DIOPHANTINE EQUATIONS $\prod_i(n_i!+p)=m^2$, $\sum_i n_i!=m^2$ AND $\prod_i(n_i!+p)=m^3$

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Abstract: In this paper, the Diophantine equations $\prod_i(n_i!+p)=m^2$, $\sum_i n_i!=m^2$ and $\prod_i(n_i!+p)=m^3$ have been discussed for n_i and m with positive integral values and 1 or prime values of p. Attempt has been made to obtain some solutions of these Diophantine equations.

Key-words: Diophantine equation, prime number and integral value.

Introduction: Brocard (1886, 1885) presented the Diophantine equation $n!+1=m^2$ for its positive integral solutions. Unaware of Brocard's query, Ramanujan (1913) formulated the problem in the form, "The number 1 + n! is a perfect square for the values 4, 5, 7 of n: Find other values." Geradin (1906) claimed that if m>71 then it must have at least 20 digits. Gupta (1935) stated that calculations of n! upto n = 63 gave no further solutions. Overholt (1993) proved that the Diophantine equation $n!+1=m^2$ has finitely many solutions if the weak form of Szpiro's conjecture is true, but this remains unproved. Dabrowski easily showed that, for each fixed k that is not a square, there is only a finite number of solutions of the Diophantine equation $n!+k=m^2$. Hari Kishan, Megha Rani and Smiti Aggarwal (1913) discussed the Diophantine Equation $n!+p=m^2$ and $n!+p=m^3$. They obtained the solution for $p=2,3,5,7,23,43,47,79,163$ and 323. They also obtained two interesting results $2!+2=2^2$ and $3!+3=3^2$ which are of the form $n!+n=n^2$ and one more interesting result $5!+5=5^3$ which is of the form $n!+n=n^3$.

In this paper, we have discussed the Diophantine equations $\prod_i(n_i!+p)=m^2$, $\sum_i n_i!=m^2$ and $\prod_i(n_i!+p)=m^3$ for different values p and obtained positive integral solutions.

Solution of Diophantine Equation $\prod_i(n_i!+1)=m^2$: The solutions of the Diophantine equation $\prod_i(n_i!+1)=m^2$ are givenbelow:

(i) We have $4!+1=5^2$. Thus (4,5) is the solution of $(n_i!+1)=m^2$.

(ii) We have $5!+1=11^2$. Thus (5,11) is the solution of $(n_i!+1)=m^2$.

(iii) We have $7!+1=71^2$. Thus (7,71) is the solution of $(n_i!+1)=m^2$.

(iv) We have $(4!+1)(5!+1)=55^2$. Thus (4,5,55) is the solution of $\prod_{i=1}^2(n_i!+1)=m^2$.

(v) We have $(5!+1)(7!+1)=781^2$. Thus (5,7,781) is the solution of $\prod_{i=1}^2(n_i!+1)=m^2$.

(vi) We have $(4!+1)(7!+1)=355^2$. Thus (4,7,355) is the solution of $\prod_{i=1}^2(n_i!+1)=m^2$.

(vii) We have $(4!+1)(5!+1)(7!+1)=3905^2$. Thus (4,5,7,3905) is the solution of $\prod_{i=1}^3(n_i!+1)=m^2$.

These are the only solutions of the Diophantine equation $\prod_i(n_i!+1)=m^2$.

Solution of Diophantine Equation $\prod_i(n_i!+3)=m^2$: The solutions of the Diophantine equation $\prod_i(n_i!+3)=m^2$ are givenbelow:

(i) We have $1!+3=2^2$. Thus (1,2) is the solution of $(n_i!+3)=m^2$.

(ii) We have $3!+3=3^2$. Thus (3,3) is the solution of $(n_i!+3)=m^2$.

(iii) We have $(1!+3)(3!+3)=6^2$. Thus (1,3,6) is the solution of $\prod_{i=1}^2(n_i!+3)=m^2$.

These are the solutions of the Diophantine equation $\prod_i(n_i!+3)=m^2$.

Solution of Diophantine Equation $\prod_i(n_i!+p_i)=m^2$: The solutions of the Diophantine equation $\prod_i(n_i!+p_i)=m^2$ are givenbelow:

(i) We have $(2!+2)(3!+3)=6^2$. Thus (2,2,3,3,6) is the solution of $\prod_{i=1}^2(n_i!+p_i)=m^2$.

(ii) We have $(3!+3)(2!+7)=9^2$. Thus (3,3,2,7,9) is the solution of $\prod_{i=1}^2(n_i!+p_i)=m^2$.

(iii) We have $(3!+19)(2!+23)=25^2$. Thus (3,19,2,23,25) is the solution of $\prod_{i=1}^2(n_i!+p_i)=m^2$.

(iv) We have $(3!+43)(2!+47)=49^2$. Thus (3,43,2,47,49) is the solution of $\prod_{i=1}^2(n_i!+p_i)=m^2$.

(v) We have $(3!+3)(2!+7)(3!+19)=45^2$. Thus (3,3,2,7,3,19,45) is the solution of $\prod_{i=1}^3(n_i!+p_i)=m^2$.

(vi) We have $(3!+3)(3!+19)(2!+23)=75^2$. Thus (3,3,3,19,2,23,75) is the solution of $\prod_{i=1}^3(n_i!+p_i)=m^2$.

(vii) We have $(3!+19)(2!+23)(3!+43)=175^2$. Thus (3,19,2,23,3,43,175) is the solution of $\prod_{i=1}^3(n_i!+p_i)=m^2$.

(viii) We have $(3!+19)(3!+43)(2!+47)=245^2$. Thus (3,19,3,43,2,47,245) is the solution of $\prod_{i=1}^3(n_i!+p_i)=m^2$.

Solution of Diophantine Equation $\sum_i n_i!=m^2$:

(i) We have $1!+2!+3!=3^2$. Thus (1,2,3,3) is the solution of $\sum_{i=1}^3 n_i!=m^2$. This is the only solution of this type.

Solution of Diophantine Equation $\prod_i(n_i!+p_i)=m^3$:

(i) We have $(3!+2)(4!+3)=6^3$. Thus (3,2,4,3,6) is the solution of $\prod_{i=1}^{2}(n_i!+p_i)=m^3$.
(ii) We have $(3!+2)(5!+5)=10^3$. Thus (3,2,5,5,10) is the solution of $\prod_{i=1}^{2}(n_i!+p_i)=m^3$.
(iii) We have $(3!+2)(1!+7)=4^3$. Thus (3,2,1,7,4) is the solution of $\prod_{i=1}^{2}(n_i!+p_i)=m^3$.
(iv) We have $(4!+3)(5!+5)=15^3$. Thus (4,3,5,5,15) is the solution of $\prod_{i=1}^{2}(n_i!+p_i)=m^3$.
(v) We have $(4!+3)(1!+7)=6^3$. Thus (4,3,1,7,6) is the solution of $\prod_{i=1}^{2}(n_i!+p_i)=m^3$.
(vi) We have $(5!+5)(1!+7)=10^3$. Thus (5,51,7,10) is the solution of $\prod_{i=1}^{2}(n_i!+p_i)=m^3$.
(vi) We have $(3!+2)(4!+3)(5!+5)=6^3$. Thus (3,2,4,3,5,5,30) is the solution of $\prod_{i=1}^{3}(n_i!+p_i)=m^3$.
(vii) We have $(4!+3)(5!+5)(1!+7)=30^3$. Thus (4,3,5,5,1,7,30) is the solution of $\prod_{i=1}^{3}(n_i!+p_i)=m^3$.

Concluding Remarks: Positive integral solutions of the Diophantine equations $\prod_i(n_i!+p)=m^2$, $\sum_i n_i!=m^2$ and $\prod_i(n_i!+p)=m^3$ have been obtained for p=2, 5, 7, 23 etc.

References:

1. Rajni Bhargav, Asymptotically Quasi- I-Nonexpansive Mapping; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 3 Issue 1 (2014), Pg 499-503
2. Brocard, H. (1876):*Question 166*, Nouv. Corresp. Math. 2, 287.
3. Brocard, H. (1885):*Question 1532*, Nouv. Ann. Math. 4, 391.
4. K. Kavitha , N.G. David, Dominator Coloring of Vertex Switching of Graphs; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 3 Issue 1 (2014), Pg 494-498
5. G´erardin, A. (1906):Contribution a l'´etude de l'´equation 1 ¢ 2 ¢ 3 ¢ 4 ¢ ¢ ¢ z + 1 = y2,
6. Hari Kishan, Megha Rani and Smiti Aggarwal(1913): On the Diophantine Equation $n!+p=m^2$ and $n!+p=m^3$. Inter, Tran. Appl. Sci. Vol. 5(3), 2013, 363-366.
7. J.R.V.Edward, P. Jaya Mary, on Gaps Between Subspaces of A Banach Space; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 3 Issue 2 (2014), Pg 842-844
8. Gupta, H. (1935):On a Brocard–Ramanujan problem. Math. Student, 3 (1935), 71.
9. Sharayu Sathe, Madhukar Kasture, Study of Magnetic Susceptibility,Geomagnetism of the Soil; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 3 Issue 2 (2014), Pg 838-841
10. Overholt, M. (1993):The diophantine equation n! + 1 = m2, Bull. London Math. Soc. 25 (1993), 104.
11. *Preethy Prabhakar, tony Thomas,* A Retrospective Study on the Key Techniques of; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 2 Issue 2 (2013), Pg 484-489
12. Ramanujan, S. (1913):Question 469, J. Indian Math. *Soc.* 5, 59.

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