FUZZY RELIABILITY ANALYSIS OF REPAIRABLE SYSTEM USING FUZZY KOLMOGOROV'S DIFFERENTIAL EQUATIONS

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Abstract: Traditionally reliability studies assume that probabilities in Markov models are accurate. However in reality, data is either insufficient or contain uncertainty which violates this assumption. Keeping this in view the reliability of Butter oil processing plant is evaluated after developing the fuzzy differential equations for the system by using its Markov model. Fourth order Runge-Kutta method has been used to evaluate fuzzy reliability of the system.

Keywords: Fuzzy differential equation, Runge-Kutta method, Fuzzy numbers, Reliability.

Introduction: The majority of industrial systems are repairable and consist of various subsystems and each subsystem is composed of many components. The probability that the system performs in expected manner depends directly on the performance of each of its components. With the advancement in technology and growing complexity of systems, the study of their reliability and availability becomes more important. For measuring the performance of the system, many techniques such as event tree, fault tree analysis (FTA), petri nets (PNs), reliability block diagrams (RBDs) and Markovian approach etc. are available in the literature [1-3]. These different techniques address reliability predictions of a system in different scenarios. Out of these Markov process is considered to be the most comprehensive technique being used today. In various engineering problems, the binary assumption in reliability theory is not acceptable. In 1965, L.A. Zadeh presented the basic concepts of fuzzy set theory [4]. Thus binary state assumption in reliability theory is replaced by fuzzy state assumption. Kumar and Kumar [5] evaluated the fuzzy reliability of stainless steel utensil manufacturing unit of constant failure and repair rates. Kumar et al. [6] discussed fuzzy reliability analysis of dual-fuel steam turbine propulsion system in LNG carriers considering data uncertainty. Garg [7] proposed an approach for analyzing the reliability of industrial system using fuzzy Kolmogorov differential equations. Knezevic and Odoom [2] discussed reliability modelling of repairable systems using petri-nets and fuzzy lambda-tau methodology. As fuzzy numbers play a significant role in performing the operations on fuzzy observations and lead the corresponding problem to solving the fuzzy differential equations. For the evaluation of reliability, the system is mathematically modelled in terms of the differential equations from its transition diagram having uncertainties in the involved parameters. These equations are then converted into fuzzy differential equations for handling the uncertainties.

Gupta et al. [8] used crisp Markov model to evaluate the reliability of Butter oil processing plant. In this paper fuzzy differential equations have been derived with the help of the existing crisp Markov model of Butter oil processing plant. These differential equations are then solved to find the probability corresponding to each α-cut with the help of Runge-Kutta fourth order method.

2. Proposed Approach: Buckley and Feuring [9]-[10] discussed fuzzy differential equations and also introduced the methods for solving such differential equations of nth order. The general nth-order fuzzy linear differential equation is represented

$$\tilde{a}_n\tilde{y}^{(n)} \oplus \tilde{a}_{n-1}\tilde{y}^{(n-1)} \oplus \ldots \oplus \tilde{a}_1\tilde{y}^{(1)} \oplus \tilde{a}_0\tilde{y} \quad = g(x)$$

with $\tilde{y}^{(j)}(0) = \tilde{\eta}_j,\ j = 0,1,\ldots,n-1$

where, $\tilde{y}^{(j)} = \frac{d^j\tilde{y}}{dx^j}$ for j = 1, 2,..., n and $\tilde{a}_j$ are fuzzy numbers while ⊕ represents fuzzy addition. A stepwise method of solution of fuzzy differential equations adopted here, is described below.

Step1: Find the α-cuts corresponding to each fuzzy parameters.

Step2: Based on these α-cuts, fuzzy differential equation is converted into the nth order differential equation:

$$\sum_{j=0}^{n}[\, a_{j(L)}(x,\alpha), a_{j(R)}(x,\alpha)]\left[y_{(L)}^{(j)}(x,\alpha), y_{(R)}^{(j)}(x,\alpha)\right] = [g(x), g(x)]$$

With $\left[y_{(L)}^{(j)}(0,\alpha), y_{(R)}^{(j)}(0,\alpha)\right] = \left[\eta_{j(L)}(0,\alpha), \eta_{j(R)}(0,\alpha)\right]$

Step3: Using the addition and multiplication of the fuzzy numbers, the above nth-order fuzzy differential equationis converted into the following differential equations.

$$\sum_{j=0}^{n} c_j\, y^{(j)} = g(x),\ y_{(L)}^{(j)}(0,\alpha) = \eta_{j(L)}(0,\alpha)$$

and $\sum_{j=0}^{n} d_j\, y^{(j)} = g(x),\ y_{(R)}^{(j)}(0,\alpha) = \eta_{j(R)}(0,\alpha)$

where, $c_j y^{(j)} = \min\,(a_{j(L)}(x,\alpha)y_{(L)}^{(j)}(x,\alpha),\ a_{j(L)}(x,\alpha)y_{(R)}^{(j)}(x,\alpha), a_{j(R)}(x,\alpha)y_{(L)}^{(j)}(x,\alpha),\ a_{j(R)}(x,\alpha)y_{(R)}^{(j)}(x,\alpha))$

And $d_j y^{(j)} = \max\,(a_{j(L)}(x,\alpha)y_{(L)}^{(j)}(x,\alpha),$

$a_{j(L)}(x,\alpha)y_{(R)}^{(j)}(x,\alpha), a_{j(R)}(x,\alpha)y_{(L)}^{(j)}(x,\alpha),$
$a_{j(R)}(x,\alpha)y_{(R)}^{(j)}(x,\alpha))$

Step4: Solve the above formulated ordinary differential equations by fourth order Runge-Kutta method to find the values of $y_{(L)}(x,\alpha)$ and $y_{(R)}(x,\alpha)$ corresponding to $x = x_0$. If $[y_{(L)}(x,\alpha),\ y_{(R)}(x,\alpha)]$ defines the α-cut of a fuzzy number then the fuzzy solution $\tilde{y}\,(x)$ of fuzzy differential equation exist.

3. Case Study: Gupta et al. [8] used Markov model with crisp parameters to calculate crisp reliability. In the present paper fuzzy parameters are used in place of crisp parameters. Markov model of Butter Oil Processing plant is shown in fig.1

3.1 System Description: A butter-oil manufacturing plant is a complex engineering system comprising of various subsystems. The main subsystems of plant are briefly described below [8]:

Separator (A): In this subsystem, motor, bearings and gearbox are connected in series. Failure of this subsystem causes the complete failure of the system.

Pasteuriser (B): Herein two units of pasteurisers are arranged in parallel configuration with one operative and other in cold standby. Complete failure of pasteuriser occurs when both the components fail.

Continuous Butter Making (C): The CBM consists of gearbox, motor and bearings in series. Failure of this subsystem causes the complete failure of the system.

Melting Vats (D): This system consists of monoblock, motors, pumps and bearings in series. Failure of this subsystem causes the complete failure of the system.

Butter-Oil Clarifier (E): The unit consists of gearbox and motor in series. Failure of this subsystem will cause the complete failure of the system.

Packaging (F): This subsystem consists of printed circuit board and pneumatic cylinder in series. Failure of this subsystem causes the complete failure of the system.

The failure and repair rates corresponding to each subsystem of the system are given as:

Failure rate (λ) =
[0.008 0.0054 0.0027 0.0009 0.0027 0.0055 0.01111]

Repair rate (μ)= [0.41 0.40 0.70 0.30 0.65 6.00]

As the data collected for evaluation of reliability contains uncertainty. So, to account for uncertainties and vagueness in data, the obtained crisp data are converted into fuzzy numbers. Crisp numbers in the given data are converted into fuzzy numbers having known spread suggested by decision makers/system analyst. An input data for fuzzy failure rate $(\tilde{\lambda}_i)$ and fuzzy repair rate $(\tilde{\mu}_i)$ for ith component of the system is in the form of triangular fuzzy numbers with ±15% in both the directions (left and right of the middle).

3.2 Assumptions:

1. Failure rates and repair rates are independent of each other and their unit is per day.
2. There is no simultaneous failure of the systems.
3. Subsystem B fails through reduced state only.
4. Repaired components function like new components and switchover devices used for standby systems are perfect.

3.3 Notations:

1. B1 indicates that subsystem B is working in reduced state.
2. $\lambda_i, i = 1,2,\dots,7$ represent the failure rates of the subsystems A, C, D, E, F, B1 and B respectively.
3. $\beta_i, i = 1,2,\dots,6$ represent the failure rates of the subsystems A, C, D, E, F and B respectively.
4. The symbols a, b, c, d, e and f represent the failed state of the subsystems A, B, C, D, E and F respectively.
5. $P_j(t),\ j = 1,2,\dots,13$ represent the probability that the system is in jth state at time t.

3.4 Transient Analysis: The Markov state transition diagram is helpful in analyzing the reliability of a repairable system. Using the concept of Markov modeling and various probability considerations, the transition diagram (fig. 1) of the Processing plant leads to the differential equations:

$$\frac{d\tilde{P}_1(t)}{dt} \oplus \tilde{\delta}_1\tilde{P}_1(t) = \sum_{j=1}^{5}\tilde{\mu}_j\tilde{P}_{j+2}(t) \oplus \tilde{\mu}_6\tilde{P}_{13}(t) \dots (1)$$

$$\frac{d\tilde{P}_2(t)}{dt} \oplus \tilde{\delta}_2\tilde{P}_2(t) = \sum_{j=1}^{5}\tilde{\mu}_j\tilde{P}_{j+7}(t) \oplus \tilde{\lambda}_6\tilde{P}_1(t) \quad \dots(2)$$

$$\frac{d\tilde{P}_{i+2}(t)}{dt} \oplus \tilde{\mu}_i\tilde{P}_{i+2}(t) = \tilde{\lambda}_i\tilde{P}_1(t) i = 1,2,\dots,5 \quad \dots(3)$$

$$\frac{d\tilde{P}_{i+7}(t)}{dt} \oplus \tilde{\mu}_i\tilde{P}_{i+7}(t) = \tilde{\lambda}_i\tilde{P}_2(t) i = 1,2,\dots,5 \quad \dots(4)$$

$$\frac{d\tilde{P}_{13}(t)}{dt} \oplus \tilde{\mu}_6\tilde{P}_{13}(t) = \tilde{\lambda}_7\tilde{P}_2(t) \quad \dots.(5)$$

with the assumed initial conditions:

$\tilde{P}_1(0) = (.94, .96, .98), \tilde{P}_2(0) = (0.004, 0.005, 0.006)$ and $\tilde{P}_j(0) = (0, 0, 0)$ for $j = 3\ to\ 13$.

Here $\tilde{\delta}_1 = \sum_{j=1}^{6}\tilde{\lambda}_j$ and $\tilde{\delta}_2 = \sum_{j=1}^{5}\tilde{\lambda}_j \oplus \tilde{\lambda}_7$

As it is difficult to find the analytic solution of the system of jumbled differential equations (1-5), we have applied Runge-Kutta fourth order method to solve this system of equations. The reliability function $\tilde{R}\,(t)$ of the system in terms of $\tilde{P}_1(t)$ and $\tilde{P}_2(t)$ can be obtained by

$$\tilde{R}\,(t) = \tilde{P}_1(t) \oplus \tilde{P}_2(t)$$

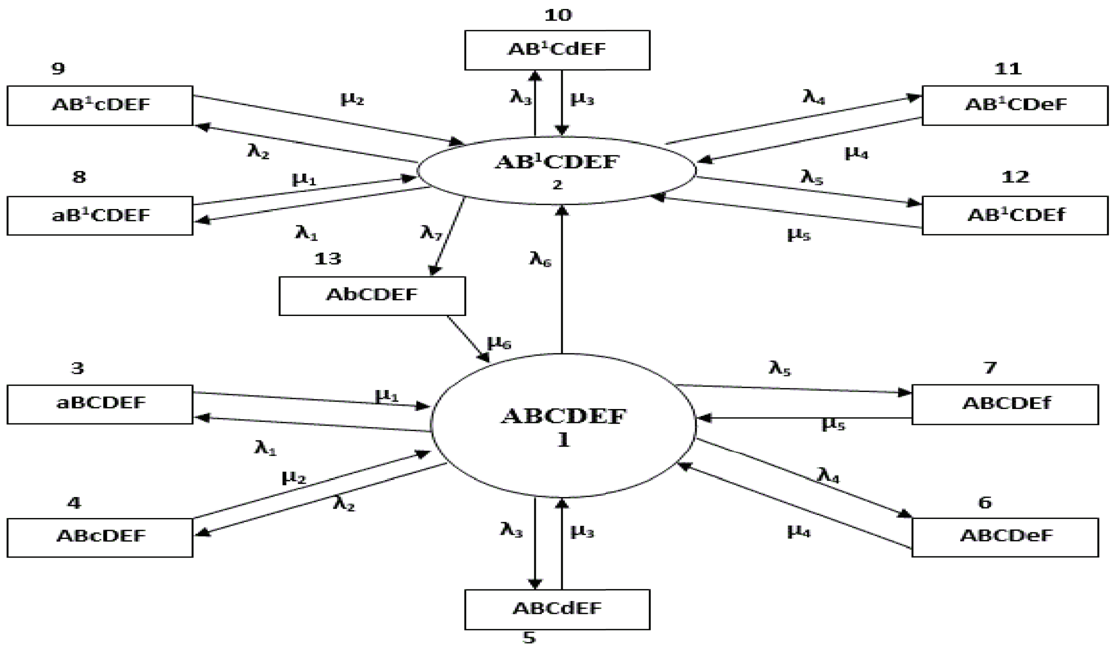

Fig. 1 Transition diagram of Butter oil processing plant

4. Results and Discussion: The solution of fuzzy differential equations of Butter-oil processing plant developed in section 3.4, is obtained by fourth order Runge-Kuttamethod for α = 0, 0.2, 0.4, 0.6, 0.8, 1 and t = 365 days. The solutions are shown in Table 1. Using fuzzy probabilities of Butter-oil processing plant, the fuzzy reliability of Butter-oil processingplant corresponding to different α-cut are computed and depicted in Table 2. The variation in reliability of the system corresponding to different presumption levels is shown with the help of figure and tables. It can be seen from the table that every fuzzy probability function $\tilde{P}_j(t)$ represents a fuzzy number. From the results, it has been concluded that the reliability of the whole system lies between 0.90366772 and 0.94387001. Fig. 2 shows the triangular fuzzy number representing fuzzy reliability of the system.

j	$\alpha = 0 \tilde{P}_{j(L)}(t,\alpha) \tilde{P}_{j(R)}(t,\alpha)$	$\alpha = 0.2$ $\tilde{P}_{j(L)}(t,\alpha) \tilde{P}_{j(R)}(t,\alpha)$	$\alpha = 0.4$ $\tilde{P}_{j(L)}(t,\alpha) \tilde{P}_{j(R)}(t,\alpha)$
1	0.60639449 0.63153144	0.60874866 0.62891536	0.61115509 0.62631268
2	0.29727322 0.31233857	0.29893900 0.31093424	0.30055262 0.30951653
3	0.01183377 0.01232287	0.01187944 0.01227188	0.01192617 0.01222117
4	0.00818751 0.00852589	0.00821911 0.00849062	0.00825144 0.00845553
5	0.00233914 0.00243594	0.00234819 0.00242586	0.00235745 0.00241583
6	0.00181954 0.00189466	0.00182655 0.00188682	0.00183373 0.00187903
7	0.00251909 0.00262332	0.00252884 0.00261246	0.00253880 0.00260166
8	0.00579878 0.00609410	0.00583154 0.00606665	0.00586325 0.00603892
9	0.00401200 0.00421635	0.00403467 0.00419735	0.00405661 0.00417817
10	0.00114643 0.00120470	0.00115289 0.00119928	0.00115914 0.00119380
11	0.00089146 0.00093695	0.00089652 0.00093272	0.00090140 0.00092846

12	0.00123461 0.00129737	0.00124156 0.00129152	0.00124829 0.00128563
13	0.00054994 0.00057782	0.00055303 0.00057523	0.00055601 0.00057260

j	$\alpha = 0.6 \widetilde{P}_{j(L)}(t,\alpha)\widetilde{P}_{j(R)}(t,\alpha)$	$\alpha = 0.8$ $\widetilde{P}_{j(L)}(t,\alpha)\widetilde{P}_{j(R)}(t,\alpha)$	$\alpha = 1$ $\widetilde{P}_{j(L)}(t,\alpha)\widetilde{P}_{j(R)}(t,\alpha)$
1	0.61360560 0.62372592	0.61609332 0.62115800	0.61861242 0.61861242
2	0.30212222 0.30808294	0.30365469 0.30663053	0.30515583 0.30515583
3	0.01197380 0.01217078	0.01202218 0.01212076	0.01207120 0.01207120
4	0.00828439 0.30808294	0.00831786 0.00838606	0.00835178 0.00835178
5	0.00236688 0.01217077	0.00237646 0.00239596	0.00238616 0.00238616
6	0.00184103 0.00187129	0.00184846 0.00186360	0.00185599 0.00185599
7	0.00254896 0.00259093	0.00255927 0.00258027	0.00256972 0.00256972
8	0.00589406 0.00601086	0.00592412 0.00598243	0.00595354 0.00595354
9	0.00407793 0.00415876	0.00409874 0.00413908	0.00411909 0.00411909
10	0.00116521 0.00118826	0.00117114 0.00118265	0.00117695 0.00117695
11	0.00090615 0.00092414	0.00091078 0.00091976	0.00091531 0.00091531
12	0.00125484 0.00127966	0.00126122 0.00127362	0.00126748 0.00126748
13	0.00055892 0.00056995	0.00056176 0.00056726	0.00056453 0.00056453

Table 1: Solution of fuzzy differential equations for the system for t = 365.

α	$R_1(t,\alpha)$	$R_2(t,\alpha)$
$\alpha = 0$	0.90366772	0.94387001
$\alpha = 0.2$	0.90768766	0.93984960
$\alpha = 0.4$	0.91170770	0.93582921
$\alpha = 0.6$	0.91572782	0.93180885
$\alpha = 0.8$	0.91974801	0.92778853
$\alpha = 1$	0.92376824	0.92376824

Table 2. Fuzzy reliability of the system for t = 365.

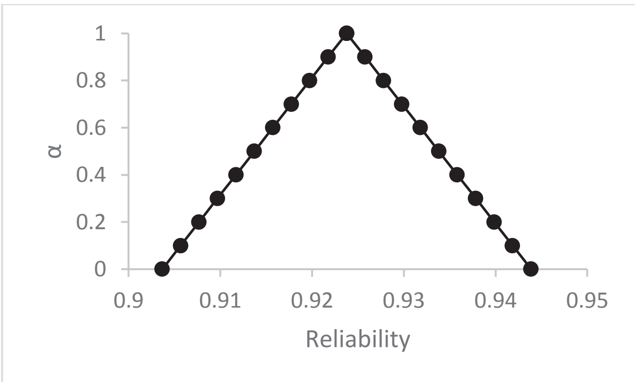

Fig 2. Triangular fuzzy number representing fuzzy reliability of the system for t = 365 corresponding to different α-cuts

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