A FUZZY CCR DEA MODEL: AN APPLICATION TO THE HEALTH SECTOR

ALKA ARYA, SHIV PRASAD YADAV

Abstract: In this paper, we focus on measuring the performance efficiencies of decision making units (DMUs) using CCR data envelopment analysis (DEA) model with fuzzy data. In conventional CCR model, the data are found as crisp quantities. However, in real life problems, the data of inputs and outputs in DEA may have fuzzy essence. In this paper, we propose a CCR DEA model for fuzzy input and fuzzy output data. This model is then reduced to a crisp LP problem by using ranking function of a fuzzy number (FN). Moreover, we present an application of the proposed model to the health sector in which two inputs (i) number of doctors (ii) number of pharmacists and two outputs (i) number of inpatients (ii) number of outpatients, and are represented as TFNs.

Keywords: CCR DEA, Dual CCR, Hospitals, TFN.

Introduction: DEA, proposed by Charnes, Cooper, and Rhodes (CCR), is a non-parametric linear programming for determining the efficiencies of homogeneous DMUs on the basis of multiple inputs and multiple outputs. The first model in DEA is CCR model. The DEA technology is mainly acceptable to evaluate the efficiency of non-profit organizations such as educational institutions, schools, hospitals, banks etc. Conventional DEA models are restricted to only crisp input/output data. Though, in real-life applications, Input and/or output data may have imprecision or fuzziness. To deal with imprecise data, the notion of fuzziness has been introduced in DEA. Kirigia and Asbu [6] determined the efficiencies and inefficiencies of African hospitals using two-stage DEA technique.

The DEA is extended to FDEA in which the imprecision is represented by fuzzy sets or FNs [5,9]. In particular there are some approaches have been developed to deal with fuzzy data in FDEA such as; α-cut approach [5], tolerance approach [9], fuzzy ranking approach [4], and possibility approach [7]. Several studies have been deal with health sector applications developing as well as developed countries using FDEA. Zavras et.al. [11] determined the relative efficiency of 133 primary health care centers of the principal Greek public insurance provider, the Social Security Institute (IKA). Dotoli et.al. [3] presented a novel cross efficiency FDEA technique for evaluating different DMUS under uncertainty in the case of health care system. In Indian context [2,8,10], there are various studies of DEA and FDEA with an application to health sector.

The paper is organized as follows: Section 2 presents preliminary, Section 3 presents the extension of DEA to Fuzzy DEA (FDEA) models, Numerical application is given in Section 4 and Conclusions of the study are given in last section of the paper.

1. Preliminary (some basic definitions)

2. Performance Efficiency: Performance efficiency of a set of n homogeneous DMUs is characterized by m inputs x_{ij}; i = 1, 2, 3, . . ., m to produce s outputs y_{rj}; r = 1, 2, 3, . . ., s. According to Cooper et.al. [1] the performance efficiency of any DMU is the ratio of virtual output (called weighted sum of outputs) and virtual input (called weighted sum of inputs).

$$\text{Performance efficiency} = \frac{\sum_{r=1}^{s} v_{rj} y_{rj}}{\sum_{i=1}^{m} u_{ij} x_{ij}}$$

where v_{rj} and u_{ij} are the weights corresponding to y_{rj} and x_{ij} .

2.2 Fuzzy Number (FN)[12]: A fuzzy number Ã is defined as a convex normalized fuzzy set Ã of the real line $\mathbb{R}$ such that

- there exists exactly one $x \in \mathbb{R}$ with $\mu_{\tilde{A}}(x)=1$. x is called the mean value of Ã.
- $\mu_{\tilde{A}}$ (membership function of Ã) is piece wise continuous function.

1.1 Triangular Fuzzy Number (TFN)

A TFN Ã, denoted by (a,b,c) and is defined by the membership function $\mu_{\tilde{A}}$ given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, \ a < x \le b, \\ \frac{c-x}{c-b}, \ b \le x < c, \\ 0, \ elsewhere, \end{cases}$$

$\forall x \in \mathbb{R}$.

1. Proposed DEA Model

1.1 CCR DEA Model:

Data envelopment analysis (DEA) [1] is a non-parametric and linear programming based technique to measures the efficiency of DMUs on the basis of multiple inputs and multiple outputs. Assume that the performance of a set of n homogeneous DMUs

(DMU$_j$; j = 1,2,...,n) is to be measured. According to cooper et.al.[1] the CCR DEA model for any DMU is maximized the ratio of virtual output and virtual input with the condition that the ratio of virtual output and virtual input for every DMU is less than or equal to unity. In this model, E_{jo} is the efficiency of DMU$_j$, and ε is the non-Archimedean infinitesimal constant. The CCR (ratio) fractional DEA program and the corresponding linear program (LP) for DMU$_j$ is given in Table 1.

Table 1: CCR DEA Models

Fractional CCR DEA model	LP CCR DEA model
$\max E_{j_o}^{\ P} = \dfrac{\sum_{r=1}^{s} v_{rj_o} y_{rj_o}}{\sum_{i=1}^{m} u_{ij_o} x_{ij_o}}$ subject to $\dfrac{\sum_{r=1}^{s} v_{rj_o} y_{rj}}{\sum_{i=1}^{m} u_{ij_o} x_{ij}} \le 1,$ $\forall j = 1,2,\ldots,n, \varepsilon > 0$ $u_{ij_o} \ge \in, i = 1,2,\ldots,m,$ $v_{rj_o} \ge \in, r = 1,2,\ldots,s.$	$\min E_{j_o}^{\ P} = \sum_{i=1}^{m} u_{ij_o} x_{ij_o}$ subject to $\sum_{r=1}^{s} v_{rj_o} y_{rjo} = 1$ $\sum_{r=1}^{s} v_{rj_o} y_{rj} - \sum_{i=1}^{m} u_{ij_o} x_{ij} \le 0,$ $\forall j = 1,2,\ldots,n, \varepsilon > 0,$ $u_{ij_o} \ge \varepsilon, i = 1,2,\ldots,m,$ $v_{rj_o} \ge \varepsilon, r = 1,2,\ldots,s.$

Where u_{ij_o} and v_{rj_o} are the weights corresponding to x_{ij} and y_{rj} respectively.

1.1 CCR DEA Model with fuzzy data

Input and output data is given in crisp form in conventional DEA, which is not always available in real world problems. However input and/or output data is not precisely measured and approximately known in real world problems, thus we use fuzzy DEA model for finding the performance efficiencies of DMUs. The FDEA model is given in Table 2

Table 2: CCR FDEA model

$$\min E_{j_o} = \sum_{i=1}^{m} u_{ij_o} x_{ij_o}$$

$$\text{subject to } \sum_{r=1}^{s} v_{rj_o} y_{rjo} = \tilde{1}$$

$$\sum_{r=1}^{s} v_{rj_o} y_{rj} - \sum_{i=1}^{m} u_{ij_o} x_{ij} \le \tilde{0},\ \forall j = 1,2,\ldots,n$$

$$u_{ij_o} \ge \varepsilon, i = 1,2,\ldots,m,$$

$$v_{rj_o} \ge \varepsilon, r = 1,2,\ldots,s, \varepsilon > 0.$$

In Table 2, x_{ij} (i =1,2,...,m) is the amount of the ith fuzzy input used by the jth DMU and y_{rj} (r =1,2,..,s) is the amount of the rth output produced, u_{ij_o} and v_{rj_o} are the weights corresponding to the ith input and rth output of DMU$_j$ respectively. Where $0 = (0,0,0)$ and $\tilde{1} = (1,1,1)$. Let us suppose that input x_{ij} and output y_{rj} be TFNs $(x_1^{ij}, x_2^{ij}, x_3^{ij})$ and $(y_1^{rj}, y_2^{rj}, y_3^{rj})$ respectively. By putting the values of the above model reduces to the following proposed model, which is crisp LPP model.

Table 3: Proposed DEA model

$$\min E_{j_o}^{\ F} = \sum_{i=1}^{m} u_{ij_o} (x_1^{ij_o} + 2x_2^{ij_o} + x_3^{ij_o}) / 4$$

subject to

$$\sum_{r=1}^{s} v_{rj_o} (y_1^{rj_o} + 2y_2^{rj_o} + y_3^{rj_o}) / 4 = (1,1,1)$$

$$\sum_{i=1}^{m} v_{rj_o} (y_1^{ij} + 2y_2^{ij} + y_3^{ij}) -$$

$$\sum_{i=1}^{m} u_{ij_o} (x_1^{ij} + 2x_2^{ij} + x_3^{ij}) \le (0,0,0),$$

$$\forall j = 1,2,\ldots,n,$$

$$u_{ij_o} \ge \varepsilon, i = 1,2,\ldots,m,$$

$$v_{rj_o} \ge \varepsilon, r = 1,2,\ldots,s, \varepsilon > 0.$$

2. **Discussion of a Numerical Example (Application):** In this section, we provide a numerical example to illustrate the proposed DEA model with fuzzy input and output data. We determine the performance efficiencies of hospitals in Meerut district, Uttar Pradesh state, India for the calendar year 2013-2014 using the proposed model. Table 4 presents the performance evaluation problem of six hospitals with two inputs x_1 (number of doctors) and x_2 (number of pharmacists), and two outputs y_1 (number of inpatients) and y_2 (number of outpatients). In this table, codes for hospitals are: H1 Mawana,H2 Sardhana, H3 Daurala, H4 Bhudbharal, H5 Jani, H6 Rohta. In Table 5, U1 and U2 are input weights corresponding to x_1 and x_2 respectively; V1 and V2 are output weights corresponding to y_1 and y_2 respectively.

Table 4: Inputs and outputs data as TFNs

Inputs and outputs	H1	H2	H3	H4	H5	H6
x_1	(8,10,12)	(8,9,12)	(10,11,13)	(6,8,10)	(9,10,11)	(8,11,12)
x_2	(4,5,7)	(3,5,6)	(2,4,5)	(1,1,1)	(3,4,5)	(2,3,4)
y_1	(3645,3650,3655)	(4155,4160,4170)	(4360,4370,4375)	(490,492,500)	(2460,2464,2470)	(1365,1368,1370)
y_2	(134135,134137,134140)	(116060,116062,116065)	(94060,94066,94070)	(24325,24329,24331)	(99745,99748,99750)	(49400,49401,49405)

Table 5: Results- performance efficiency, input and output weights

DMU	Efficiency	U1	U2	V1	V2
1	1	0.18×10^{-1}	0.16	0.47×10^{-4}	0.62×10^{-5}
2	1	0.19×10^{-1}	0.17	0.52×10^{-4}	0.67×10^{-5}
3	1	0.1×10^{-5}	0.27	0.21×10^{-3}	0.1×10^{-5}
4	0.95	0.1×10^{-5}	1.05	0.1×10^{-5}	0.41×10^{-4}
5	0.70	0.1×10^{-5}	0.47	0.38×10^{-3}	0.1×10^{-5}
6	0.64	0.1×10^{-5}	0.52	0.19×10^{-4}	0.2×10^{-4}

Conclusion: In this paper, we determined the efficiencies and crisp weights for fuzzy inputs and outputs. The CCR FDEA model is then reduced to crisp LP by using ranking function. The proposed model determines the crisp efficiencies and crisp weights corresponding to inputs and outputs as TFNs. These efficiencies and crisp weights afford extra information to the decision maker, which helps to deal with uncertainty in real world applications.

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Alka Arya/Research Scholar/
Dr. Shiv Prasad Yadav/Professor/
Department of Mathematics/ Indian Institute of Technology Roorkee/Roorkee 247667.