MAGNETO HYDRODYNAMIC BOUNDARY LAYER FLOW OF A DISSIPATING NANOFLUID PAST AN EXPONENTIAL STRETCHING SHEET WITH CONVECTIVE BOUNDARY CONDITION

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Abstract: This study investigates the influence of viscous dissipation on MHD two dimensional steady boundary layer flow of nanofluid over an exponential stretching sheet with convective boundary condition. The governing partial differential equations with the appropriate boundary conditions are reduced to a set of ordinary differential equations by using similarity transformations. The resultant equations are then solved numerically using Runge - Kutta fourth order method along with shooting technique. Two types of nanofluids, namely, cupper-water and alumina-water are considered. The velocity and temperature as well as the shear stress and heat transfer rates are computed. The influence of various pertinent parameters on the flow and heat transfer characteristics is discussed. The present study helps to understand the efficiency of heat transfer transport in nanofluids which are likely to be the smart coolants of the next generation.

Keywords: Exponential Stretching sheet, MHD,Nanofluid,Viscous dissipation.

Introduction: Thermal conductivities of fluids plays an important role in the heat transfer coefficient, so many methods have been used to enhance the thermal conductivity of fluids. However, it is observed that the enhancement of the thermal conductivity of poor heat transfer fluids such as oil, water, and ethylene glycol is possible in view of the addition of nanoparticles in the base fluid. The term “nanofluid” was first introduced by Choi[1-3].The magnetic nanofluids are important to guide the particles up the bloodstream to a tumor with magnets.In fact, magnetic nanoparticles are more adhesive to tumor cells than nano-malignant cells and therefore absorb much more power than microparticles in alternating current magnetic fields tolerable in humans [4-6]. Viscous Dissipation is the rate at which the work done against viscous forces is irreversibly converted into internal energy and the literature has many shades of problem on that [7-8]. Gebhart [9] was the first who studied the problem taking into account the viscous dissipation.Recently, Habibi and Jahangiri [10] studied the forced convection boundary layer magneto hydrodynamic flow of nanofluid over a permeable stretching plate with viscous dissipation.

With the above awareness, the present paper similarity solution of two-dimensional MHD boundary layer flow of an incompressible dissipating nanofluid past an exponential stretching surface with convective boundary condition is investigated.

2. Mathematical Analysis: A steady laminar two-dimensional flow of an incompressible dissipating nanofluid past an exponential stretching sheet is considered. The x -axis is taken along the surface and y - axis is perpendicular to it. A variable magnetic field $B(x)$is applied normal to the sheet. The transverse applied magnetic field and magnetic Reynolds number are assumed to be very small, so that the induced magnetic field is negligible. The physical model and co-ordinate system is shown in Fig.1. Under these assumptions the boundary layer equations governing the flow and temperature field in the presence of viscous dissipation are

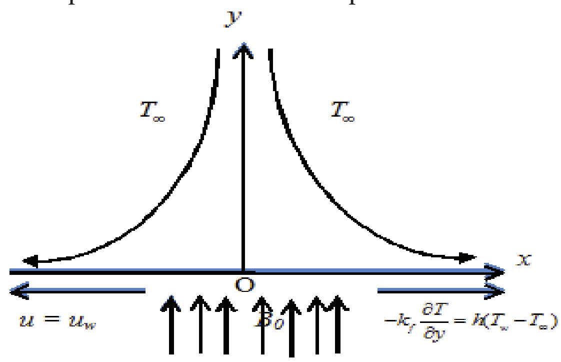

Fig. 1 Physical model and coordinate system

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho_{nf}}\left[\mu_{nf}\frac{\partial^2 u}{\partial y^2} - \sigma B^2 u\right] \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho Cp)_{nf}}\frac{\partial^2 T}{\partial y^2} + \frac{\nu_{nf}}{(\rho Cp)_{nf}}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

The respective boundary conditions of the flow are

$$u=u_w,\ v=0,\ k_f\frac{\partial T}{\partial y}=h_f\left(T_f-T_\infty\right)\quad at\quad y=0$$
$$u\to 0,\ T\to T_\infty\quad as\quad y\to\infty \tag{4}$$

where u, v are the velocity components in the x and y directions, respectively, T – the temperature of the nanofluid, T_∞- the ambient fluid temperature, σ is the electric conductivity, and q_r is the radiative heat flux. T_f is the temperature of the fluid and k_f is the thermal conductivity of ordinary fluid. The effective density ρ_{nf}, thermal diffusivity α_{nf}, the heat capacitance $(\rho C_p)_{nf}$, the effective dynamic viscosity μ_{nf} and the thermal conductivity k_{nf} of nanofluid from the literature are given by

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s,\ \alpha_{nf}=\frac{k_{nf}}{(\rho Cp)_{nf}},$$
$$(\rho C_p)_{nf}=(1-\phi)(\rho C_p)_f+\phi(\rho C_p)_s, \tag{5}$$
$$\frac{k_{nf}}{k_f}=\frac{(k_s+2k_f)-2\phi(k_f-k_s)}{(k_s+2k_f)+\phi(k_f-k_s)},\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}}$$

Here the subscripts nf, f and s represent the thermo physical properties of the nanofluids, base fluid and nano-solid particles respectively. ϕ is the nanoparticle volume fraction.

Table 1 Thermo physical properties of fluid and nanoparticles [11]

Physical properties	Fluid phase (water)	Cu	Al_2O_3
$Cp(J/kg\,K)$	4179	385	765
$\rho(kg/m^3)$	997.1	8933	3970
$k(W/m\,K)$	0.613	401	40

To obtain similarity solutions, the magnetic field B(x)assumes the form

$$B=B_0e^{x/2L} \tag{6}$$

To get similarity solutions of equations (1) - (3) subject to the boundary conditions (4), we introduce the following similarity transformations.

$$\eta=\left(\frac{U_0}{2\nu_f L}\right)^{1/2}e^{\frac{x}{2L}}y,\ u=U_0e^{\frac{x}{2L}}f'(\eta),v=-\left(\frac{U_0\nu_f}{2L}\right)^{1/2}e^{\frac{x}{2L}}\{f(\eta)+\eta f'(\eta)\}, \tag{7}$$
$$\theta=\frac{T-T_\infty}{T_f-T_\infty},M=\left(\frac{2\sigma B_0^2L}{U_0\rho_f}\right)^{1/2},\ \Pr=\frac{\nu_f}{\alpha_f},Ec=\frac{U_0^2}{(Cp)_f\left(T_f-T_\infty\right)},\mathrm{Re}_x=\frac{u_wx}{\nu_f}$$

where ν_f is the kinematic viscosity of the base fluid.

In view of equation (7), equations (1), (2) and (3) take the following dimensionless form.

$$f'''+(1-\phi)^{2.5}\left[\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)\left(ff''-f'^2\right)-Mf'\right]=0 \tag{8}$$

$$\frac{1}{\Pr}\frac{k_{nf}}{k_f}\theta''+\left(1-\phi+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)f\theta'+\frac{Ec}{(1-\phi)^{2.5}}f''^2=0 \tag{9}$$

where prime denotes the differentiation with respect to η.

The corresponding boundary conditions are

$$f'(0)=1,\ f(0)=0,\theta'(0)=-\gamma\left(1-\theta(0)\right) \tag{10}$$
$$f'(\infty)\to 0,\ \theta(\infty)\to 0$$

Using equation (6), the skin friction coefficient and local Nusselt number can be expressed as

$$\sqrt{\mathrm{Re}_x}C_f=\sqrt{\frac{x}{2L}}\frac{1}{(1-\phi)^{2.5}}f''(0) \tag{11}$$

$$\frac{Nu_L}{\sqrt{\mathrm{Re}_x}}=-\sqrt{\frac{x}{2L}}\frac{k_{nf}}{k_f}\theta'(0) \tag{12}$$

3. Results and Discussion: In order to get a physical insight into the problem, a representative set of numerical results is shown graphically in Figs.2-9, to illustrate the influence of physical parameters viz., The volume fraction parameter ϕ, the magnetic parameter M,Eckert number $Ec$and convective parameter γon the velocity and temperature.

Figs. 2 and 3 represent the velocity and temperature profiles for different values of the nanoparticle volume fraction parameter ϕ respectively, in the case of Cu nanofluid. It is found that the nanofluid velocity decreases and the temperature increases with an increase in the

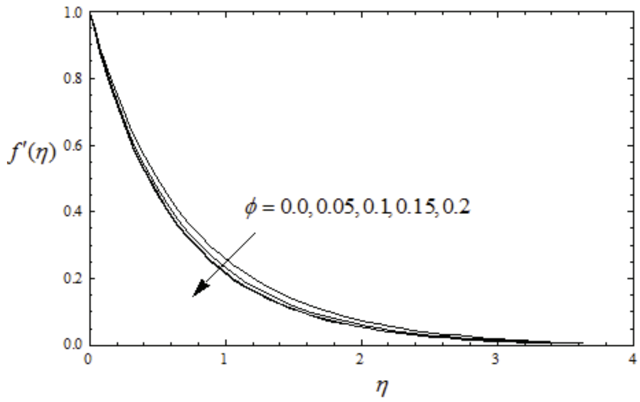

Fig.2 Velocity profiles for different values of ϕ

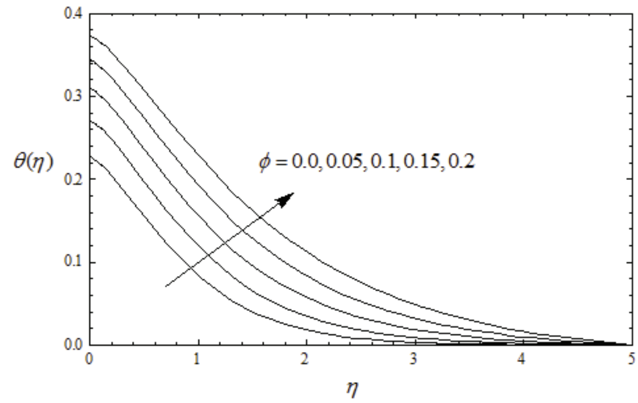

Fig.3 Temperature profiles for different values of ϕ

nanoparticle volume fraction. These figures illustrate this agreement with the physical behavior that the thermal conductivity increases and then the thermal boundary layer thickness increases as increase in volume of nanoparticles.

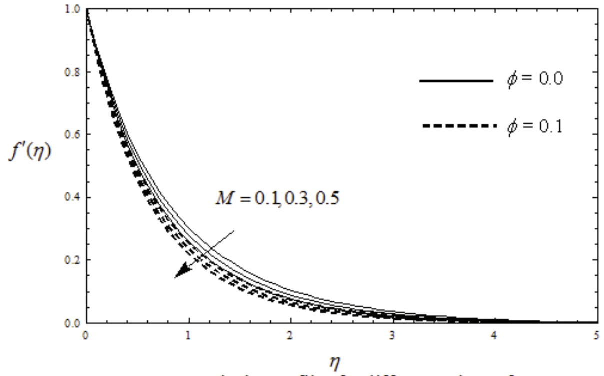

Fig.4 Velocity profiles for different values of M

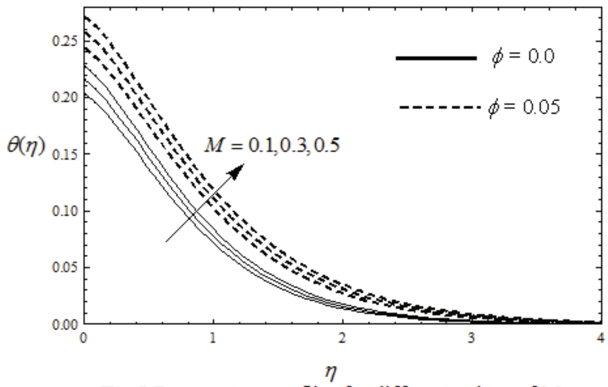

Fig.5 Temperature profiles for different values of M

The effect of magnetic parameter Mon the velocity and temperature profiles is shown in Figs. 4 and 5, respectively. It is observed that the velocity decreases as the magnetic parameter increases (Fig.4). This is because of the Lorentz force which tends to resist the fluid flow and thus reducing its velocity. Also, the boundary layer thickness decreases with an increase in the magnetic parameter. Fig. 5 portrays that an increase in the magnetic parameter results in an increase in the temperature of the nanofluid.

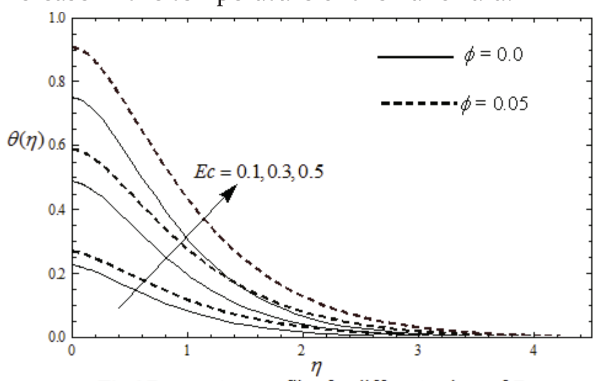

Fig.6 Temperature profiles for different values of Ec

For different values of the viscous dissipation parameter i.e., the Eckert number Ec on the temperature is shown in Fig.6. The Eckert number Ecexpress the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conservation of kinetic energy into internal energy by work done against the viscous fluid stress. The positive Eckert number implies cooling of the sheet i.e., loss of heat from the sheet to the fluid. It is found that the translation velocity and temperature as well as thermal boundary layer thickness increase slightly with an increase in Ec.

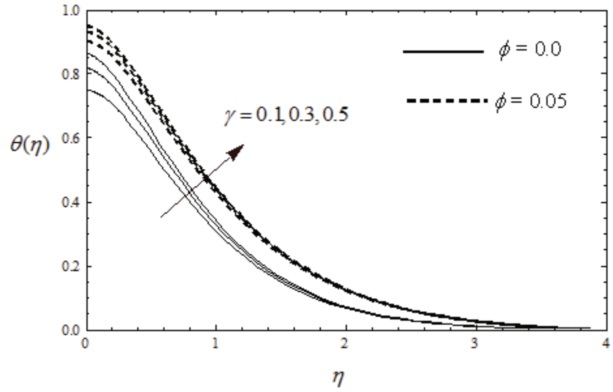

Fig.7 Temperature profiles for different values of γ

Fig. 7represents the effect of Biot number γ on temperature. The Biot number involves the heat transfer coefficient. Higher Biot number implies an enhancement in the heat transfer coefficient. This enhancement in the heat transfer coefficient give rise to the temperature and thermal boundary layer thickness.

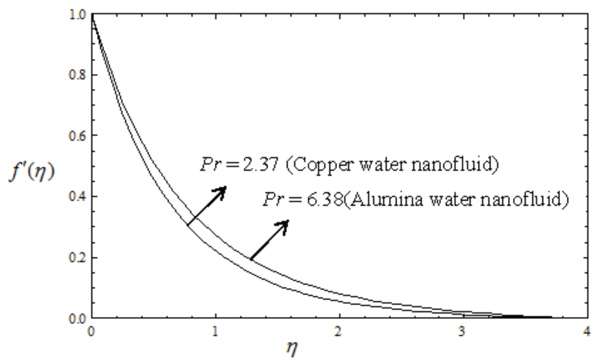

Fig .8 Velocity profiles of Copper water and Alumina water Nanofluid

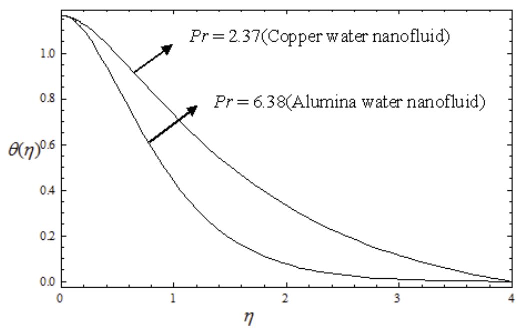

Fig .9 Velocity profiles of Copper water and Alumina water Nanofluid

The effect of Prandtl number on the velocity for both copper-water nanofluid (Pr = 2.37) and alumina-water nanofluid (Pr = 6.38) is shown in Fig.8. It is noticed that the velocity increases as the Prandtl number increases, which shows that the velocity is higher for alumina-water nanofluid than that of copper-water nanofluid. The effect of Prandtl number on the temperature for both copper-water nanofluid (Pr = 2.37) and alumina-water nanofluid (Pr = 6.38) is shown in Fig. 9. It is noticed that the temperature decreases as the Prandtl number increases, which shows that the temperature is higher for copper-water nanofluid than that of alumina-water nanofluid.

Table. 2 Values of $f''(0)$ and $-\theta'(0)$ for various values of ϕ, M and Ec

ϕ	M	Ec	γ	$f''(0)$	$-\theta'(0)$
0	0.1	0.1	0.1	− 1.32791	0.079464
0.05	0.1	0.1	0.1	−1.46506	0.0758863
0.1	0.1	0.1	0.1	−1.54604	0.0728535
0.1	0.5	0.1	0.1	− 1.6776	0.0704936
0.1	1.0	0.1	0.1	−1.82924	0.0676703
0.1	0.1	0.3	0.1	−1.54604	0.0425577
0.1	0.1	0.5	0.1	−1.54604	0.0122619
0.1	0.1	0.1	0.3	−1.54604	0.176263
0.1	0.1	0.1	0.5	−1.54604	0.246136

parameter$M$ or Eckert numberEcor Biot number γ. Also it is observed that the temperature of the fluid is much higher for Cu-water than for pure water (regular fluid, ϕ = 0).The rate of heat transfer decreases with an increase in volume fraction

The variations of $f''(0)$ and $-\theta'(0)$ which are proportional to the local skin-friction coefficient and rate of heat transfers are shown in Table 2. From Table 2, it is found that both the wall skin friction coefficient and heat transfer rate at the surface decreases as ϕincreases. This happens due to the fact that with the increase in ϕ, the thermal conductivity increases which lowers the temperature gradient at the plate surface. The same behaviour observed in the case of M or Ec, but the opposite behaviour is observed in heat transfer with an increase in γ.

5. Conclusions: The effect of various governing parameters on velocity and temperature profiles are shown graphically. Numerical results for the skin-friction and Nusselt number are presented in tabular form. The results in summary have shown that the velocity of the fluid as well as the surface skin-friction decreases with an increase in volume fraction parameter ϕ or Magnetic parameter M. The temperature of the fluid increases with an increase in volume fraction parameter ϕ or Magnetic

parameter ϕ or Magnetic parameter M or Eckert numberEc, but it shows an opposite behaviour with an increase in Biot number.

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