EFFICIENT TRIPLE CONNECTED EDGE DOMINATION NUMBER OF A GRAPH

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Abstract: The concept of triple connected graphs with real life application was introduced in [2] by considering the existence of a path containing any three vertices of a graph G. In [1], G Mahadevan et,al., introduced the concept of triple connected domination number of a graph. A subset S of E of a nontrivial connected graph G is said to be triple connected edge dominating set, if S is a edge dominating set and the induced subgraph <S> is triple edge connected . The minimum cardinality taken over all triple connected edge dominating set is called the triple connected edge domination number and is denoted by r'_{tc} . A subset S of E of a nontrivial graph G is said to be an efficient edge dominating set, if every edge is dominated exactly once. The minimum cardinality taken over all efficient edge dominating sets is called the efficient edge domination number & is denoted byr'_e. In this paper we introduce new domination parameter efficient triple connected edge domination number of a graph with real life application. A subset S of E of a nontrivial connected graph G is said to be an efficient triple connected edge dominating set, if S is an efficient edge dominating set & the induced subgraph <S> is triple edge connected. The minimum cardinality taken over all efficient triple connected edge dominating set is called the efficient triple connected edge domination number & is denoted by γ'_{etc}. We also determine this number for some standard graphs & obtain bounds for general graph.

Keywords : Edge Domination number, Triple edge connected graph, triple connected edge domination number, Efficient triple connected edge domination number.

Introduction: All graphs considered here are finite, undirected without loops and multiple edges. As usual p and q denote the number of vertices and edges of a graph G=(V,E) respectively.

A set F of edges in a graph G=(V,E) is called an edge dominating set of G if every edge in E-F is adjacent to at least one edge in F. The edge domination number γ'(G) of a graph G is the minimum cardinality of an edge dominating set of G. This concept was introduced by [5].

An edge dominating set F of graph G is a connected edge dominating set if the induced subgraph <F> is connected. The connected edge domination number $\gamma'_c(G)$of G is the minimum cardinality of a connected edge dominating set. This concept was introduced by Kulli and Sigarkanti[7].

The concept of efficient edge domination number was studied by S.C.Sigarkanti[4]. A set F of edges in a graph G is an efficient edge dominating set if every edge not in F is adjacent to exactly one edge in F . The efficient edge domination number γ'_e(G) of G is the order of a smallest efficient edge dominating set of G.

A subset S of V of a nontrivial connected graph G is said to be an efficient triple connected dominating set, if S is an efficient dominating set and the induced subgraph <S> is triple connected. The minimum cardinality taken over all efficient triple connected dominating sets is called the efficient triple connected domination number and is denoted by γ_{etc}. This concept was introduced by [2].

An edge cover is a set F of edges which covers every vertex in V. The edge covering number α_1(G) is the minimum cardinality of an edge cover.

A set of edges in G is an independent set of edges known as matching of G has no two of its edges adjacent and the maximum number of edges in such a set is the edge independence number β_1(G).

Recently, the concept of triple connected graphs has been introduced by Paulraj Joseph et. Al. [3] by considering the existence of a path containing any three vertices of G. They have studied the properties of triple connected graphs and established many results on them. A graph G is said to be triple connected if any three vertices lie on a path in G. All paths, cycles, complete graphs and wheels are some standard examples of triple connected graphs. A subset S of V of a nontrivial connected graph G is said to be a Smarandachely triple connected dominating set, if S is a dominating set and the induced subgraph <S> is triple connected. The minimum cardinality taken over all Smarandachely triple connected dominating sets is called the Smarandachely triple connected dominating set with γ_{tc}(G). Any Smarandachely triple connected dominating set with γ_{tc}vertices is called aγ_{tc}-set of G. This concept was introduced by [1]. A subset S of V of a nontrivial connected graph G is said to be triple connected edge dominating set, if S is a edge dominating set and the induced subgraph <S> is triple edge connected. The minimum cardinality taken over all triple connected edge dominating sets is called the triple connected edge domination number of G and is denoted by $\gamma'_{tc}(G)$. This concept was introduced by [6]. A subset S of E of a nontrivial connected graph G is said to be an efficient triple connected edge dominating set, if S is an efficient edge dominating set & the induced subgraph <S> is triple edge connected. The minimum cardinality taken over all efficient triple connected edge dominating set is called the efficient triple connected edge domination number & is denoted by γ'_{etc}.

Efficient Triple Connected Edge Domination Number Of A Graph:

Definition 2.1 : A subset S of E of a nontrivial graph G is said to be a efficient triple connected edge dominating set, if S is a efficient edge dominating set and the induced subgraph <S> is triple edge connected. The minimum cardinality taken over all efficient triple connected edge dominating sets is called the efficient triple connected edge domination number of G and is denoted by γ'_{etc}(G). Any efficient triple connected edge dominating set with γ'_{etc} edges is called a γ'_{etc}-set of G.

Example 2.2: For the graph G_1 in figure S = { e_4, e_5, e_7} forms a γ'_{etc}-set of G_1. Hence $\gamma'_{etc}(G_1)$ =3.

G_1:

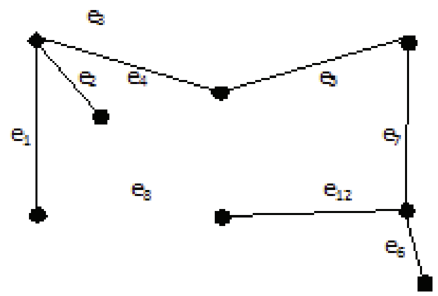

Figure 2.1 : Graph with γ'_{etc}=3

Observation 2.3 : Efficient triple connected edge dominating set does not exists for all graphs and if exists, then $\gamma'_{etc} \geq 3$.

Remark 2.4 : Throughout this paper we consider only connected graphs for which efficient triple connected edge dominating set exists.

Observation 2.5 : The complement of the efficient triple edge connected dominating set need not be a efficient triple connected edge dominating set.

Example 2.6 : For the figure 2.2 S ={ e_2, e_3, e_4 } forms a efficient triple connected edge dominating set of G_2 .But the complement E-S ={e_1, e_5, e_6, e_7, e_8} is not a efficient triple edge connected dominating set.

G_2:

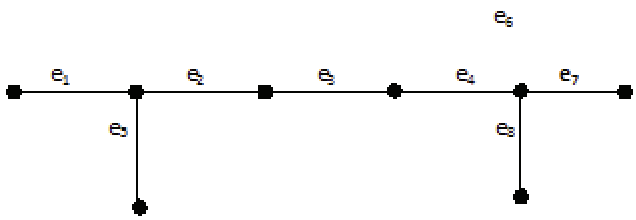

Figure 2.2 : Graph in which E-S is not an γ'_{etc}-set

Observation 2.7 : Every efficient triple connected edge dominating set is a edge dominating set but converse is not true .

Observation 2.8 : Every efficient triple connected edge dominating set is a triple connected edge dominating set but converse is not true .

Example 2.9 : For the graph G_3 in figure , S = {e_2, e_3, e_4} forms the efficient triple connected edge dominating set.

G_3:

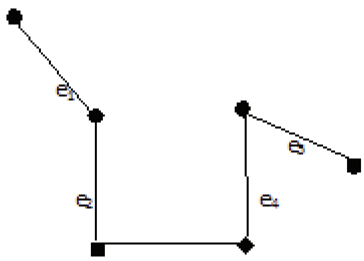

Figure 2.3

Exact value for some standard graphs :

1. For any cycle of order p≥3, γ'_{etc}(G) = $\begin{cases} p if p < 5 \\ p-2\ if p \geq 5 \end{cases}$
2. For any path of order p≥ 3, γ'_{etc}(G) = $\begin{cases} p if p = 4 \\ p-1\ if p = 5 \\ p-2\ if p \geq 6 \end{cases}$
3. For any complete graph of order p≥ 3, $\gamma'_{etc}(K_p)$ = p-1.
4. For any complete bipartite graph of order p≥ 4, , $\gamma'_{etc}(K_{m,n})$ = m.
5. For any star of order p≥3, , $\gamma'_{etc}(K_{1,n})$ = 3.
6. For any wheel of order p≥ 4, $\gamma'_{etc}(W_p)$ = p-1.

Theorem 2.10 : For any connected graph G with p≥3, we have 3≤ γ'_{etc} ≤p and the bounds are sharp.

Proof : The lower and upper bound follows from the definition.

Theorem 2.11 : For any connected graph G,

γ'_{etc}(G)≤ q - Δ′(G)

Proof : Let e be an edge of degree Δ′ . Then clearly E-N(e) is efficient triple connected edge dominating set of G.

Hence, γ'_{etc}(G)≤ q - Δ′(G).

Theorem 2.12 : For any graph G with p≥ 3 vertices,

$\gamma'_{etc}(G) \leq \beta_1(G)$

Proof : Let F be minimum efficient triple connected edge dominating set of G and let S be maximum independent edge set in E-F. Then every edge in E-(F∪S) must be adjacent to exactly one edge in S, then S is efficient triple connected edge dominating set of G. Otherwise ,let us suppose that $F' \subset$F such that no edge in F' is adjacent to an edge in S. Then every edge in F' is adjacent to exactly one edge in E –(F∪S). Then for each edge in F', pick E-(F∪S) edges adjacent to it and let R be the set of all such edges,

Then, $|R| \leq |F'|$

$\leq |S \cup R|$

$\leq |S| + |F'|$

$\gamma'_{etc}(G) \leq \beta_1(G)$

Theorem 2.13 : For any nontrivial tree T,

γ'_{etc}(T) ≤ n, where n is number of cutvertices of T.

Proof : The result follows from the fact that every edge is adjacent with a cut vertex. Hence the proof.

Theorem 2.14 : For any graph G with q edges ,

$\gamma'_{etc}(G) \leq \frac{q}{2}$

Proof : Let G be a graph with q edges. Let S be efficient triple connected edge dominating set of G.

Then, $|S| \leq \frac{q}{2}$

Hence, $\gamma'_{etc}(G) \leq \frac{q}{2}$

Theorem 2.15 : For any graph G,

$\gamma'_{etc}(G) \leq \alpha_1(G)$, where $\alpha_1(G)$ is line covering number of G.

Proof : Let G be a graph with q edges and S be line cover of G.

Suppose , $|S| \leq \frac{q}{2}$

Then S is efficient triple connected edge dominating set of G.

From theorem(2.14)

$\gamma'_{etc}(G) \leq \frac{q}{2}$

$\leq \alpha_1(G)$.

Hence the proof .

Theorem 2.15 : For any connected graph G with p≥3 vertices , $\gamma'_{etc}(G) + \kappa(G) \leq$ 2p-1.

Proof : Let G be a connected graph with p≥ 3 vertices. We know that $\kappa(G) \leq$ p-1 and from theorem (2.10), γ'_{etc}(G) ≤p. Hence $\gamma'_{etc}(G) + \kappa(G) \leq$ 2p-1.

Theorem 2.16: For any connected graph G with p≥3 vertices , $\gamma'_{etc}(G) + \chi(G) \leq$ 2p.

Proof : Let G be a connected graph with p≥ 3 vertices . We know that $\chi(G) \leq$ p and from theorem(2.10) , γ'_{etc}(G) ≤p. Hence $\gamma'_{etc}(G) + \chi(G) \leq$ 2p.

Conclusion: Real life application of triple connected edge domination number & efficient triple connected edge domination number.

The concept of triple connected graphs with real life application was introduced by [3] by considering the existence of a path containing any three vertices of a graph G. In [6] introduced the concept of triple connected edge domination number of a graph. In this we introduce new domination parameter efficient triple connected edge domination number of a graph with real life application.

Suppose we want to locate four servers in four different organizations in a city. First we construct a graph whose nodes (vertices) indicating the servers and the edges indicating the paths which are connecting the servers. Check whether this graph is triple edge connected or not.

Case 1 : If the connected graph is triple edge connected, then any of the four servers can be located in any of the four nodes so that if anyone of the four server is not working then the other three can be used immediately.

Case 2 : If the connected graph is not triple edge connected, then construct a triple edge connected dominating set in the graph & then the four servers can be located in any of the four nodes in the triple connected edge dominating set of the graph so that the edges dominated by the triple edge connected dominating set can be linked to one server directly and the other three servers whenever necessary.

Case 3 : If the constructed graph is neither triple edge connected nor we can find a triple connected edge dominating set in the graph, then do some necessary corrections in the construction (either by increasing edge or by increasing nodes) and make the constructed graph a triple edge connected or a triple connected edge dominating set .

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