INVESTIGATION OF DISPERSION EFFECTS OVER A VERTICAL PLATE SATURATED WITH NON-DARCY POROUS MEDIUM

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Abstract: The present investigation deals with the study of unsteady, free convective, MHD, chemically reacting fluid flow over a vertical flat plate saturated with non- Darcy porous medium in the presence of double dispersion effect. The numerical computation for the governing equations of two dimensional boundary layer flow has been performed using implicit finite difference method of Crank- Nicolson type. The behavior of velocity, temperature and concentration distributions for various controlling parameters of this problem are graphically illustrated and discussed in detail. The average skin friction, Nusselt number and Sherwood number for sundry parameters are presented in table.

Keywords: chemical reaction, double dispersion effect, MHD, non-Darcy porous medium.

Introduction: The study of Magnetohydrodynamic flow has been a subject of intense research due to its overwhelming importance in numerous fields ranging from several natural phenomena like meteorology, solar physics, cosmic fluid dynamics, astrophysics, geophysics and in the motion of earth's core. The velocity fluctuations are suppressed due to the damping nature of the external magnetic field. [1-2].

The development of dispersion theory has been mainly related to miscible displacement and solute spreading in porous media. These areas are of major interest in secondary and tertiary oil recovery operations and pollution control in water resources engineering [3-4].

The Darcy - Forchheimer model describes the effect of inertia as well as viscous forces in porous media. In many cases, porous media with high permeability, the viscous effects due to frictional drag at the boundary and the inertia effects within a porous medium become significant. The analysis of convective transport in a porous medium subject to a non-uniform porosity distribution and high flow rate near the wall has a variety of applications in many biomechanical problems, filtration transpiration cooling and geothermal [5-6].

The combined heat and mass transfer with chemical reaction effects plays an important role in designs of many chemical processing equipments, polymer production, manufacturing of ceramics and distributions of temperature moisture over agricultural fields [7-8].

The motivation of the present study is to bring out the effect of double dispersion on chemically reacting viscous fluid flow over a vertical flat plate saturated with non-Darcy porous medium. The governing equations are solved by using Crank-Nicolson method and the results obtained are displayed in form of graphs and table.

Formulation of the problem: Consider a two-dimensional, unsteady, viscous incompressible electrically conducting fluid flow over a vertical flat plate saturated with non-Darcy porous medium. *x*-axis is assumed to be taken along the plate and *y*-axis normal to the plate. The wall $y=0$ is maintained at constant temperature T_w and concentration C_w, higher than the ambient temperature T_∞ and ambient concentration C_∞, respectively. The fluid properties are assumed to be constant except for density variations in the buoyancy force term. The flow is influenced by a uniform transverse magnetic field of strength B_0. The energy and mass diffusion equations are influenced by double dispersion effect. Assuming that the Boussinesq and boundary layer approximation hold and using the Darcy-Forchheimer model, the basic equations, which govern the problem, are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$\frac{\partial u}{\partial t^*}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0{}^2}{\rho}u-\frac{C_b}{\sqrt{k_1}}u^2-\frac{\nu}{k_1}u$$
$$+g\beta_T(T-T_\infty)+g\beta_C(C-C_\infty) \qquad (2)$$

$$\frac{\partial T}{\partial t^*}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha_e\frac{\partial T}{\partial y}\right) \qquad (3)$$

$$\frac{\partial C}{\partial t^*}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{\partial}{\partial y}\left(D_e\frac{\partial C}{\partial y}\right)$$
$$-K_R\left(C-C_\infty\right) \qquad (4)$$

The appropriate initial and boundary conditions of the problem are

$t^*\le 0: u=0, v=0, T=T_\infty, C=C_\infty$ for all *x, y*

$t^*>0: u=0, v=0, T=T_w, C=C_w$ at $y=0$

$u=0$, $T=T_\infty$, $C=C_\infty$ at $x=0$

$u\to 0, T\to T_\infty, C\to C_\infty$ as $y\to\infty$ (5)

The quantities α_e and D_e are defined as $\alpha_e = \alpha + \gamma \, du$ and $D_e = D + \xi \, du$, where α and D are the constant thermal and molecular diffusivities, respectively, whereas γ and ξ are the coefficients of the thermal and solutal dispersions, respectively and d is the pore diameter.

Introducing the following non-dimensional quantities

$$X = \frac{x}{L}, Y = \frac{y}{L}(Gr_T)^{1/4}, U = \frac{uL}{\nu}(Gr_T)^{-1/2},$$

$$V = \frac{\nu L}{\nu}(Gr_T)^{-1/4}, t = \frac{\nu t^*}{L^2}(Gr_T)^{1/2}, \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$\phi = \frac{C - C_\infty}{C_w - C_\infty}, Gr_T = \frac{g\beta_T (T_w - T_\infty) L^3}{\nu^2}, F_I = \frac{C_b L}{\sqrt{k_1}},$$

$$M = \frac{\sigma B_0^2 L^2}{\mu (Gr_T)^{1/2}}, \frac{1}{K} = \frac{L^2}{K_1 (Gr_T)^{1/2}}, N = \frac{\beta_T (C_w - C_\infty)}{\beta_C (T_w - T_\infty)},$$

$$\mathrm{P_r} = \frac{\mu C_P}{k}, \gamma = \frac{\gamma^* (Gr_T)^{1/2} d}{L}, K_r = \frac{K_R L^2}{\nu (Gr_T)^{1/2}}$$

$$Sc = \frac{\nu}{D}, \xi = \frac{\xi^* (Gr_T)^{1/2} d}{L} \quad (6)$$

In view of Eqn. (6), the basic equations can be written as

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (7)$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \frac{\partial^2 U}{\partial Y^2} - F_I U^2 - \left(M + \frac{1}{K}\right)U + \theta + N\phi \quad (8)$$

$$\frac{\partial \theta}{\partial t} + U\frac{\partial \theta}{\partial X} + \left(V - \gamma\frac{\partial U}{\partial Y}\right)\frac{\partial \theta}{\partial Y} = \left(\frac{1}{\mathrm{Pr}} + \gamma U\right)\frac{\partial^2 \theta}{\partial Y^2} \quad (9)$$

$$\frac{\partial \phi}{\partial t} + U\frac{\partial \phi}{\partial X} + \left(V - \xi\frac{\partial U}{\partial Y}\right)\frac{\partial \phi}{\partial Y} = \left(\frac{1}{Sc} + \xi U\right)\frac{\partial^2 \phi}{\partial Y^2} - K_r \phi \quad (10)$$

The dimensionalized initial and boundary conditions are

$t \le 0 : U = 0, V = 0, \theta = 0, \phi = 0$ for all X, Y

$t > 0 : U = 0, V = 0, \theta = 1, \phi = 1$ at Y = 0

$U = 0, \theta = 0, \phi = 0$ at X = 0

$U \to 0, \theta \to 0, \phi \to 0$ as $Y \to \infty$ (11)

The local skin Friction, local Nusselt numbers and local Sherwood number at the wall, which is defined as follows

$$\tau = -\left(\frac{\partial U}{\partial Y}\right)_{Y=0}, Nu = -X\left(\frac{\partial \theta}{\partial Y}\right)_{Y=0}, Sh = -X\left(\frac{\partial \varphi}{\partial Y}\right)_{Y=0} \quad (12)$$

Average skin friction, Nusselt number and Sherwood number is expressed as:

$$\overline{\tau} = -\int_0^1 \left(\frac{\partial U}{\partial Y}\right)_{Y=0} dX, \ \overline{Nu} = -\int_0^1 \frac{1}{\theta_Y = 0}\left(\frac{\partial \theta}{\partial Y}\right)_{Y=0} dX,$$

$$\overline{Sh} = -\int_0^1 \frac{1}{\phi_Y = 0}\left(\frac{\partial \phi}{\partial Y}\right)_{Y=0} dX \quad (13)$$

Finite Difference Scheme: The set of coupled non-linear differential Eqns. (7) - (10) subjected to the initial and boundary conditions (11) are solved by implicit finite difference scheme of Crank– Nicolson type. We consider a rectangular region with X varying from 0 to 1 and Y varying from 0 to 22, where Y_{max} is regarded as $Y \to \infty$. It is ensured that Y_{max} lies well outside the dynamic, thermal and mass diffusion boundary layers. The mesh spacing in the X and Y directions $\Delta X = 0.05$, $\Delta Y = 0.05$ and time interval $\Delta t = 0.01$ are selected to obtain the tolerance limit within 10^{-5}. Computations are repeated until the steady state is reached. A convergence criterion based on the relative difference between the two consecutive iteration values is employed. When the difference reaches less than 10^{-5} at all grid points, the solution is assumed to have converged and the iterative process is terminated. The scheme is unconditionally stable. The local truncation error is $O(\Delta t^2 + \Delta X^2 + \Delta Y^2)$ and it tends to zero as Δt, ΔX and ΔY tend to zero. It follows that the Crank-Nicolson method is compatible. Stability and compatibility ensure the convergence.

Result and discussion: This section provides a manifestation of numerical calculations for various governing parameters on the velocity, temperature and concentration distributions which are illustrated in form of figures 1-8. Based on the previous investigations, the governing parameters are fastened with following default parametric values: M = 3, K = 2, F_I = 1, N = 0.5, P_r = 7, γ = 0.3, S_c = 2, K_r = 3, ξ = 0.3 unless specifically indicated on the appropriate graph.

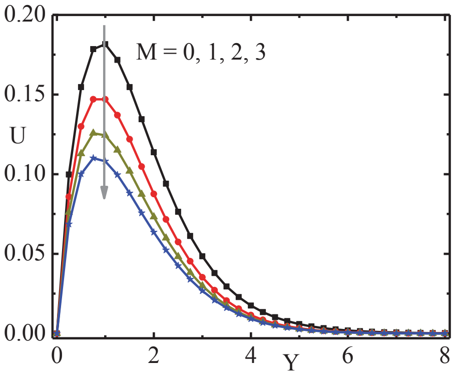

Fig 1: Effect of M on U

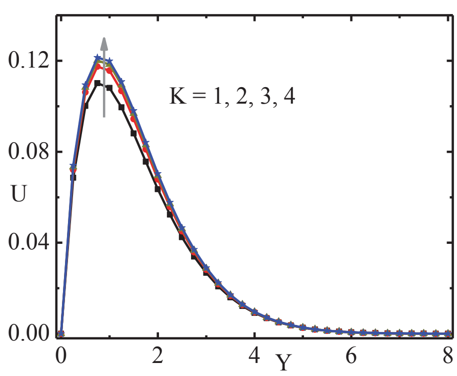

Fig 2: Effect of K on U

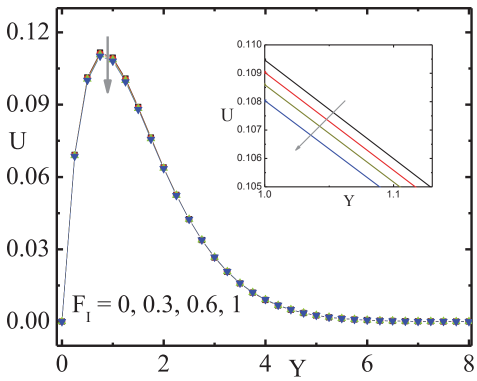

Fig 3: Effect of F_I on U

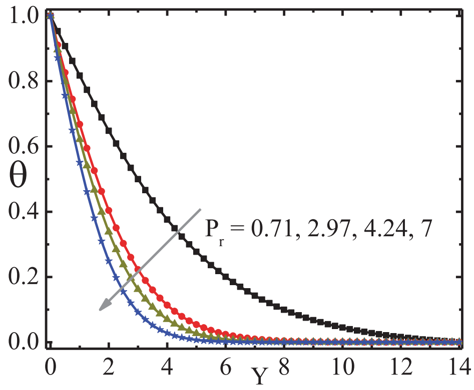

Fig 4: Effect of P_r on θ

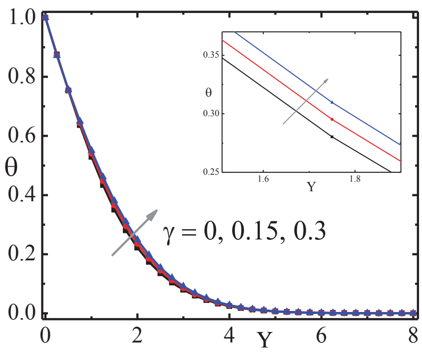

Fig 5: Effect of γ on θ

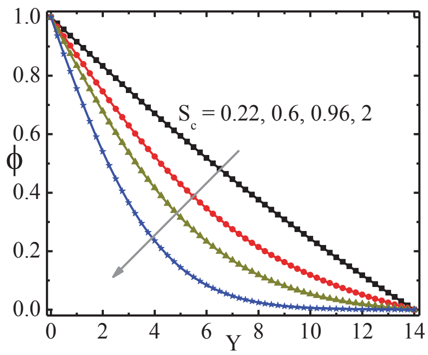

Fig 6: Effect of S_c on ϕ

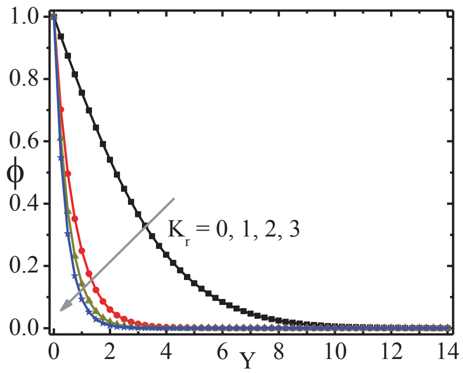

Fig 7: Effect of K_r on ϕ

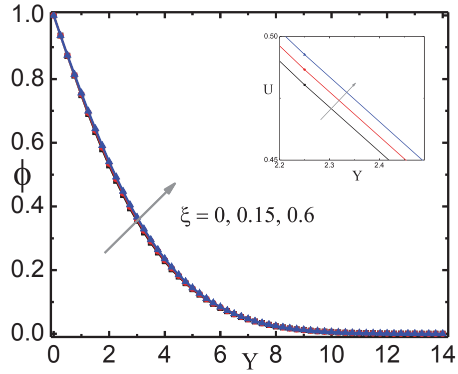

Fig 8: Effect of ξ on ϕ

Figures 1-3 presents the effect of M , K and F_I on velocity profiles respectively.

Figure 1 shows that fluid velocity decreases for strengthening the transverse magnetic field. An increase in porous permeability parameter accelerates the velocity along the boundary layer by deteriorating the resistivity of porous medium which is demonstrated in Fig. 2. Fig. 3 elucidates that an increase in Forchheimer number increases the resistance to the flow which diminishes the velocity.

Fig. 4 and 5 are plotted to show the influence of Pr and γ on temperature profiles respectively. It is clear from Fig. 4 that higher values of Prandtl number have the tendency to reduce the thermal conductivity of the fluid in the boundary layer as well as the thermal boundary layer thickness. Fig. 5 elucidates that the temperature profiles increase for increasing the thermal dispersion parameter. It seems the dispersion effects are important for sensitive control of the heat transfer in a small region near the wall.

The effect of S_c, K_r and ξ on concentration profiles are presented in Figs. 6 - 8, respectively. It is observed from Fig. 6 that the mass transfer is diminished for increasing the values of Schmidt number. Physically it is true, since the higher values of S_c have the tendency to dilute molecular diffusivity and therefore decrease in thickness of the concentration boundary layer. It is apparent from Fig. 7 that the presence of a destructive chemical reaction within the boundary layer has the tendency to decrease the concentration. Figure 8 elucidates that the concentration profiles increase for increasing the solutal dispersion parameter. Table 1 presents the effect of M , K , F_I , γ , K_r and ξ on average skin friction, Nusselt number and Sherwood number.

Table 1:

Physical parameters	Values	$\bar{\tau}$	$\overline{Nu}$	$\overline{Sh}$
M	0	0.41583	1.04294	0.52055
	1	0.37604	0.96356	0.46918
	2	0.34640	0.90298	0.43395
K	1	0.32283	0.85401	0.40770
	2	0.33401	0.87731	0.41996
	3	0.33799	0.88558	0.42441
F_I	0	0.32416	0.85722	0.40917
	0.3	0.32375	0.85625	0.40872
	0.6	0.32335	0.85528	0.40828
γ	0	0.32173	0.79909	0.40003
	0.15	0.32226	0.82820	0.40391
	0.3	0.32283	0.85401	0.40770
K_r	0	0.32283	0.85401	0.40770
	1	0.32283	0.85401	1.44387
	2	0.32283	0.85401	1.98113

ξ	0	0.32283	0.85401	0.40214
	0.15	0.32283	0.85401	0.40502
	0.3	0.32283	0.85401	0.40770

Conclusions: The main findings of the present study are summarized as follows: Velocity profiles decreases for increasing the value of the magnetic field parameter and Forchheimer number while it increases for increasing the value of porous permeability parameter. The heat transfer enhances with an increase in the thermal dispersion parameter whereas it diminishes with an increase in the values of Prandtl number. The concentration of the fluid decreases for increasing the Schmidt number and chemical reaction parameter while the reverse trend is observed for the solutal dispersion parameter. It seems the dispersion effects are important for sensitive control of the heat transfer in a small region near the wall.

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References:

1. A.J. Chamkha, "Unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption", Int. J. Engrg. Sci., Vol. 42, 2004, pp. 217-230.
2. Fahad Al Basir , Sumit Nandi , Priti Kumar Roy, Optimal Control on Saponification to Maximize; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 3 Issue 1 (2014), Pg 441-446
3. R. Sivaraj and B. RushiKumar, "Viscoelastic fluid flow over a moving vertical cone and flat plate with variable electric conductivity", Int. J. Heat Mass Trans., Vol. 61, 2013, pp. 119-128.
4. R.R. Kairi, and P.V.S.N. Murthy, "Effect of double dispersion on mixed convection heat and mass transfer in a non-Newtonian fluid-saturated non-Darcy porous medium", J. Porous Med., Vol. 13, 2010, pp. 749-757.
5. B. Rushikumar and R. Sivaraj, "MHD viscoelastic fluid non-Darcy flow over a vertical cone and a flat plate", Int. Commun. Heat Mass Transf., Vol. 40, 2013, pp. 1-6.
6. *M.Anbuchelvi,* Wearer form of Contra Super Continuity; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 2 Issue 2 (2013), Pg 490-493
7. A.A. Afify, "Effects of variable viscosity on non-Darcy MHD free convection along a non-isothermal vertical surface in a thermally stratified porous medium", Appl. Math. Model., Vol. 31, 2007, pp. 1621-1634.
8. D.A. Nield and A. Bejan, "Convection in Porous Media" Springer-Verlag, Berlin, 1992.
9. D. Pal and H. Mondal, "Influence of chemical reaction and thermal radiation on mixed convection heat and mass transfer over a stretching sheet in Darcian porous medium with Soret and Dufour effect", Energ. Convers. Manage., Vol. 62, 2012, pp. 102-108.
10. Dr. M. Jeyaraman, S. Chinthamani Vasthirani, Dr. R. Muthuraj, Study of **(Λ, M-A)**-Closed Sets; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 3 Issue 2 (2014), Pg 829-832
11. M.M. Rashidi, N. Rahimzadeh, M. Ferdows, M.J. Uddin, O.A Beg, "Group theory and differential transform analysis of mixed convective heat and mass transfer from a horizontal surface with chemical reaction effects", Chem. Engrg. Commun., Vol. 199, 2012, pp. 1012-1043.

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