LINE CORPORATE DOMINATION GRAPH

S. PETHANACHI SELVAM, S. PADMASHINI

Abstract: In this paper, we introduce Line corporate graph $\mathcal{C}_l(G)$ and line corporate dominating set, inverse line corporate dominating set. We establish line corporate domination number, inverse line corporate domination number for some standard graphs. We prove some properties connecting line corporate dominating set and inverse line corporate dominating set. Further we prove bounds and properties related to line and inverse line corporate domination number of $\mathcal{C}_l(G)$.

Keywords: Inverse line domination set, Line corporate graph, Line corporate dominating set.

Introduction: All graphs in this paper are finite, simple and undirected. Let $G=(V,E)$ be a graph where the symbols V and E denote the vertex set and edge set of G. For all other terminology and notations, we follow Harray [1] and the definitions related to dominations are referred from T.W. Haynes & Kulli[2][3].

M.H. Muddebihal[4] introduced inverse Line Domination graph. S.Pethanachi Selvam, S.Padmashini[5] introduced Inverse Complementary domination graph.

By the motivation of the papers, we introduce Line corporate graph, line corporate domination number and Inverse line corporate domination number.

In this paper we prove the bounds and properties related to line and inverse line corporate domination number of $\mathcal{C}_l(G)$. Also, we find line & inverse line corporate domination number for some standard graphs such as cycle, star, wheel and complete bipartite graphs.

Definitions:

Definition 2.1: Let **G**(=(p,q))be a graph. A graph obtained by joining the maximum degree vertex of **G** with the maximum degree vertex of $L(G)$ (=(q,r)), the line graph of G, by a bridge is called a Line Corporate graph having p+q vertices and q+r edges and is denoted by $\mathcal{C}_l(G)$.

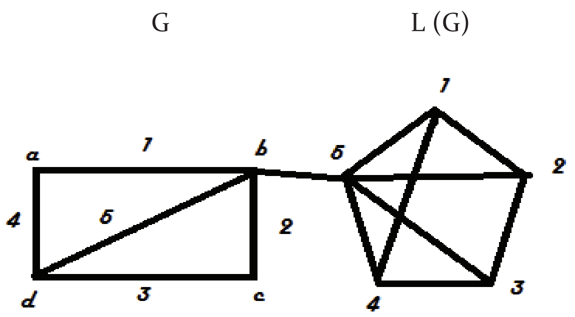
Definition 2.2: Let $\mathcal{C}_l(G)$ be a line corporate graph. A set $D \subseteq V(\mathcal{C}_l(G))$ is said to be Line Corporate Dominating Set (shortly written as $C_l DS$) if every vertex in $V-D$ is adjacent to some vertex in D .The minimum cardinality of vertices in such a set is called Line Corporate domination number of $\mathcal{C}_l(G)$ and is denoted by $\gamma(\mathcal{C}_l(G))$.

A graph having $C_l DS$ is called a Line Corporate Domination graph.

Definition 2.3: Let D be a line corporate dominating set $(C_l DS)$. If $V(\mathcal{C}_l(G))$ - D contains another $C_l DS$ namely D^{-1}, then D^{-1} is called the Inverse Line Corporate Dominating Set (shortly by $IC_l DS$) w.r.t D. The minimum cardinality of vertices in $IC_l DS$ is called Inverse Line corporate domination number and is denoted by $\gamma^{-1}(\mathcal{C}_l(G))$.

A graph having $IC_l DS$ is called an Inverse line corporate domination graph.

Example: Consider the Line corporate graph of G



$\mathcal{C}_l(G)$

Fig 2.1

Here the line corporate dominating set is {b,5} and the inverse line corporate dominating set is {d,3,1}

Results: The following propositions can be easily proved.

Proposition 3.1

For any graph G, $\gamma(\mathcal{C}_l(G)) \le \gamma^{-1}(\mathcal{C}_l(G))$ and $\gamma(\mathcal{C}_l(G)) = 1$ if and only if G is K_2.

Proposition 3.2

1. Every graph has a line corporate domination graph.
2. Every inverse line corporate dominating set is also a line corporate dominating set. Converse need not be true
3. $\gamma^{-1}(\mathcal{C}_l(G))$ = $\gamma^{-1}(G) + \gamma_l^{-1}(G)$

Main Results

Theorem 4.1

For any cycle C_p where $p \geq 3$, we have

$$\begin{cases} \gamma(\mathcal{C}_l(C_p)) = \frac{2p}{3} \; if \; p \equiv 0 \pmod 3 \\ \left\lceil \frac{2p}{3} \right\rceil otherwise \end{cases}$$

And
$$\begin{cases} \gamma^{-1}(\mathcal{C}_l(C_p)) = \frac{2p}{3} \; if \; p \equiv 0 \pmod 3 \\ 2\left\lceil \frac{p}{3} \right\rceil otherwise \end{cases}$$

Proof:

Let C_p be a cycle, where $p \equiv 0(\bmod 3)$. Then the line corporate graph C_p have two cycles joined by an edge. As any vertex of C_p dominates two vertices and $\mathcal{C}_l(C_p)$ has 2p vertices, $\gamma(\mathcal{C}_l(C_p)) = 2\left(\frac{p}{3}\right)$

Next we prove that, $\gamma(\mathcal{C}_l(C_p)) = \left\lceil \frac{2p}{3} \right\rceil$ if $p \equiv 0(\bmod 3)$.

As every vertex of C_p dominate two vertices, we can consider the following two cases.

Case1: Let $p \equiv 2(\bmod 3)$ ie, $p \equiv 3k+2$. where $k \geq 1$. Then $2p \equiv 1(\bmod 3)$ (ie,) $2p = 3k+1$, since $\mathcal{C}_l(C_p)$ has 2p vertices. Thus $\mathcal{C}_l(C_p)$ has k vertices which dominate 3k vertices and one vertex which dominate the remaining vertex. Therefore, we have $\gamma(\mathcal{C}_l(C_p)) = k+1 = \frac{2p}{3} + \frac{2}{3}$

Case 2: Let $p \equiv 1(\bmod 3)$ (ie ,) $p = 3k+1$. Then $2p \equiv 2(\bmod 3)$ (ie,) $2p \equiv 3k+2$ .Thus $\mathcal{C}_l(C_p)$ has k vertices which dominate 3k vertices and one vertex which dominate the remaining two vertices. Therefore,. $\gamma(\mathcal{C}_l(C_p)) = k+1 = \frac{2p}{3} + \frac{1}{3}$

From the above two cases, we have $\gamma(\mathcal{C}_l(C_p)) = \left\lceil \frac{2p}{3} \right\rceil$.

The proof of $\gamma^{-1}(\mathcal{C}_l(C_p)) = 2\left(\frac{p}{3}\right)$ if $p \equiv 0(\bmod 3)$ is similar to the result $\gamma(\mathcal{C}_l(C_p)) = 2\left(\frac{p}{3}\right)$ if $p \equiv 0(\bmod 3)$

As , $\gamma^{-1}(C_p) = \left\lceil \frac{p}{3} \right\rceil = \gamma_l^{-1}(C_p)$, we have

$$\gamma^{-1}(\mathcal{C}_l(C_p)) = \gamma^{-1}(C_p) + \gamma_l^{-1}(C_p) = \left\lceil \frac{p}{3} \right\rceil + \left\lceil \frac{p}{3} \right\rceil = 2\left\lceil \frac{p}{3} \right\rceil$$

Theorem 4.2:

(a) For any Star graph $K_{1,p}$,

(i) $\gamma(\mathcal{C}_l(K_{1,p})) = 2$ (ii) $\gamma^{-1}(\mathcal{C}_l(K_{1,p})) = p+1$

(b) For any Complete bipartite graph $K_{p,p}$,

$\gamma(\mathcal{C}_l(K_{p,p})) = \gamma^{-1}(\mathcal{C}_l(K_{p,p})) = p+2$

Proof:

(a) (i) Proof is obvious.

(ii) Let $K_{1,p}$ be a Star graph with p+1 vertices. Then $\mathcal{C}_l(K_{1,p})$ is formed by joining the maximum degree vertex of $K_{1,p}$ and the maximum degree vertex of K_p by an edge. Then by (i) IC_lDS of $\mathcal{C}_l(K_{1,p})$ contains p vertices of $K_{1,p}$ and one of the vertices of K_p.

(b)Let $K_{p,p}$ be a complete graph having 2p vertices. As $\gamma(K_{p,p}) = 2 = \gamma^{-1}(K_{p,p})$ and $\gamma_l(K_{p,p}) = p = \gamma_l^{-1}(K_{p,p})$, the result follows.

Theorem 4.3

Let W_p $(p \geq 3)$ be a wheel graph with $q(=2(p-1))$ edges. Then

(i) $\gamma(\mathcal{C}_l(W_p)) = 2 + \left\lceil \frac{q-4}{6} \right\rceil$

(ii)
$$\gamma^{-1}(\mathcal{C}_l(W_p)) = \begin{cases} 1 + \left\lceil \frac{p}{3} \right\rceil + \left\lceil \frac{q+2}{6} \right\rceil if \; \frac{q}{2} - 2 \equiv 0 \pmod 3 \\ 1 + \left\lceil \frac{p}{3} \right\rceil + \left\lceil \frac{q-4}{6} \right\rceil otherwise \end{cases}$$

Proof: Let W_p be a wheel graph having p vertices and 2(p-1) edges and its line graph having 2(p-1) vertices and $\frac{p^2+3p-4}{2}$ edges. Then the line corporate graph have 3p-2 vertices and $\frac{p^2+7p-6}{2}$ edges.

(i) In $\mathcal{C}_l(W_p)$, the line corporate dominating set of W_p contains one vertex of W_p which dominate at least (p-1) vertices and one of the vertices having highest degree in $L(W_p)$ dominate at least p vertices. Choose the remaining $p-3\left(=\frac{q}{2}-2\right)$ vertices of $\mathcal{C}_l(W_p)$ in the form of path. Therefore, $\left\lceil \frac{\frac{q}{2}-2}{3} \right\rceil$ vertices dominate the remaining p-3 vertices. If we choose the disjoint path, then it may not give the minimum cardinality. Thus we have

$$\gamma(\mathcal{C}_l(W_p)) = 1+1+\left\lceil \frac{\frac{q}{2}-2}{3} \right\rceil = 2+\left\lceil \frac{q-4}{6} \right\rceil$$

(ii) In $\mathcal{C}_l(W_p)$, the inverse line corporate dominating set contains $\left\lceil \frac{p}{3} \right\rceil$ vertices which dominate all the vertices of W_p, since it is in of the form of a cycle. Also $\mathrm{IC}_l DS$ contains one of the $\frac{q}{2}-1$ vertices of highest degree which dominate p vertices in $L(W_p)$. Then, the remaining $\frac{q}{2}-2$ vertices of $L(W_p)$ which are in the form of path. Considered the following two cases

Case (i): Let if $\left(\frac{q}{2}-2\right) \equiv 0 (\mathrm{mod}\, 3)$. Then $\mathrm{IC}_l DS$ contains $\left\lceil \frac{\frac{q}{2}-2}{3}+1 \right\rceil$ vertices which dominated all the remaining vertices i.e) $\frac{q-4}{2}$ vertices. Thus, $\gamma^{-1}(\mathcal{C}_l(W_p)) = 1+\left\lceil \frac{p}{3} \right\rceil+\left\lceil \frac{q+2}{6} \right\rceil$.

Case (ii): Let $\left(\frac{q}{2}-2\right) \equiv 0 (\mathrm{mod}\, 3)$. As in the proof of (i), obviously $\mathrm{IC}_l DS$ contains $\left\lceil \frac{q-4}{6} \right\rceil$ vertices which dominate the remaining vertices of $L(W_p)$.

Hence $\gamma^{-1}(\mathcal{C}_l(W_p)) = 1+\left\lceil \frac{p}{3} \right\rceil+\left\lceil \frac{q-4}{6} \right\rceil$.

Theorem 4.4: Let G be a connected graph. Then

(i) $\gamma(\mathcal{C}_l(G)) \le \gamma(G)+\gamma_l(G) \le \gamma^{-1}(\mathcal{C}_l(G))$

(ii) $\gamma(\mathcal{C}_l(G))+\gamma^{-1}(\mathcal{C}_l(G) \le 2(\gamma^{-1}(G)+\gamma_l^{-1}(G))$

(iii) $\gamma(\mathcal{C}_l(G))+\gamma^{-1}(\mathcal{C}_l(G) \le p+q$ and equality holds for p_3 & p_4.

Proof: Proof is obvious.

Theorem 4.5

For any graph G, we have

$$\left\lfloor \frac{\delta(\mathcal{C}_l(G))}{\Delta(\mathcal{C}_l(G))} \right\rfloor \le \gamma\ (\mathcal{C}_l(G)) \le \left\lceil \frac{p+q}{2} \right\rceil.$$

If $\delta(\mathcal{C}_l(G)) = \Delta(\mathcal{C}_l(G))$ then the result holds, as $\gamma(\mathcal{C}_l(G)) \ge 1$

Suppose $\delta(\mathcal{C}_l(G)) < \Delta(\mathcal{C}_l(G))$, Then its range is zero and hence $\left\lfloor \frac{\delta(\mathcal{C}_l(G))}{\Delta(\mathcal{C}_l(G))} \right\rfloor \le \gamma(\mathcal{C}_l(G))$

As $\gamma(\mathcal{C}_l(G))+\gamma^{-1}(\mathcal{C}_l(G)) \le p+q$ and $\gamma(\mathcal{C}_l(G)) \le \gamma^{-1}(\mathcal{C}_l(G))$, we have $\gamma(\mathcal{C}_l(G))+\gamma\ (\mathcal{C}_l(G)) \le p+q$ .Thus $\gamma(C_l(G)) \le \frac{p+q}{2} \le \left\lceil \frac{p+q}{2} \right\rceil$ and hence proved.

The following theorems can be easily proved.

Theorem 4.6: For any graph G, the inverse line corporate domination graph exist if and only if neither the graph G nor L (G) has isolated vertices.

Theorem 4.7: For any connected line corporate graph $\mathcal{C}_l(G)$, $\gamma(\mathcal{C}_l(G)) = 2$ if and only if G has at least one vertex of degree (p-1) and $L(G)$ has at least one vertex of degree $(q-1)$.

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S. Pethanachi Selvam/Associate Professor/Departmentof Mathematics/
The Standard Fireworks Rajaratnam College for Women/Sivakasi-626123/TamilNadu/India.
S. Padmashini/M.Phil Scholar/Department of Mathematics/
Sri Ramasamy Naidu Memorial College/Sattur/TamilNadu/India.