{n}\left[\begin{array}{l} -K_{1i_L}\, y_i^{a_i} + \dfrac{K_{2i_L}\, y_i^{a_i}}{Q_i} \\ +K_{3i_L}\, Q_i + K_{4i_L}\, S_i \\ -K_{5i_L}\, y_i^{a_i}\, Q_i - K_{6i_L}\, y_i^{a_i}\, S_i \end{array}\right]$$

-------(9)

Subject to:

$$\sum_{i=1}^{n} w_i\, S_i \le W$$

$$\sum_{i=1}^{n} C_i\, y_i\, Q_i \le \frac{M}{2}$$

This primal problem (9) is a constrained signomial problem with 5n-1 degree of difficulty. The corresponding dual problem is

$\max\ D_L$

$$= -\prod_{i=1}^{n}\left[\begin{array}{l} \left(\dfrac{K_{1i_L}}{w_{1i}}\right)^{-w_{1i}} \left(\dfrac{K_{2i_L}}{w_{2i}}\right)^{w_{2i}} \left(\dfrac{K_{3i_L}}{w_{3i}}\right)^{w_{3i}} \\ \left(\dfrac{K_{4i_L}}{w_{4i}}\right)^{w_{4i}} \left(\dfrac{K_{5i_L}}{w_{5i}}\right)^{-w_{5i}} \left(\dfrac{K_{6i_L}}{w_{6i}}\right)^{-w_{6i}} \left(\dfrac{K_{7i_L}}{w_{7i}}\right)^{w_{7i}} \\ \left(\dfrac{K_{8i_L}}{w_{8i}}\right)^{w_{8i}} \left(\sum_{i=1}^{n} w_{7i}\right)^{\sum_{i=1}^{n} w_{7i}} \left(\sum_{i=1}^{n} w_{8i}\right)^{\sum_{i=1}^{n} w_{8i}} \end{array}\right]^{-1}$$

------- (10)

Subject to:

$$-w_{1i} + w_{2i} + w_{3i} + w_{4i} - w_{5i} - w_{6i} = -1$$

$$-w_{2i} + w_{3i} - w_{5i} + w_{8i} = 0$$

$$w_{4i} - w_{6i} + w_{7i} = 0$$

$$-a_i w_{1i} + a_i w_{2i} - a_i w_{5i} - a_i w_{6i} + w_{8i} = 0$$

By using geometric programming theorem [4], the analytical expression for the decision variables Q_i, y_i and S_i are obtained.

$$Q_i^* = \sqrt{\frac{K_{2i_L}\, w_{5i}}{K_{5i_L}\, w_{2i}}}$$

$$S_i^* = \frac{K_{5i_L}\, w_{6i}\, Q_i^*}{K_{6i_L}\, w_{5i}} \qquad \text{--------- (11)}$$

$$y_i^* = \frac{\left(K_{7i_L}\, w_{8i}\, S_i^*\right)}{\left(K_{8i_L}\, Q_i^* w_{7i}\right)}$$

Sub Q_i^* and S_i^* in $PF_L(S,Q,y)$ and $PF_C(S,Q,y)$ and the optimal values of $PF_L^{\,1}(S,Q,y)$ and $PF_C^{\,1}(S,Q,y)$ are obtained.

In a similar way, optimal values of $PF_C(S,Q,y)$ subject to the given constraint are obtained. The problem is then written to standard geometric programming problem as

$$\min\ PF_C(S,Q,y) = \sum_{i=1}^{n}\left[\begin{array}{l} -K_{1i_C}\, y_i^{a_i} + \dfrac{K_{2i_C}\, y_i^{a_i}}{Q_i} \\ +K_{3i_C}\, Q_i + K_{4i_C}\, S_i \\ -K_{5i_C}\, y_i^{a_i}\, Q_i - K_{6i_C}\, y_i^{a_i}\, S_i \end{array}\right] -$$

---------(12)

Subject to:

$$\sum_{i=1}^{n} w_i\, S_i \le W$$

$$\sum_{i=1}^{n} C_i\, y_i\, Q_i \le \frac{M}{2}$$

This primal problem (12) is a constrained signomial problem with 5n-1 degree of difficulty. The corresponding dual problem is

$\max\ D_L$

$$= -\prod_{i=1}^{n}\left[\begin{array}{l} \left(\dfrac{K_{1i_C}}{w_{1i}}\right)^{-w_{1i}} \left(\dfrac{K_{2i_C}}{w_{2i}}\right)^{w_{2i}} \left(\dfrac{K_{3i_C}}{w_{3i}}\right)^{w_{3i}} \left(\dfrac{K_{4i_C}}{w_{4i}}\right)^{w_{4i}} \\ \left(\dfrac{K_{5i_C}}{w_{5i}}\right)^{-w_{5i}} \left(\dfrac{K_{6i_C}}{w_{6i}}\right)^{-w_{6i}} \left(\dfrac{K_{7i_C}}{w_{7i}}\right)^{w_{7i}} \\ \left(\dfrac{K_{8i_C}}{w_{8i}}\right)^{w_{8i}} \left(\sum_{i=1}^{n} w_{7i}\right)^{\sum_{i=1}^{n} w_{7i}} \left(\sum_{i=1}^{n} w_{8i}\right)^{\sum_{i=1}^{n} w_{8i}} \end{array}\right]^{-1}$$

---------- (13)

Subject to:

$$-w_{1i} + w_{2i} + w_{3i} + w_{4i} - w_{5i} - w_{6i} = -1$$

$$-w_{2i} + w_{3i} - w_{5i} + w_{8i} = 0$$

$$w_{4i} - w_{6i} + w_{7i} = 0$$

$$-a_i w_{1i} + a_i w_{2i} - a_i w_{5i} - a_i w_{6i} + w_{8i} = 0$$

By using geometric programming theorem [4], the analytical expression for the decision variables Q_i,y_i and S_i are obtained.

$$Q_i^* = \sqrt{\frac{K_{2i_C}\, w_{5i}}{K_{5i_C}\, w_{2i}}}$$

$$S_i^* = \frac{K_{5i_C}\, w_{6i}\, Q_i^*}{K_{6i_C}\, w_{5i}} \qquad \text{---------- (14)}$$

$$y_i^* = \frac{\left(K_{7i_C}\, w_{8i}\, S_i^*\right)}{\left(K_{8i_C}\, Q_i^* w_{7i}\right)}$$

Sub Q_i^*, y_i^* and S_i^* in $PF_L(S,Q,y)$ and $PF_C(S,Q,y)$ and the optimal values of $PF_L^{\,2}(S,Q,y)$ and $PF_C^{\,2}(S,Q,y)$ are obtained.

Using the optimal solutions, a pay off matrix of size 2x2 is formed $\begin{bmatrix} PF_L^{\,1} & PF_C^{\,1} \\ PF_L^{\,2} & PF_C^{\,2} \end{bmatrix}$

From the pay off matrix, lower bounds are

$$L_L = \min[PF_L^{\,1}, PF_L^{\,2}]$$

$$L_C = \min[PF_C^{\,1}, PF_C^{\,2}]$$

and the upper bounds are

$$U_L = \max[PF_L^{\,1}, PF_L^{\,2}]$$

$$U_C = \max[PF_C^{\,1}, PF_C^{\,2}]$$

Now solve the problem (6) by fuzzy geometric programming technique. The linear membership functions are taken as follows.

$$\mu_{PF_L}(S,Q) = \begin{cases} 1 & if \quad PF_L(S,Q,y) \le U_L \\ 1 - \dfrac{U_L - PF_L}{U_L - L_L} & if \quad L_L \le PF_L \le U_L \\ 0 & if \quad otherwise \end{cases}$$

$$\mu_{PF_C}(S,Q) = \begin{cases} 1 & if \quad PF_C(S,Q,y) \le U_C \\ 1 - \dfrac{U_C - PF_C}{U_C - L_C} & if \quad L_C \le PF_C \le U_C \\ 0 & if \quad otherwise \end{cases}$$

The problem (6) can be formulated as

$$\max\ V(S,Q,y) = [\,\mu_{PF_L}(S,Q,y)] + [\,\mu_{PF_C}(S,Q,y)]$$

That is,

$$\max\ V(S,Q,y) = \sum_{i=1}^{n} \begin{bmatrix} K_{1i_F}\, y_i^{a_i} - \dfrac{K_{2i_F}\, y_i^{a_i}}{Q_i} - K_{3i_F}\, Q_i - \\ K_{4i_F}\, S_i + K_{5i_F}\, y_i^{a_i}\, Q_i + K_{6i_F}\, y_i^{a_i}\, S_i \end{bmatrix}$$

Subject to:

$$\sum_{i=1}^{n} K_{7i}\, S_i \le 1$$

$$\sum_{i=1}^{n} K_{8i}\, y_i\, Q_i \le 1$$

Where

$$K_{1i_F} = \frac{C_{i_L}(P_{i_L} - C_{i_R})}{(U_L - L_L)} + \frac{C_{i_C}(P_{i_C} - C_{i_C})}{(U_C - L_C)}$$

$$K_{2i_F} = \frac{C_{i_L}\, C_{3i_L}}{(U_L - L_L)} + \frac{C_{i_C}\, C_{3i_C}}{(U_C - L_C)}$$

$$K_{3i_F} = \frac{C_{1i_R} + g_{i_R}}{2(U_L - L_L)} + \frac{C_{1i_C} + g_{i_C}}{2(U_C - L_C)}$$

$$K_{4i_F} = \frac{C_{1i_R} + g_{i_R} + C_{2i_R}}{(U_L - L_L)} + \frac{C_{1i_C} + g_{i_C} + C_{2i_C}}{(U_C - L_C)}$$

$$K_{5i_F} = \frac{(C_{1i_L} + g_{i_L})C_{i_L}}{2P_{i_R}(U_L - L_L)} + \frac{(C_{1i_C} + g_{i_C})C_{i_C}}{2P_{i_C}(U_C - L_C)}$$

$$K_{6i_F} = \frac{(C_{1i_L} + g_{i_L})C_{i_L}}{(U_L - L_L)} + \frac{(C_{1i_C} + g_{i_C})C_{i_C}}{(U_C - L_C)}$$

The standard geometric programming problem is

$$\min\ V(S,Q,y) = \sum_{i=1}^{n} \begin{bmatrix} -K_{1i_F}\, y_i^{a_i} + \dfrac{K_{2i_F}\, y_i^{a_i}}{Q_i} + K_{3i_F}\, Q_i \\ +K_{4i_F}\, S_i - K_{5i_F}\, y_i^{a_i}\, Q_i - K_{6i_F}\, y_i^{a_i}\, S_i \end{bmatrix}$$

--------(15)

Subject to:

$$\sum_{i=1}^{n} K_{7i}\, S_i \le 1$$

$$\sum_{i=1}^{n} K_{8i}\, y_i\, Q_i \le 1$$

This primal problem (15) is a constrained signomial problem with 5n-1 degree of difficulty. The corresponding dual problem is

$$\max\ V_L = -\prod_{i=1}^{n} \begin{bmatrix} \left(\dfrac{K_{1i_F}}{w_{1i}}\right)^{-w_{1i}} \left(\dfrac{K_{2i_F}}{w_{2i}}\right)^{w_{2i}} \left(\dfrac{K_{3i_F}}{w_{3i}}\right)^{w_{3i}} \\ \left(\dfrac{K_{4i_F}}{w_{4i}}\right)^{w_{4i}} \left(\dfrac{K_{5i_F}}{w_{5i}}\right)^{-w_{5i}} \left(\dfrac{K_{6i_F}}{w_{6i}}\right)^{-w_{6i}} \\ \left(\dfrac{K_{7i_F}}{w_{7i}}\right)^{w_{7i}} \left(\dfrac{K_{8i}}{w_{8i}}\right)^{w_{8i}} \left(\sum_{i=1}^{n} w_{7i}\right)^{\sum_{i=1}^{n} w_{7i}} \\ \left(\sum_{i=1}^{n} w_{8i}\right)^{\sum_{i=1}^{n} w_{8i}} \end{bmatrix}^{-1}$$

---------- (16)

Subject to:

$$-w_{1i}+w_{2i}+w_{3i}+w_{4i}-w_{5i}-w_{6i}=-1$$
$$-w_{2i}+w_{3i}-w_{5i}+w_{8i}=0$$
$$w_{4i}-w_{6i}+w_{7i}=0$$
$$-a_i w_{1i}+a_i w_{2i}-a_i w_{5i}-a_i w_{6i}+w_{8i}=0$$

By using geometric programming theorem [4], the analytical expression for the decision variables Q_i, y_i and S_i are obtained.

$$Q_i^*=\sqrt{\frac{K_{2i_F}\,w_{5i}}{K_{5i_F}\,w_{2i}}}$$
$$S_i^*=\frac{K_{5i_F}\,w_{6i}\,Q_i^*}{K_{6i_F}\,w_{5i}} \qquad \text{--------- (17)}$$
$$y_i^*=\frac{\left(K_{7i}\,w_{8i}\,S_i^*\right)}{\left(K_{8i}\,Q_i^* w_{7i}\right)}$$

Case 2:

The cost parameters are deterministic, W and M are pentagonal fuzzy number, then the problem is

$$\max\ PF(S,Q,y)=\sum_{i=1}^{n}\left[\begin{array}{l} C_i y_i^{a_i}\left(p_i-C_i\right)-\dfrac{C_{3i}C_i y_i^{a_i}}{Q_i}-\dfrac{(C_{1i}+g_i)Q_i}{2} \\ -(C_{1i}+g_i+C_{2i})S_i+\dfrac{(C_{1i}+g_i)C_i y_i^{a_i}Q_i}{2P_i}+ \\ \dfrac{(C_{1i}+g_i)C_i y_i^{a_i}S_i}{P_i}\end{array}\right]$$

Subject to:

$$\sum_{i=1}^{n} w_i S_i \le \tilde{W}$$
$$\sum_{i=1}^{n} C_i\, y_i\, Q_i \le \frac{\tilde{M}}{2}$$

Using the Nearest Interval Approximation, the above model is defuzzified

$$\max\ PF(S,Q,y)=\sum_{i=1}^{n}\left[\begin{array}{l} K_{1i}y_i^{a_i}-\dfrac{K_{2i}y_i^{a_i}}{Q_i}-K_{3i}Q_i \\ -K_{4i}S_i+K_{5i}y_i^{a_i}Q_i+K_{6i}y_i^{a_i}S_i\end{array}\right] \qquad \text{------- (18)}$$

Subject to:

$$\sum_{i=1}^{n} K_{7i_C}\,S_i \le 1$$
$$\sum_{i=1}^{n} K_{8i_C}\,y_i\,Q_i \le 1$$

Where

$$K_{1i}=C_i(P_i-C_i)\ K_{2i}=C_{3i}C_i\ K_{3i}=\frac{(C_{1i}+g_i)}{2}$$
$$K_{4i}=(C_{1i}+g_i+C_{2i})$$
$$K_{5i}=\frac{(C_{1i}+g_i)C_i}{2P_i}\ K_{6i}=\frac{(C_{1i}+g_i)C_i}{P_i}$$
$$K_{7i_C}=\frac{w_i}{W_C}\ K_{8i_C}=\frac{2C_i}{M_C}$$

The standard geometric programming is

$$\min\ PF(S,Q,y)=\sum_{i=1}^{n}\left[\begin{array}{l} -K_{1i}y_i^{a_i}+\dfrac{K_{2i}y_i^{a_i}}{Q_i}+K_{3i}Q_i+ \\ K_{4i}S_i-K_{5i}y_i^{a_i}Q_i-K_{6i}y_i^{a_i}S_i\end{array}\right] \qquad \text{------- (19)}$$

Subject to:

$$\sum_{i=1}^{n} K_{7i_C}\,S_i \le 1$$
$$\sum_{i=1}^{n} K_{8i_C}\,y_i\,Q_i \le 1$$

This primal problem (19) is a constrained signomial problem with 5n-1 degree of difficulty. The corresponding dual problem is

$$\max\ D_L=-\prod_{i=1}^{n}\left[\begin{array}{l} \left(\dfrac{K_{1i}}{w_{1i}}\right)^{-w_{1i}}\left(\dfrac{K_{2i}}{w_{2i}}\right)^{w_{2i}}\left(\dfrac{K_{3i}}{w_{3i}}\right)^{w_{3i}} \\ \left(\dfrac{K_{4i}}{w_{4i}}\right)^{w_{4i}}\left(\dfrac{K_{5i}}{w_{5i}}\right)^{-w_{5i}}\left(\dfrac{K_{6i}}{w_{6i}}\right)^{-w_{6i}} \\ \left(\dfrac{K_{7i}}{w_{7i}}\right)^{w_{7i}}\left(\dfrac{K_{8i}}{w_{8i}}\right)^{w_{8i}}\left(\sum_{i=1}^{n}w_{7i}\right)^{\sum_{i=1}^{n}w_{7i}} \\ \left(\sum_{i=1}^{n}w_{8i}\right)^{\sum_{i=1}^{n}w_{8i}}\end{array}\right]^{-1} \qquad \text{---------- (20)}$$

Subject to:

$$-w_{1i}+w_{2i}+w_{3i}+w_{4i}-w_{5i}-w_{6i}=-1$$
$$-w_{2i}+w_{3i}-w_{5i}+w_{8i}=0$$
$$w_{4i}-w_{6i}+w_{7i}=0$$
$$-a_i w_{1i}+a_i w_{2i}-a_i w_{5i}-a_i w_{6i}+w_{8i}=0$$

By using geometric programming theorem [4], the analytical expression for the decision variables Q_i,y_i and S_i are obtained.

$$Q_i^* = \sqrt{\frac{K_{2i}\, w_{5i}}{K_{5i}\, w_{2i}}}$$

$$S_i^* = \frac{K_{5i}\, w_{6i}\, Q_i^*}{K_{6i}\, w_{5i}} \qquad \text{---------- (21)}$$

$$y_i^* = \frac{\left(K_{7i_C}\, w_{8i}\, S_i^*\right)}{\left(K_{8i_C}\, Q_i^* w_{7i}\right)}$$

Case 3:

All the parameters are consider as pentagonal fuzzy numbers,

$$\max\ P\tilde{F}(S,Q,y) = \sum_{i=1}^{n} \left[\begin{array}{l} \tilde{C}_i\, y_i^{a_i} \left(\tilde{p}_i - \tilde{C}_i\right) - \dfrac{\tilde{C}_{3i}\, \tilde{C}_i\, y_i^{a_i}}{Q_i} \\ - \dfrac{(\tilde{C}_{1i} + \tilde{g}_i)Q_i}{2} - \\ (\tilde{C}_{1i} + \tilde{g}_i + \tilde{C}_{2i})S_i \\ + \dfrac{(\tilde{C}_{1i} + \tilde{g}_i)\tilde{C}_i\, y_i^{a_i} Q_i}{2\tilde{P}_i} + \\ \dfrac{(\tilde{C}_{1i} + \tilde{g}_i)\tilde{C}_i\, y_i^{a_i} S_i}{\tilde{P}_i} \end{array} \right]$$

----- (22)

Subject to:

$$\sum_{i=1}^{n} w_i\, S_i \le \widetilde{W}$$

$$\sum_{i=1}^{n} C_i\, y_i\, Q_i \le \frac{\widetilde{M}}{2}$$

Using the nearest interval approximation, the above model is defuzzified as follows

$$\max\ PF(S,Q,y) = [PF_L, PF_R]$$

Subject to:

$$\sum_{i=1}^{n} w_i\, S_i \le W_C$$

$$\sum_{i=1}^{n} C_i\, y_i\, Q_i \le \frac{M_C}{2}$$

The model is converted into a multi-objective non-linear programming problem

$$\max\ PF_L(S,Q,y) = \sum_{i=1}^{n} \left[\begin{array}{l} K_{1i_L} y_i^{a_i} - \dfrac{K_{2i_L} y_i^{a_i}}{Q_i} - K_{3i_L}\, Q_i \\ -K_{4i_L} S_i + K_{5i_L} y_i^{a_i}\, Q_i + K_{6i_L} y_i^{a_i}\, S_i \end{array} \right]$$

-------(23)

$$\max\ PF_C(S,Q,y) = \sum_{i=1}^{n} \left[\begin{array}{l} K_{1i_C} y_i^{a_i} - \dfrac{K_{2i_C} y_i^{a_i}}{Q_i} - K_{3i_C}\, Q_i - \\ K_{4i_C} S_i + K_{5i_C} y_i^{a_i}\, Q_i + K_{6i_C} y_i^{a_i}\, S_i \end{array} \right]$$

--------(24)

Subject to:

$$\sum_{i=1}^{n} K_{7i_C}\ S_i \le 1$$

$$\sum_{i=1}^{n} K_{8i_C}\ y_i\, Q_i \le 1$$

By using the same procedure as in case 1and 2, multi-objective inventory problem is solved, and the analytical expression for the decision variables Q_i,y_i and S_i are obtained.

$$Q_i^* = \sqrt{\frac{K_{2i_F}\ w_{5i}}{K_{5i_F}\ w_{2i}}}$$

$$S_i^* = \frac{K_{5i_F}\ w_{6i}\, Q_i^*}{K_{6i_F}\ w_{5i}} \qquad \text{--------- (25)}$$

$$y_i^* = \frac{\left(K_{7i_C}\ w_{8i}\, S_i^*\right)}{\left(K_{8i_C}\, Q_i^* w_{7i}\right)}$$

7. Numerical Example:

Assuming an apparel showroom they sell 3 items. The shop has a total available storage space of 7500 m^2. The relevant data for the three items is given below:

C_1 = Rs.250 P_1 = 500 units p_1 = Rs.450 C_{31} = Rs.70 C_{11} = Rs.0.5 C_{21} = Rs.3 g_1 = Rs.1 ω_1 = 5m^2 a_1 = 0.5 M = Rs.50000 C_2 = Rs.150 P_2 = 400 units p_2 = Rs.300 C_{32} = Rs.50 C_{12} = Rs.2.5 C_{22} = Rs.2 g_1 = Rs.0.5 ω_2 = 1m^2 a_1 = 0.6 C_3 = Rs.300 P_3 = 500 units p_3 = Rs.600 C_{33} = Rs.100 C_{13} = Rs.10 C_{23} = Rs.7 g_3 = Rs.5 ω_3 = 7m^2 a_3 = 0.9

$$\widetilde{C}_1 = (250, 300, 350, 400, 450)$$

$$\widetilde{C}_{11} = (0.5, 0.7, 0.9, 1.1, 1.3)\ \widetilde{C}_{21} = (3, 4, 5, 6, 7)$$

$$\widetilde{C}_{31} = (70, 71, 72, 73, 74)$$

$$\widetilde{P}_1 = (300, 400, 500, 600, 700)$$

$$\widetilde{p}_1 = (450, 460, 470, 480, 490)\ \widetilde{g}_1 = (1, 2, 3, 4, 5)$$

$$\widetilde{C}_2 = (150, 170, 190, 210, 230)$$

$$\widetilde{C}_{12} = (2.5, 2.6, 2.7, 2.8, 2.9)$$

$$\widetilde{C}_{22} = (2, 2.5, 3, 3.5, 4)$$

$$\widetilde{C}_{32} = (50, 55, 60, 65, 70)$$

$$\widetilde{P}_2 = (200, 300, 400, 500, 600)$$

$$\widetilde{p}_2 = (300, 400, 500, 600, 700)$$

$\tilde{g}_2 = (0.5, 0.6, 0.7, 0.8, 0.9)$

$\tilde{P}_3 = (500, 550, 560, 570, 580)$

$\tilde{C}_{13} = (10, 10.5, 11, 11.5, 12)$

$\tilde{C}_{23} = (7, 7.1, 7.2, 7.3, 7.4)$

$\tilde{C}_{33} = (100, 110, 120, 130, 140)$

$\tilde{C}_3 = (300, 310, 320, 330, 340)$

$\tilde{p}_3 = (600, 605, 610, 615, 620)$ $\tilde{g}_1 = (5, 6, 7, 8, 9)$

$\omega_A = 0.6$ $\tilde{W} = (6000, 6500, 7000, 7500, 8000)$

$\tilde{M} = (200000, 210000, 220000, 230000, 240000)$

Using the analytical expression (5),(17),(21) and (26) for Q_i^*, S_i^* and y_i^* in crisp and fuzzy environment, the following results are obtained.

Table 1: Left and Right Branches of Fuzzy Parameters

BR	$\tilde{C}_1$	$\tilde{C}_{11}$	$\tilde{C}_{21}$	$\tilde{C}_{31}$	$\tilde{p}_1$	$\tilde{P}_1$	$\tilde{g}_1$	$\tilde{W}$	$\tilde{M}$	$\tilde{C}_2$	$\tilde{C}_{12}$	$\tilde{C}_{22}$	$\tilde{C}_{32}$	$\tilde{p}_2$	$\tilde{P}_2$	$\tilde{g}_2$
Left	H	P	L	P	L	H	L	P	H	L	H	P	L	H	P	H
Right	P	L	H	H	P	L	L	P	H	P	L	H	L	P	H	L

BR	$\tilde{C}_3$	$\tilde{C}_{13}$	$\tilde{C}_{23}$	$\tilde{C}_{33}$	$\tilde{p}_3$	$\tilde{P}_3$	$\tilde{g}_3$
Left	P	L	H	H	P	L	P
Right	H	P	L	P	L	H	P

Here P, L and H stands for Parabolic, Linear and hyperbolic pentagonal fuzzy membership function respectively.

Table 2: Nearest interval approximation to pentagonalfuzzy numbers for item 1, 2 and 3

BR	$\tilde{C}_1$	$\tilde{C}_{11}$	$\tilde{C}_{21}$	$\tilde{C}_{31}$	$\tilde{p}_1$	$\tilde{P}_1$	$\tilde{g}_1$	$\tilde{W}$
Left	287.2426	7.133	3.99	71.0667	459	375.1031	1.9	6533.3
Right	396.6667	10.8667	6.2775	73.2676	479.3333	610	4.1	7466.7
Centre	341.9546	8.99	5.13	72.16715	469.1665	492.55155	3	7000
BR	$\tilde{C}_2$	$\tilde{C}_{12}$	$\tilde{C}_{22}$	$\tilde{C}_{32}$	$\tilde{p}_2$	$\tilde{P}_2$	$\tilde{g}_2$	$\tilde{M}$
Left	168	2.5736	2.533	54.5	375.1031	306.6667	0.5745	207370
Right	208.6667	2.5736	2.5333	54.5	375.1031	306.6667	0.5745	232700
Centre	188.3333	2.6918	3.0856	60	484.2182	417.3175	30.692	220035
BR	$\tilde{C}_3$	$\tilde{C}_{13}$	$\tilde{C}_{23}$	$\tilde{C}_{33}$	$\tilde{p}_3$	$\tilde{P}_3$	$\tilde{g}_3$	
Left	310.6667	10.45	7.0734	107.3950	605.3333	537	6.0667	
Right	332.6868	11.4667	7.31	129.3333	614.6667	572.6783	7.9333	
Centre	321.6767	10.9583	7.1917	118.3641	610	554.83915	7	

Table 3: Optimal Solutions

Cases	i	S_i^* (Units)	Q_i^* (Units)	y_i^* (Percentage)	Average PF (S^*, Q^*, y^*) (Rupees)	Profit for per unit
Crisp	1	23.1134	53.8593	49.69	96973	440.78
	2	25.9832	43.4836	47.52		
	3	25.2304	49.0575	51.01		
Case 1	1	31.3843	45.1483	59.39	[97764,145940 ,208480]	547.55
	2	52.0937	56.0041	76.5		
	3	38.1407	45.5906	73.16		
Case 2	1	46.1037	45.9735	52.54	110380	511.0185
	2	27.76	36.9553	50.73		
	3	24.0172	34.7409	65.22		
Case 3	1	49.2089	36.8698	59.68	[97383, 146300, 205250]	550
	2	41.0850	53.6301	72.92		
	3	47.5906	38.1407	75.16		

Observation: In Table-3, the optimal values are given for the fuzzy models as well as the crisp model, from the same, the following are observed.

1. In case 1, the optimal value of the average profit is more than that of crisp model.
2. In case 2, the optimal value of the average profit is less compared to that of Cases 1 and 3.
3. In case 3, the optimal value of the average profit is more compared to that of cases 1,2 and crisp model.
4. Among the above three cases, Case 3 gives the best optimal solution.

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