

SOFT β COMPACTNESS IN SOFT TOPOLOGICAL SPACES

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Abstract: D.Molodtsov 1999 introduced the concept of a soft set as a new approach for modeling uncertainties. Arockiarani and Arockialancy defined soft β open sets and continued study weaker form of soft open sets in soft topological spaces. The present paper introduces the concepts of soft β compactness and soft β closed spaces in soft topological spaces. Soft filter bases are used to characterize these concepts. A comparison between these types and some different types of compactness in soft topology is established.

Keywords: Soft β compact, soft filter base, soft β closed spaces, Soft β open set

Introduction: Molodtsov [5] initiated a novel concept of soft set theory, which is a completely new approach for modelling vagueness and uncertainty. Mathematics is based on exact concepts and there is not vagueness for mathematical concepts. In fields like medicine, engineering, economics and sociology the notions are vague, researchers need to define some new concepts for vagueness. Researchers proposed several methods such as fuzzy set theory, rough set theory and soft set theory to overcome the difficulty of vagueness.

Fuzzy set theory [22] proposed by L.Zadeh in 1965, provides an appropriate framework for processing and representing vague concepts. The basic idea of fuzzy set theory hinges on fuzzy membership function. Rough set theory [21] which is proposed by Pawlak in 1982 is another mathematical approach to vagueness in information systems. The theory of rough sets and fuzzy sets are the tools for dealing with vagueness but both the theories have their own difficulties. The reason for these difficulties is possibly, the inadequacy of the parametrization tool of the theory as mentioned by Molodtsov [5] in 1999. He successfully applied the soft set theory into several directions, such as smoothness of functions, game theory, theory of measurement, Riemann integration and so on. He also showed how soft set theory is free from the parametrization in adequacy syndrome of fuzzy set theory, rough set theory, probability theory and game theory.

Soft set theory provides a general framework with involment of parameters. In recent years research work on soft set theory is taking place rapidly. As a result of this many researchers [1],[2],[8],[14] are working about soft set theory and its applications in various fields. Shabir and Naz [10] introduced the soft topological spaces which are defined over an initial universe with fixed set of parameters. Aygunoglu and Aygun [1] defined soft continuity of soft mapping, soft product topology soft compactness etc in soft topological spaces. Min[11] gave some results on soft topological spaces. Hussain and Ahmad[4] continued and discussed the properties of soft interior, soft exterior and soft boundary on soft topology. Saziye

Yüksel, Naime Tozlu and Zehra Güzel Ergül[17] introduced the soft filters in soft topological spaces. Arockiarani and Arokialancy[7] defined soft β open sets and continued to study weaker form of soft open sets in soft topological spaces. Recently S.S.Benchalli et al [18],[19], [20] introduced the soft β connected spaces, soft β separation axiom spaces and on some weaker forms of soft closed sets in soft topological spaces

The main purpose of this paper is to introduce soft β -compactness and soft β closed spaces in soft topological spaces. These notions generalize basic classical results. Using soft filterbases, we characterize soft β -compactness and soft β closed spaces. We also study some expected basic properties of these concepts.

Preliminaries: Throughout this paper (X, τ, E) will be a soft topological space.

Definition 2.1 [15] Let X be an initial universe and let E be a set of parameters. Let $P(X)$ denote the power set of X and let A be a nonempty subset of E . A pair (F, A) is called a soft set over X , where F is a mapping given by $F : A \rightarrow P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X . For $e \in A$, $F(e)$ may be considered as the set of e -approximate elements of the soft set (F, A) .

Definition 2.2 [15] A soft set (F, A) over X is called a null soft set, denoted by Φ , if $e \in A$, $F(e) = \Phi$.

Definition 2.3 [16] A soft set (F, A) over X is called an absolute soft set, denoted by \tilde{A} , if $e \in A$, $F(e) = X$. If $A = E$, then the A -universal soft set is called a universal soft set, denoted by \tilde{X} .

Definition 2.4 [16] Let Y be a nonempty subset of X ; then \tilde{Y} denotes the soft set (Y, E) over X for which $Y(e) = Y$, for all $e \in E$.

Definition 2.5 [16] The union of two soft sets of (F, A) and (G, B) over the common universe X is

the soft set (H, C) , where $C = A \cup B$ and for all $e \in C$

$$H(e) = F(e), \text{ if } e \in A - B,$$

$$G(e), \text{ if } e \in B - A,$$

$$F(e) \cup G(e) \text{ if } e \in A \cap B$$

We write $(F, A) \tilde{\cap} (G, B) = (H, C)$

Definition 2.6 [16] The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe X , denoted by $(F, A) \cap (G, B)$, is defined as $C = A \cap B$, and $H(e) = F(e) \cap G(e)$ for all $e \in C$

Definition 2.7 [16] Let (F, A) and (G, B) be two soft sets over a common universe X . $(F, A) \tilde{\subset} (G, B)$, if $A \subset B$ and

$$H(e) = F(e) \subset G(e) \text{ for all } e \in A.$$

Definition 2.8 [10] Let τ be the collection of soft sets over X ; then τ is said to be a soft topology on X if it satisfies the following axioms:

- (1) Φ, X belongs to τ .
- (2) The union of any number of soft sets in τ belongs to τ .
- (3) The intersection of any two soft sets in τ belongs to τ .

The triplet (X, τ, E) is called a soft topological space over X . Let (X, τ, E) be a soft topological space over X ; then the members of τ are said to be soft open sets in X . The relative complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$.

Definition 2.9 Let (X, τ, E) be a soft topological space over X and let (F, A) be a soft set over X .

(1) Soft Interior [13] The soft interior of (F, A) is the soft set $int((F, A)) = \cup \{(O, A) : (O, A) \text{ is soft open and } (O, A) \tilde{\subset} (F, A)\}$

(2) Soft Closure [13] : The soft closure of (F, A) is the soft set $cl((F, A)) = \cap \{(F, E) : (F, E) \text{ is soft closed and } (F, A) \subset (F, E)\}$

Definition 2.10 [12] A soft set (F, A) of a soft topological space (X, τ, E) is said to be

1. Soft open if its complement is soft closed,
2. Soft α -open if, $(F, A) \tilde{\subset} Int(Cl(Int((F, A))))$,
3. Soft preopen if $(F, A) \tilde{\subset} Int(Cl((F, A)))$
4. Soft semi open if $(F, A) \tilde{\subset} Cl(Int((F, A)))$,
5. Soft β - open if $(F, A) \tilde{\subset} Cl(int(Cl((F, A))))$.

The complement of soft open, (resp. soft α open, soft preopen, soft semiopen, soft β open)sets are said to be soft closed (resp.soft α closed, soft

preclosed, soft semiclosed, soft β closed). The intersection of soft closed(resp. soft α closed, soft preclosed, soft semiclosed, soft β closed)sets containing (F, A) is called the soft closure(resp. soft α closure, soft pre-closure, soft semi-closure, soft β closure)of (F, A) and is denoted by $scl(F, A)$ (resp. $s\alpha\text{ cl}(F, A)$, $sPcl(F, A)$, $sScl(F, A)$, $s\beta\text{ cl}(F, A)$). The soft interior of (F, A) is defined by the union of all soft open (resp.soft α open, soft preopen, soft semiopen, soft β open) sets contained in (F, A) and is denoted by $sInt(F, A)$ (resp. $s\alpha\text{ int}(F, A)$, $sPInt(F, A)$, $sSInt(F, A)$, $s\beta\text{ Int}(F, A)$).

Definition 2.11 [17] Let (X, τ, E) be a soft topological space, X is called soft compact if every soft open cover of X has a finite subcover.

Example 2.12 Let $X = \{x_1, x_2, x_3, \dots\}$, $E = \{e_1, e_2\}$ and consider the family of soft sets $\{(F_n)_E / n \in \mathbb{N}\}$ where $(F_n)_E = \{e_1(x_1, x_2, \dots, x_n), e_2(X)\}$. The family $\tau = \{(F_n)_E / n \in \mathbb{N}\} \cup \{\phi, E\}$ is a soft topology on X . However, the soft open cover $\{(F_n)_E / n \in \mathbb{N}\}$ has no finite sub cover, that is (X, τ) is not soft compact.

Example 2.13 Let $I = \{e_1, e_2, \dots\}$ consider the family $S = \{(F_A)_n / n \in \mathbb{N}\}$ where

$$(F_A)_n = \{(e_1, \phi), (e_2, \phi), \dots, (e_{n-1}, \phi), (e_n, X), (e_{n+1}, \phi), \dots\}$$

and $A_n = \{e_n\}$ Then the family S is a subbase for a soft topology τ .

Soft β -Compactness in Soft Topological Spaces

Definition 3.1 A cover of a soft set is said to be a soft β open cover if every member of the cover is a soft β open set.

Definition 3.2 A soft topological space (X, τ, E) is said to be soft β -compact if each soft β open cover of X has a finite subcover.

Definition 3.3 Let (X, τ, E) and (Y, τ', E) be two soft topological spaces. A function

$$f: (X, \tau, E) \rightarrow (Y, \tau', E) \text{ is said to be}$$

- (i) Soft β continuous: If $f^{-1}(G, E)$ is soft β open in (X, τ, E) for every soft open set (G, E) of (Y, τ', E) .
- (ii) Soft β irresolute if $f^{-1}(G, E)$ is soft β open in (X, τ, E) for every soft β open set (G, E) of (Y, τ', E) .

Lemma 3.4 Let $f: (X, \tau, E) \rightarrow (Y, \tau', E)$ be a function then the following are equivalent:

- (i) f is soft β irresolute

(ii) $f(s\beta cl(U,E)) \subseteq s\beta cl f(U,E)$, for every soft set (U,E) in X .

Proof: (i) \Rightarrow (ii)

Let (U,E) be a soft set $s\beta cl f(U,E)$ is soft β closed by

(i) $f^{-1}[s\beta cl f(U,E)]$ is soft β closed and so $f^{-1}[s\beta cl f(U,E)] = s\beta cl f^{-1}[s\beta cl f(U,E)]$. Hence $f[s\beta cl f(U,E)] = s\beta cl f(f^{-1}[s\beta cl f(U,E)])$.

(ii) \Rightarrow (i)

(V,E) be a soft β closed set in Y . By (ii) if $(V,E) = f^{-1}((V,E))$ then $s\beta cl f^{-1}((V,E)) \subseteq f^{-1}(s\beta cl f(f^{-1}((V,E)))) \subseteq f^{-1}(s\beta cl((V,E))) = f^{-1}((V,E))$.

Since $f^{-1}((V,E)) \subseteq s\beta cl f^{-1}((V,E))$, then $f^{-1}((V,E)) = s\beta cl f^{-1}((V,E))$. Hence $f^{-1}((V,E))$ is soft set.

Lemma 3.5 Let $f:(X,\tau,E) \rightarrow (Y,\tau',E)$ be a function then the following are equivalent.

(i) f is soft β continuous

(ii) $f(s\beta cl((U,E))) \subseteq scl f(U,E)$, for every soft set (U,E) in X .

Proof Obvious.

Theorem 3.6 If $(X,\tau,E) \rightarrow (Y,\tau',E)$ is soft open function then $f^{-1}(s\beta cl((U,E))) \subseteq scl f(U,E)$, for every soft set (U,E) in Y . If (X,τ,E) is soft topological space, then τ_β stands for the soft topology on X having $s\beta o(X,\beta,E)$ as a soft subbase.

Theorem 3.7 (X,τ,E) is a soft β -compact iff (X,τ_β,E) is soft compact.

Proof: If (X,τ_β,E) is soft compact, then (X,τ,E) is soft β compact. Since $s\beta o(X,\tau,E) \subseteq \tau_\beta$. The converse is consequence of the famous soft Alexanders subbase theorem [17] soft topological spaces.

Lemma 3.8 A soft β closed subset of soft β compact space is soft β -compact.

Theorem 3.9 Let $f:(X,\tau,E) \rightarrow (Y,\tau',E)$ be a soft β irresolute and (G,E) is soft β -compact, then $f((G,E))$ is soft β -compact.

Proof: Let Δ be a soft β open cover of $f((G,E))$. Then $f((G,E)) \subseteq \bigcup_{(F_i,E) \in \Delta} (F_i,E)$ and $(G,E) \subseteq f^{-1}(f((G,E))) \subseteq f^{-1}(\bigcup_{(F_i,E) \in \Delta} (F_i,E)) = \bigcup_{(F_i,E) \in \Delta} f^{-1}(F_i,E)$. As f is soft β irresolute $f^{-1}((F_i,E))$ is soft β open for all $(F_i,E) \in \Delta$. As (G,E) is soft β -compact $f^{-1}(\bigcup_{(F_i,E) \in \Delta} (F_i,E)) \supseteq (G,E)$. Where Δ_i

is soft finite subcollection of Δ . Hence $f((G,E)) \subseteq \bigcup_{(F_i,E) \in \Delta_i} (F_i,E)$ that is $f((G,E))$ is soft β -compact.

Theorem 3.10 If $f:(X,\tau,E) \rightarrow (Y,\tau',E)$ is soft open function then $f^{-1}(scl((G,E))) \subseteq scl(f^{-1}(G,E))$, for every soft set (G,E) in Y .

Definition 3.11 Let $\mathcal{w} \subseteq s(X,E)$, then \mathcal{w} is said to be a soft filter base on X if

(i) $\mathcal{w} \neq \emptyset$ and $\emptyset \notin \mathcal{w}$

(ii) $\forall F_A, G_B \in \mathcal{w}, \exists H_C \subseteq F_A \cap G_B$ or A collection of soft subsets \mathcal{w} of a soft topological space X is said to form a soft filterbase iff for every finite collection

$$\{(F_i, E_i) : i=1,2,3,\dots,n\} \cap_{i=1}^n (F_i, E_i) \neq \emptyset.$$

Theorem 3.12 A soft topological space X is soft β compact iff for every collection $\{(F_i, E_i) : i \in \Delta\}$ of soft β closed sets of \tilde{X} having the finite intersection property

$$\bigcap_{i \in \Delta} (F_i, E_i) \neq \emptyset$$

Proof: Let $\{(F_i, E_i) : i \in \Delta\}$ be a collection of soft β closed sets with the finite intersection property. Suppose that $\bigcap_{i \in \Delta} (F_i, E_i) = \emptyset$, then $\bigcup_{i \in \Delta} (F_i, E_i)^c = X$. Since $\{(F_i, E_i)^c : i \in \Delta\}$ is a collection of soft β open cover of \tilde{X} , then from the soft β -compactness of \tilde{X} it follows that there exists a finite soft subset $\Delta_i \subset \Delta$ such that

$$\bigcup_{i \in \Delta_i} (F_i, E_i)^c = X. \text{ Then } \bigcap_{i \in \Delta_i} (F_i, E_i) = \emptyset \text{ which gives a contradiction and therefore } \bigcap_{i \in \Delta} (F_i, E_i) \neq \emptyset$$

Conversely, Let $\{(F_i, E_i) : i \in \Delta\}$ be a collection of soft β open cover of \tilde{X} . Suppose that for every finite subset

$$\Delta_i \subset \Delta, \text{ we have } \bigcup_{i \in \Delta_i} (F_i, E_i) \neq X. \text{ Then } \bigcap_{i \in \Delta_i} (F_i, E_i)^c \neq \emptyset.$$

Hence $\{(F_i, E_i)^c : i \in \Delta\}$ satisfies the finite intersection property. Then from the hypothesis, we have $\bigcap_{i \in \Delta_i} (F_i, E_i)^c \neq \emptyset$, which implies $\bigcap_{i \in \Delta_i} (F_i, E_i) \neq X$ and this contradicting that $\{(F_i, E_i) : i \in \Delta\}$ is soft β open cover of X . Thus \tilde{X} is soft β -compact.

Theorem 3.13 A soft continuous image of a soft β -compact set is soft β -compact.

Proof: Let $f:(X, \tau_1, E) \rightarrow (Y, \tau_2, E)$ be a soft β continuous function of a soft β topological space into an arbitrary soft topological space (Y, τ_2, E) . Also let (G, E) be a soft β -compact subset of \tilde{X} , then we have to show that the image of (G, E) that is $f[(G, E)]$ is soft β -compact subset of (Y, τ_2, E) . Let $C^* = \{(F_i, E_i) : i \in I\}$ be a soft open cover of $f[(G, E)]$, so that $f[(G, E)] \subset \cup_{i \in I} (F_i, E_i)$ implies $(G, E) \subset f^{-1}[f[(G, E)]] \subset f^{-1}[\cup_{i \in I} (F_i, E_i)] = \cup_{i \in I} f^{-1}(F_i, E_i)$ implies $(G, E) \subset f^{-1}\{(F_i, E_i) : i \in I\}$ implies $f^{-1}\{(F_i, E_i) : i \in I\}$ is soft β cover of (G, E) and f is soft continuous and $\{(F_i, E_i) : i \in I\}$ is soft β open set implies $f^{-1}(F_i, E_i)$ is soft β open set. Conclusively, $f^{-1}(F_i, E_i)$ is soft β open cover of (G, E) where (G, E) is soft β -compact and hence $f^{-1}[(G, E)]$ is reducible to soft finite cover say $(G, E) \subset f^{-1}[(F_1, E_1)] \cup f^{-1}[(F_2, E_2)] \dots \cup f^{-1}[(F_m, E_m)]$ implies $f[(G, E)] \subset f[f^{-1}[(F_1, E_1)] \cup f^{-1}[(F_2, E_2)] \dots \cup f^{-1}[(F_m, E_m)]]$ implies $f[(G, E)] \subset [(F_1, E_1)] \cup [(F_2, E_2)] \dots \cup [(F_m, E_m)]$ implies $f[(G, E)]$ is soft β -compact.

Theorem 3.14 Every Soft β -compact subset of a soft Hausdorff space is soft β closed.

Proof: Let (G, E) be a soft β -compact of soft β Hausdorff space (X, τ, E) . Then $(G, E) \subset \tilde{X}$ is soft β closed if its complement $\tilde{X} - (G, E)$ is soft β open. Let an arbitrary $e_x \in (G, E)$ and a fixed $e_y \in \tilde{X} - (G, E)$ such that $e_x \neq e_y$ that is $e_x \in (G, E)$ and $(G, E) \subset \tilde{X}$ which implies $e_x \in \tilde{X}$ but $e_x \neq \tilde{X} - (G, E)$, $e_y \in \tilde{X} - (G, E)$ which implies $e_y \in \tilde{X}$. Also, Now (X, τ, E) is soft β Hausdorff space and $e_x, e_y \in \tilde{X}$ such that $e_x \neq e_y$ implies there exist soft β open sets (G_i, E) and $(G_j, E) \in (X, \tau, E)$ such that $e_x \in (G_i, E)$ and $e_y \in (G_j, E)$ and $(G_i, E) \cap (G_j, E) = \emptyset$ which is

true for all $e_x \in (G, E)$ The class $\{(G_i, E) : i \in (G, E)\}$ is clearly a soft β open cover of the soft β -compact set (G, E) and hence this cover must be reducible to soft β finite subcover.

Now we give some results of soft β -compactness in terms of soft filterbases.

Theorem 3.15 A soft topological space is soft β -compact iff every soft filterbases ξ in $X \cap_{(G, E) \in \xi} s\beta cl(G, E) \neq \emptyset$.

Proof: Let (U, E) be a soft β open set cover of \tilde{X} and (U, E) has no finite subcover. Then for every finite subcollection $\{(G_1, E_1), (G_2, E_2), \dots, (G_n, E_n)\}$ of (U, E) there exists $e_x \in \tilde{X}$ such that $(G_i, E_i)(e_x) \subset X$ for every $i=1, 2, 3, \dots, n$. Then $\{(G_i, E_i)(e_x)\}' \supset \emptyset$, so that $\cap_{i=1}^n (G_i, E_i)(e_x) \neq \emptyset$. Thus $\{(G_i, E_i)(e_x) : (G_i, E_i) \in (F, E)\}$ forms a soft filter base in \tilde{X} . Since (F, E) is soft β open cover of \tilde{X} , then $\{\bigcup_{(G_i, E_i) \in (F, E)} (G_i, E_i)(e_x)\} = X$ for every $e_x \in X$ and hence $\cap_{(G_i, E_i) \in (F, E)} s\beta cl\{(G_i, E_i)(e_x)\}' = \cap_{(G_i, E_i) \in (F, E)} \{(G_i, E_i)(e_x)\}' = \emptyset$ which is contradiction. Then every soft β open set cover of \tilde{X} has a finite subcover and hence (\tilde{X}, τ, E) is soft β -compact.

conversly, Suppose there exists a soft filter bases ϖ such that $\cap s\beta cl(G, E) = \emptyset$ so that $\bigcup_{(G, E) \in \varpi} \{s\beta cl(G, E)'(e_x)\} = X$, for every $e_x \in X$ and hence $\bigcup_{i=1}^n (G_j, E_j)'(e_x) = \tilde{X}$. So that $\bigcap_{i=1}^n (G_j, E_j) = \emptyset$ which is contradiction. Since (G_j, E_j) are members of filter bases ϖ . Therefore $\cap_{(G, E) \in \varpi} s\beta cl(G, E) \neq \emptyset$ for every filterbase ϖ

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