

SLIGHTLY GENERALIED STAR $\omega\alpha$ -CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

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Abstract: The purpose of this paper is to introduce the concept of slightly generalized star $\omega\alpha$ -continuous (briefly slightly $g^*\omega\alpha$ -continuous) functions in topological spaces. Further, the basic properties and preservation theorems of slightly $g^*\omega\alpha$ -continuous functions are studied.

Keywords: $g^*\omega\alpha$ -closed sets, $g^*\omega\alpha$ -continuous functions, $g^*\omega\alpha$ -irresolute maps.

1. Introduction: The concept of slightly continuous functions and their properties were first studied by Jain [5] in 1980. Later Nour [7], Baker [1] and Ekiri and Caldas [4] introduced and studied slightly semi continuity, slightly pre continuity and slightly γ -continuity in topological spaces.

The object of this paper is to introduce a new generalization of slightly continuity, which we call slightly $g^*\omega\alpha$ -continuity using $g^*\omega\alpha$ -closed sets in topological spaces.

2. Preliminary: Throughout this paper, the spaces (X, τ) and (Y, σ) (or simply X and Y) always denote topological spaces on which no separation axioms are assumed unless explicitly stated.

Definition 2.1 [8]: A subset A of a topological space X is called a generalized star $\omega\alpha$ -closed (briefly $g^*\omega\alpha$ -closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\omega\alpha$ -open in X .

The family of all $g^*\omega\alpha$ -closed subsets of X is denoted by $G^*\omega\alpha C(X)$.

Definition 2.2: A subset A of a topological space X is called

(i) $T_{g^*\omega\alpha}$ -space [9] if every $g^*\omega\alpha$ -closed set is closed.

Definition 2.3: A topological space X is said to be a

- (i) Mildly compact [11] if every clopen cover of X has a finite subcover.
- (ii) Clopen T_1 [11] if for each pair of distinct points x and y in X , there exist a disjoint clopen sets U containing x but not y and V containing y but not x .
- (iii) Clopen T_2 [11] if for each pair of distinct points x and y in X , there exist disjoint clopen sets U and V such that $x \in U$ and $y \in V$.
- (iv) Ultra Hausdroff [11] if every distinct points of X can be separated by disjoint clopen sets.

Definition 2.4: A function $f: X \rightarrow Y$ is called

- (i) slightly continuous [5] if for each $x \in X$ and every clopen subset V of Y containing $f(x)$ there exists an open set U of X with $x \in U$ and $f(U) \subseteq V$.
- (ii) $g^*\omega\alpha$ -continuous [10] if $f^{-1}(V)$ is $g^*\omega\alpha$ -closed in X for every closed set V of Y .
- (iii) $g^*\omega\alpha$ -irresolute [10] if $f^{-1}(V)$ is $g^*\omega\alpha$ -closed in X for every $g^*\omega\alpha$ -closed set V of Y .
- (iv) $g^*\omega\alpha$ -open [10] if $f(V)$ is $g^*\omega\alpha$ -open in Y for every open set V of X .

3. Slightly Generalized Star $\omega\alpha$ -Continuous Functions in Topological Spaces:

Definition 3.1: A function $f: X \rightarrow Y$ is said to be slightly $g^*\omega\alpha$ -continuous at a point $x \in X$ if for each clopen subset V of Y containing $f(x)$, there exists a $g^*\omega\alpha$ -open subset U of X containing x such that $f(U) \subseteq V$.

Remark 3.2: If the above property holds for each point $x \in X$, then f is said to be slightly $g^*\omega\alpha$ -continuous functions.

Lemma 3.3: The following statements are equivalent for a function $f: X \rightarrow Y$:

- (i) f is slightly $g^*\omega\alpha$ -continuous.
- (ii) for every $x \in X$ and each clopen set $V \subseteq Y$ containing $f(x)$, there exists $U \in G^*\omega\alpha O(X, x)$ such that $f(U) \subseteq V$.

Proof: Suppose (i) holds. Let $x \in X$ and let V be a clopen set in Y containing $f(x)$. By hypothesis, f is slightly $g^*\omega\alpha$ -continuous, then $f^{-1}(V)$ is $g^*\omega\alpha$ -open in X and $x \in f^{-1}(V)$. Let $U = f^{-1}(V)$. Then U is $g^*\omega\alpha$ -open set in X , such that $x \in U$ and $f(U) \subseteq V$. Hence (ii) holds.

Suppose (ii) holds. Let $x \in X$ and V be a clopen in Y such that $x \in f^{-1}(V)$. Then $f(x) \in V$, from the hypothesis, there exist $g^*\omega\alpha$ -open set U_x in X such that $x \in U_x$ and $f(U_x) \subseteq V$. Now $x \in U_x \subseteq f^{-1}(V)$ and $f^{-1}(V) = \cup \{U_x : x \in f^{-1}(V)\}$. Since arbitrary union of $g^*\omega\alpha$ -open set is $g^*\omega\alpha$ -open, thus $f^{-1}(V)$ is $g^*\omega\alpha$ -open in X . Therefore f is slightly $g^*\omega\alpha$ -continuous. Hence (i) hold.

Theorem 3.4: The following statements are equivalent for a function $f: X \rightarrow Y$:

- (i) f is slightly $g^*\omega\alpha$ -continuous.
- (ii) for every clopen set $V \subseteq Y$, $f^{-1}(V)$ is $g^*\omega\alpha$ -open.
- (iii) for every clopen set $V \subseteq Y$, $f^{-1}(V)$ is $g^*\omega\alpha$ -closed.
- (iv) for every clopen set $V \subseteq Y$, $f^{-1}(V)$ is $g^*\omega\alpha$ -clopen.

Proof: (i) \rightarrow (ii) Let V be a clopen set in Y . Let $x \in X$ and $x \in f^{-1}(V)$. Since f is slightly $g^*\omega\alpha$ -continuous and by the lemma 3.3, there exist $g^*\omega\alpha$ -open set U_x in X containing x such that $f(U_x) \subseteq V$, this implies $U_x \subseteq f^{-1}(V)$. Therefore $f^{-1}(V) = \cup \{U_x : x \in f^{-1}(V)\}$. Since arbitrary union of $g^*\omega\alpha$ -open set is $g^*\omega\alpha$ -open. Thus $f^{-1}(V)$ is $g^*\omega\alpha$ -open set in X .

(ii) \rightarrow (iii) Let V be a clopen set in Y , then $Y-V$ is clopen in Y . From (ii), $f^{-1}(Y-V)$ is $g^*\omega\alpha$ -open. That is

$f^{-1}(Y-V) = X-f^{-1}(V)$ is $g^*\omega\alpha$ -open, implies $f^{-1}(V)$ is $g^*\omega\alpha$ -closed.

(iii) \rightarrow (iv) It follows from (ii) and (iii).

(iv) \rightarrow (i) Let V be a clopen set in Y containing $f(x)$. Then by hypothesis, $f^{-1}(V)$ is $g^*\omega\alpha$ -clopen in X . Put $U=f^{-1}(V)$, implies $f(U)\subseteq V$. Therefore for each $x\in X$ and each clopen set $V\subseteq Y$, there exist $g^*\omega\alpha$ -open set U in X such that $f(U)\subseteq V$. Thus f is slightly $g^*\omega\alpha$ -continuous.

Theorem 3.5: Every slightly continuous function is slightly $g^*\omega\alpha$ -continuous.

Proof: Let $f: X \rightarrow Y$ be a slightly $g^*\omega\alpha$ -continuous. Let U be a clopen set in Y , then $f^{-1}(U)$ is $g^*\omega\alpha$ -open in X . Since every open set is $g^*\omega\alpha$ -open [8], $f^{-1}(U)$ is $g^*\omega\alpha$ -open. Hence f is slightly continuous.

The converse of the above theorem need not be true in general as seen from the following example.

Example 3.6: Let $X=Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Then the identity function $f: X \rightarrow Y$ is slightly $g^*\omega\alpha$ -continuous but not slightly continuous, since for the set $A = \{b\}$ is clopen in Y , $f^{-1}(\{b\}) = \{b\}$ is $g^*\omega\alpha$ -open but not an open set in X .

Remark 3.7: The converse of the theorem 3.5 holds, if X is $T_{g^*\omega\alpha}$ space.

Definition 3.8[6]: A space X is called Locally Indiscrete space if every open set is closed in X .

Theorem 3.9: If a function $f: X \rightarrow Y$ is slightly $g^*\omega\alpha$ -continuous and Y is locally indiscrete space, then f is $g^*\omega\alpha$ -continuous.

Proof: Let U be an open set in Y . Since Y is locally indiscrete, U is closed in Y , implies U is clopen in Y . Since f is slightly $g^*\omega\alpha$ -continuous, $f^{-1}(U)$ is $g^*\omega\alpha$ -open in X . Hence f is $g^*\omega\alpha$ -continuous.

Theorem 3.10: Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ are any two functions. Then, the following properties hold:

- (i) If f is $g^*\omega\alpha$ -irresolute and g is slightly $g^*\omega\alpha$ -continuous, then gof is slightly $g^*\omega\alpha$ -continuous.
- (ii) If f is $g^*\omega\alpha$ -continuous and g is slightly continuous, then gof is slightly $g^*\omega\alpha$ -continuous.

Proof: (i) Let V be a clopen set in Z . Since g is slightly $g^*\omega\alpha$ -continuous, $g^{-1}(V)$ is $g^*\omega\alpha$ -open in Y . Since f is $g^*\omega\alpha$ -irresolute, $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is $g^*\omega\alpha$ -open in X . Therefore gof is slightly $g^*\omega\alpha$ -continuous.

Let V be clopen set in Z . Then $g^{-1}(V)$ is open in Y as g is slightly continuous function. Since f is $g^*\omega\alpha$ -continuous, $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is $g^*\omega\alpha$ -open in X . Therefore, gof is slightly $g^*\omega\alpha$ continuous.

Corollary 3.11: Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ are any two functions. Then, the following properties hold:

- (i) If f is $g^*\omega\alpha$ -irresolute and g is slightly continuous, then gof is slightly $g^*\omega\alpha$ -continuous.
- (ii) If f is $g^*\omega\alpha$ -continuous and g is slightly continuous, then gof is slightly $g^*\omega\alpha$ -continuous.

Definition 3.12: A topological space X is said to be $g^*\omega\alpha$ -compact if every $g^*\omega\alpha$ -open cover of X has a finite subcover.

Theorem 3.13: If $f: X \rightarrow Y$ is slightly $g^*\omega\alpha$ -continuous surjection and X is $g^*\omega\alpha$ -compact then Y is mildly compact.

Proof: Let $\{V_\alpha : V_\alpha \in CO(Y), \alpha \in I\}$ be an cover of Y . Since f is slightly $g^*\omega\alpha$ -continuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is $g^*\omega\alpha$ -open cover of X . Since X is $g^*\omega\alpha$ -compact, there exist a finite subset I_0 of I such that $X = \cup \{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Therefore $Y = \cup \{V_\alpha : \alpha \in I_0\}$, since f is surjective. Thus, every clopen cover of Y has finite subcover. Hence Y is mildly compact.

Definition 3.14: A space X is called $g^*\omega\alpha$ -connected provided that X is not the union of two disjoint non empty $g^*\omega\alpha$ -open sets.

Theorem 3.15: If $f: X \rightarrow Y$ is slightly $g^*\omega\alpha$ -continuous surjection and X is $g^*\omega\alpha$ -connected then Y is connected.

Proof: Suppose, on the contrary Y is disconnected space. Then, there exist two non empty disjoint open sets U and V such that $Y=U\cup V$. Therefore U and V are clopen sets in Y . Since f is slightly $g^*\omega\alpha$ -continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are $g^*\omega\alpha$ -open sets in X . Moreover $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint non empty and $X= f^{-1}(U) \cup f^{-1}(V)$ as X is $g^*\omega\alpha$ -compact and f is surjective. Therefore X is not $g^*\omega\alpha$ -connected, which is a contradiction. Hence Y is connected.

Recall that, a space X is said to be (1)extremely disconnected [2] if the closure of every open set of X is open. (2) o-dimensional space if its topology has a base consisting of clopen sets.

Theorem 3.16: If $f: X \rightarrow Y$ is slightly $g^*\omega\alpha$ -continuous and Y is extremally disconnected space then f is $g^*\omega\alpha$ -continuous.

Proof: Let $x\in X$ and V be a clopen set in Y containing $f(x)$. Since f is extremally disconnected, $cl(V)$ is open and hence clopen. Since f is slightly $g^*\omega\alpha$ -continuous, there exist $g^*\omega\alpha$ -open set U in X with $x\in U$ and $f(U)\subseteq cl(V)$. Thus, f is $g^*\omega\alpha$ -continuous.

Theorem 3.17: If $f: X \rightarrow Y$ is slightly $g^*\omega\alpha$ -continuous and Y is locally indiscrete space then f is $g^*\omega\alpha$ -continuous.

Proof: Let V be an open set in Y . Since f is locally indiscrete space, V is closed in Y and hence clopen in Y . Since f is slightly $g^*\omega\alpha$ -continuous, $f^{-1}(V)$ is $g^*\omega\alpha$ -open in X . Therefore f is $g^*\omega\alpha$ -continuous.

Theorem 3.18: If $f: X \rightarrow Y$ is slightly $g^*\omega\alpha$ -continuous and Y is o-dimensional space then f is $g^*\omega\alpha$ -continuous.

Proof: Let $x\in X$ and V be an open set in Y containing $f(x)$. Since Y is o-dimensional space, there exist clopen set U in Y containing $f(x)$ such that $U\subseteq V$. Since f is slightly $g^*\omega\alpha$ -continuous, there exist $g^*\omega\alpha$ -open set G in X such that $f(G)\subseteq U$, that is $f(x)\in f(G)\subseteq U\subseteq V$. Therefore f is $g^*\omega\alpha$ -continuous.

4. Separation Axioms Related to Generalized Star $\omega\alpha$ -Open Sets:

Theorem 4.1: Let $f: X \rightarrow Y$ be a function and $g: X \rightarrow X \times Y$ be the graph of f , defined by $g(x) = (x, f(x))$ for every $x \in X$. Then g is slightly $g^*\omega\alpha$ -continuous if and only if f is slightly $g^*\omega\alpha$ -continuous.

Proof: Let V be a clopen set in Y , then $X \times V$ is clopen in $X \times Y$. Since g is slightly $g^*\omega\alpha$ -continuous, then $f^{-1}(V) = g^{-1}(X \times V) \in G^*\omega\alpha O(X)$. Thus f is slightly $g^*\omega\alpha$ -continuous.

Conversely, let $x \in X$ and F be a clopen set in $X \times Y$ containing $g(x)$. Then $F \cap (\{x\} \times Y)$ is clopen in $\{x\} \times Y$ containing $g(x)$. Also $\{x\} \times Y$ is homeomorphic to Y . Hence $\{y \in Y: (x, y) \in F\}$ is a clopen set in Y . Since f is slightly $g^*\omega\alpha$ -continuous, $\cup\{f^{-1}(y): (x, y) \in F\}$ is a $g^*\omega\alpha$ -open set in X . Further $x \in \cup\{f^{-1}(y): (x, y) \in F\} \subseteq g^{-1}(F)$. Hence $g^{-1}(F)$ is $g^*\omega\alpha$ -open. Then g is slightly $g^*\omega\alpha$ -continuous.

Definition 4.2 [10]: A topological space X is said to be

- (i) $g^*\omega\alpha$ - T_1 space if for each pair of distinct points x and y of X , there exist disjoint $g^*\omega\alpha$ -open sets U containing x but not y and V containing y but not x .
- (ii) $g^*\omega\alpha$ - T_2 space if for each pair of distinct points x and y of X , there exist disjoint $g^*\omega\alpha$ -open sets U and V such that $x \in U$ and $y \in V$.

Theorem 4.3: If $f: X \rightarrow Y$ is slightly $g^*\omega\alpha$ -continuous injection and Y is ultra Hausdroff space then X is $g^*\omega\alpha$ - T_2 space.

Proof: Let x_1 and x_2 be any two distinct points in X . Then $f(x_1) \neq f(x_2)$ as f is injective. Since Y is ultra Hausdroff, there exist clopen sets V_1 and V_2 in Y such that $f(x_1) \in V_1$ and $f(x_2) \in V_2$ and $V_1 \cap V_2 = \emptyset$. Since f is slightly $g^*\omega\alpha$ -continuous, $x_i \in f^{-1}(V_i) \in G^*\omega\alpha O(X)$ for $i=1, 2$ and $f^{-1}(V_1) \cap f^{-1}(V_2) = \emptyset$. Thus, X is $g^*\omega\alpha$ - T_2 space.

Theorem 4.4: If $f: X \rightarrow Y$ is slightly $g^*\omega\alpha$ -continuous injective and Y is clopen T_1 then X is $g^*\omega\alpha$ - T_1 space.

Proof: Let x and y be any two distinct points in X . Since f is injective, $f(x) \neq f(y)$. Since Y is clopen T_1 , then there exist disjoint clopen sets V and W in Y such that $f(x) \in V$, $f(y) \notin V$ and $f(x) \notin W$, $f(y) \in W$. Since f is slightly $g^*\omega\alpha$ -continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are disjoint $g^*\omega\alpha$ -open sets in X such that $x \in f^{-1}(V)$, $y \notin f^{-1}(V)$ and $x \notin f^{-1}(W)$, $y \in f^{-1}(W)$. This shows that X is $g^*\omega\alpha$ - T_1 .

Theorem 4.5: If $f: X \rightarrow Y$ is slightly $g^*\omega\alpha$ -continuous injective and Y is clopen T_2 then X is $g^*\omega\alpha$ - T_2 space.

Proof: Let x and y be any two distinct points in X , $f(x) \neq f(y)$ as f is injective. Since Y is clopen T_2 , then there exist disjoint clopen sets V and W in Y such that $f(x) \in V$ and $f(y) \in W$. Since f is slightly $g^*\omega\alpha$ -continuous, $f^{-1}(V) \in G^*\omega\alpha O(X, x)$ and $f^{-1}(W) \in G^*\omega\alpha O(X, y)$ and $f^{-1}(V) \cap f^{-1}(W) = \emptyset$. Therefore for each distinct points x, y in X , there exist two disjoint $g^*\omega\alpha$ -open sets $f^{-1}(V)$ and $f^{-1}(W)$ such that $f^{-1}(V) \cap f^{-1}(W) = \emptyset$. Hence X is $g^*\omega\alpha$ - T_2 .

Definition 4.6: A topological space X is said to be

- (i) $g^*\omega\alpha$ normal if for any pair of disjoint $g^*\omega\alpha$ -closed sets A and B in X , there exist disjoint open sets U and V in X such that $A \subseteq U, B \subseteq V$.
- (ii) $g^*\omega\alpha$ -regular if for each $x \in X$ and for each $g^*\omega\alpha$ -closed set F not containing x there exist disjoint open sets U and V such that $x \in U$ and $F \subseteq V$.

Theorem 4.7: Let Y be a 0-dimensional space and $f: X \rightarrow Y$ be a slightly $g^*\omega\alpha$ -continuous injection. Then the following properties hold:

- (i) If Y is T_1 (respectively T_2) then X is $g^*\omega\alpha$ - T_1 (respectively $g^*\omega\alpha$ - T_2).
- (ii) If f is either open or closed then X is $g^*\omega\alpha$ -regular.
- (iii) If f is closed and Y is normal then X is $g^*\omega\alpha$ -regular.

Proof: (i) The first part is obvious, we prove the second part. Let Y be T_2 space. Since f is injective, for any pair of distinct points $x, y \in X$, $f(x) \neq f(y)$. Since Y is T_2 , then there exist disjoint open sets V_1 and V_2 in Y such that $f(x) \in V_1, f(y) \in V_2$ and $V_1 \cap V_2 = \emptyset$. From hypothesis Y is 0-dimensional, then there exists U_1 and $U_2 \in CO(Y)$ such that $f(x) \in U_1 \subseteq V_1, f(y) \in U_2 \subseteq V_2$. Consequently $x \in f^{-1}(U_1) \subseteq f^{-1}(V_1)$ and $y \in f^{-1}(U_2) \subseteq f^{-1}(V_2)$ and $f^{-1}(U_1) \cap f^{-1}(U_2) = \emptyset$. Since f is slightly $g^*\omega\alpha$ -continuous, $f^{-1}(U_1)$ and $f^{-1}(U_2)$ are $g^*\omega\alpha$ -open sets and so X is $g^*\omega\alpha$ - T_2 space.

(ii) Suppose f is open. Let $x \in X$ and U be an open set containing x . Then $f(x) \in f(U)$ as f is open. On the other hand, 0-dimensional of Y gives the existence of $V \in CO(Y)$ such that $f(x) \in V \subseteq f(U)$. So, $x \in f^{-1}(V) \subseteq U$ as f is injective. Again, f is slightly $g^*\omega\alpha$ -continuous and $f^{-1}(V)$ is $g^*\omega\alpha$ -clopen set in X from theorem 3.4, hence $x \in f^{-1}(V) = cl(f^{-1}(V)) \subseteq U$. This implies X is $g^*\omega\alpha$ -regular.

Suppose f is closed. Let $x \in X$ and F be a closed set in X such that $x \notin F$. Then $f(x) \notin f(F)$ and $f(x) \in X - f(F)$, which is open in Y . But Y is 0-dimensional, then there exist clopen set V in Y such that $f(x) \in V \subseteq Y - f(F)$. Since f is slightly $g^*\omega\alpha$ -continuous, we have $x \in f^{-1}(V) \in G^*\omega\alpha CO(X), F \subseteq X - f^{-1}(V) \in G^*\omega\alpha CO(X)$. Therefore X is $g^*\omega\alpha$ -regular.

(iii) Let F_1 and F_2 be any two closed sets in X such that $F_1 \cap F_2 = \emptyset$. Since f is closed and injective, we have $f(F_1)$ and $f(F_2)$ are closed sets in Y with $f(F_1) \cap f(F_2) = \emptyset$. By normality of Y , there exist two open set U and V in Y such that $f(F_1) \subseteq U, f(F_2) \subseteq V$ and $U \cap V = \emptyset$. Let $y \in f(F_1)$, then $y \in U$. Since Y is 0-dimensional and U is open in Y , there exists a clopen set U_y such that $y \in U_y \subseteq U$. Then $f(F_1) \subseteq \cup\{U_y : U_y \in CO(Y), y \in f(F_1)\} \subseteq U$ and thus $F_1 \subseteq \cup\{f^{-1}(U_y) : U_y \in CO(Y), y \in f(F_1)\} \subseteq f^{-1}(U)$. Since f is slightly $g^*\omega\alpha$ -continuous, $f^{-1}(U_y)$ is $g^*\omega\alpha$ -open for each $U_y : CO(Y)$, so that $G = \cup\{f^{-1}(U_y) : y \in f(F_1)\}$ is $g^*\omega\alpha$ -open in X and $F_1 \subseteq G \subseteq f^{-1}(U)$. Similarly, there exist $g^*\omega\alpha$ -open set H in X such that

$F_2 \subseteq H \subseteq f^{-1}(V)$ and $G \cap H \subseteq f^{-1}(U \cap V) = \phi$. This shows that X is $g^*\omega\alpha$ -normal.

Definition 4.8: A space X is said to be $g^*\omega\alpha$ -connected between subsets A and B provided there is no $g^*\omega\alpha$ -clopen set F for which $A \subseteq F$ and $F \cap B = \phi$.

Definition 4.9: A function $f : X \rightarrow Y$ is said to be set $g^*\omega\alpha$ -connected if whenever X is $g^*\omega\alpha$ -connected between $f(A)$ and $f(B)$ with respect to the relative topology on $f(X)$.

Theorem 4.10: A function $f : X \rightarrow Y$ is set $g^*\omega\alpha$ -connected if and only if $f^{-1}(F)$ is $g^*\omega\alpha$ -clopen for every clopen subsets F of $f(X)$ (w.r.t. the relative topology on $f(X)$).

Proof: Necessity: Assume the F is clopen subsets of $f(X)$ w.r.t the relative topology on $f(X)$. Suppose that $f^{-1}(F)$ is not $g^*\omega\alpha$ -closed in X . Then there exist $x \in X - f^{-1}(F)$ such that for every $g^*\omega\alpha$ -open set U with $x \in U$ and $U \cap f^{-1}(F) \neq \phi$. We claim that the space X is set $g^*\omega\alpha$ -connected between x and $f^{-1}(F)$. Suppose, there exist $g^*\omega\alpha$ -clopen set A such that $f^{-1}(F) \subseteq A$ and $x \notin A$. Then $x \in X - A \subseteq X - f^{-1}(F)$ and equivalently $X - A$ is $g^*\omega\alpha$ -open set containing x and disjoint from $f^{-1}(F)$, this contradiction implies that X is set $g^*\omega\alpha$ -connected between x and $f^{-1}(F)$. Since f is set $g^*\omega\alpha$ -connected, $f(X)$ is connected between $f(x)$ and $f(f^{-1}(F))$. But $f(f^{-1}(F)) \subseteq F$, which is a contradiction.

Therefore $f^{-1}(F)$ is $g^*\omega\alpha$ -closed in X and from the argument will show that $f^{-1}(F)$ is also $g^*\omega\alpha$ -open.

Sufficiency: Suppose X is $g^*\omega\alpha$ -connected between A and B and also $f(X)$ is not connected between $f(A)$ and $f(B)$ (in relative topology on $f(X)$). Thus, there is a set $F \subseteq f(X)$ that is clopen in the relative topology on $f(X)$ such that $f(A) \subseteq F$ and $F \cap f(B) = \phi$. Then $A \subseteq f^{-1}(F)$, $B \cap f^{-1}(F) = \phi$ and $f^{-1}(F)$ is $g^*\omega\alpha$ -clopen, which shows that X is not $g^*\omega\alpha$ -connected between A and B . It follows that f is set $g^*\omega\alpha$ -connected.

Corollary 4.11: Every slightly $g^*\omega\alpha$ -continuous surjection is set $g^*\omega\alpha$ -connected.

Theorem 4.12: Every set $g^*\omega\alpha$ -connected function is slightly $g^*\omega\alpha$ -continuous.

Proof: Assume that $f : X \rightarrow Y$ is set $g^*\omega\alpha$ -connected. Let F be a clopen subset of Y . Then $F \cap f(X)$ is clopen in the relative topology on $f(X)$. Since f is set $g^*\omega\alpha$ -connected, by theorem 4.10, $f^{-1}(F) = f^{-1}(F \cap f(X))$ is $g^*\omega\alpha$ -clopen in X .

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