# RADIATION EFFECTS ON MHD CONVECTIVE DISSIPATING BOUNDARY LAYER FLOW OF EG-BASED CU NANOTUBES DUE TO AN EXTENSIBLE MOVING SURFACE

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**Abstract:** Not only, the usage of nanofluids, the shape of the nanoparticles present in the nanofluid is also an essential factor for the investigation. This paper examines the upshots of the effects of viscous dissipation in the presence of radiation on *Cu-EG* nanotubes over a moving extensible sheet which obeys more general stretching law. The sheet is in such a way that it occupies the negative *x*-axis also and is moving persistently in the positive *x*-direction. In this paper, a comparison between the spherical and cylindrical nanoparticles is made. The fluid is considered to be a gray, absorbing emitting radiation but non-scattering medium. The fluid is electrically conducting and a transverse variable magnetic field is applied parallel to *y*-axis. The governing boundary layer equations are transformed to a system of ODE using similarity variables and then solved numerically using Runge-Kutta fourth order method along with shooting technique. The behavior of flow and heat-transfer characteristics for varied material parameters are analyzed and discussed. The results of the present problem is compared with that of existing literature and found to be a good agreement. Nanofluids are better coolants for the heating world.

**Keywords:** MHD, Stretching sheet, Thermal Radiation, Viscous Dissipation.

**Introduction:** Present world is fond of miniaturization and transferring heat from those small objects is a challenging deal today. As a result, a finest fluid is invented whose thermal conductivity can be increased abnormally by taking it as a mixture of poor conductivity fluid and nanoparticles, which transfers the heat more. Choid did the pioneering works in nanofluids [1-2]. Attractive analyses are there on nanofluids due to stretching sheet [3-4 ] reported in the literature. Thermal radiation effects are extremely important in the context of flow processes involving high temperature especially in designing equipments. Thermal radiation effects of on the boundary layer flow have also been considerably researched [5,6].Viscous dissipation is the transformation of kinetic energy into internal energy due to viscosity. This field has received attention of many scholars researchers because of its wide applications in polymer industries(for molding and extrusion)[7-8]. Recently, number of works was made on MHD nanofluid flow past non-linear stretching sheet were reported. Kuiken [9] has proved through his investigation that when the Reynolds number is assumed to be large, a "backward" boundary layer exists along the moving sheet. However, the studies on boundary layer flows using nanofluids due to an elongated moving surface is very limited [10-11].With the above awareness, it can be noted that no attempt is made to examine viscous dissipation effects on MHD flow of Cu-EG based nanofluid with cylindrical particles. Thus the problem is investigated. Our objective is extending the analysis of Aly and Syed [12].

**2.Problem Formation:** A steady two-dimensional flow of an incompressible viscous electrically conducting and radiating fluid due to a movable and extensible sheet is considered. The extensible sheet occupies the negative *x*-axis and moves continuously towards the positive *x*-direction with a velocity given by $u_s(x)$. The stretching sheet of temperature $T_w(x)$ is placed in an ambient fluid of uniform temperature $T_\infty$. We assume that the temperature of the sheet is which corresponds to a heated sheet. A uniform transverse variable magnetic field *B(x)* is applied normal to the sheet. The base fluid is the Ethylene Glycol containing the cylindrical Copper nanoparticle.The base fluid and nanoparticles are in a thermal equilibrium and no slip occurs between them. Under these assumptions, the governing boundary layer equations are

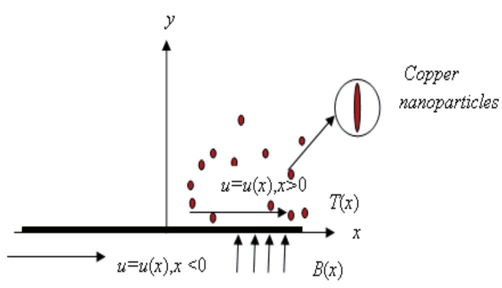

Fig.1 Physical model of the problem

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2(x)}{\rho_{nf}}u \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}+\frac{\mu_{nf}}{(\rho C_p)_{nf}}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{(\rho C_p)_{nf}}\frac{\partial q_r}{\partial y} \qquad (3)$$

where, $u$, $v$ are the velocity components in the $x$and y directions, respectively. $T$ is the temperature of the fluid, $T_\infty$ is the temperature of the fluid far away from the plate, $q_r$is the radiative heat flux.$\mu_{nf}$ is the effective dynamic viscosity,$\rho_{nf}$is the effective density. Furthermore $\sigma$, $B$(x) and $(C_p)_{nf}$are the fluid electric conductivity, variable magnetic induction and the specific heat at constant pressure, respectively.

The appropriate boundary conditions are

$$u=u_s(x),\ v=0, T=T_w(x)\ at\quad y=0$$
$$u\to 0, T\to T_\infty \qquad as\quad y\to\infty \qquad (4)$$

The effective density, thermal diffusivity, heat capacitance and dynamic viscosity of the nanofluid are given by

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s;\ \alpha_{nf}=\frac{k_{nf}}{(\rho Cp)_{nf}};\ \mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}};$$

$$(\rho C_p)_{nf}=(1-\phi)(\rho C_p)_f+\phi(\rho C_p)_s;\ \nu_f=\frac{\mu_f}{\rho_f} \qquad (5)$$

And the effective thermal conductivity can be incorporated from the following expression Rana and Bhargava [13]

$$\frac{k_{nf}}{k_{f_\infty}}=\frac{(k_s+(s-1)k_{f_\infty})-(s-1)\phi(k_{f_\infty}-k_s)}{(k_s+(s-1)k_{f_\infty})+\phi(k_{f_\infty}-k_s)} \qquad (6)$$

where$s$ is the nanoparticles empirical shape factor. In particular, $s$ = 3 stood for spherical shaped nanoparticles and $s$ = 3/2 for cylindrical ones (nanotubes).

**Table 1:**Thermophysical properties of base fluid and nanoparticles.

| Physical Properties | Fluid Phase (*E-G*) | *Cu* |
|---|---|---|
| $Cp(J/kgK)$ | 114.4 | 385 |
| $\rho(kg/m^3)$ | 2415 | 8933 |
| $\beta\times10^{-5}(1/K)$ | 65 | 1.67 |
| $K(W/mK)$ | 0.252 | 401 |

By using the Rosseland approximation, the radiative heat flux is given by

$$q_r=-\frac{4}{3}\frac{\sigma}{k^*}\frac{\partial T^4}{\partial y} \qquad (7)$$

where $\sigma$ is the Stefan-Boltzmann constant and $k^*$ is the mean absorption coefficient,then equation (7) can be linearized by expanding $T^4$ into the Taylor series about $T_\infty$, which after neglecting higher-order terms takes the form

$$T^4\cong 4T^3_\infty-3T^4_\infty \qquad (8)$$

In view of equations (7) and (8), Equation (3) becomes

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\left(\alpha_{nf}+\frac{1}{(\rho C_p)_{nf}}\frac{16\sigma T_\infty^3}{3k^*}\right)\frac{\partial^2 T}{\partial y^2}+\frac{\mu_{nf}}{(\rho C_p)_{nf}}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (9)$$

Introducing the similarity transformations,

$$\eta=y\sqrt{\frac{u_s}{2\nu_f|x|}},\quad \psi=f(\eta)\sqrt{2\nu_f u_s|x|},$$
$$u=u_sf',\quad v=-\sqrt{\frac{2\nu_f u_s}{|x|}}\left[\frac{n+1}{2}\eta f'+\frac{n-1}{2}f\right],$$
$$\theta=\frac{T-T_\infty}{T_w-T_\infty},T_w(x)=T_\infty+T_0\left(\frac{x_0}{|x|}\right)^m,$$
$$u_s(x)=\left(\frac{x_0}{|x|}\right)^n u_0,B(x)=B_0\sqrt{\left(\frac{x_0}{|x|}\right)^{n+1}},M=\frac{\sigma B_0^2x_0}{u_0\rho_f},$$
$$\Pr=\frac{(\rho C_p)_f\nu_f}{k_f},\quad Br=\frac{\mu_f u_s^2}{\nu_f(\rho C_p)_f T_0\left(\frac{x_0}{|x|}\right)^m},Nr=\frac{4\sigma T_\infty^3}{k^*k_f} \qquad (10)$$

Equations (1), (2) and (9) take the following dimensionless form.

$$f'''+(1-\phi)^{2.5}\left[\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)\left((n-1)ff''-2nf'^2\right)-Mf'\right]=0 \qquad (11)$$

$$\left(1+\frac{4}{3}Nr\right)\theta''+\Pr\left(\frac{k_f}{k_{nf}}\right)\left[\left(1-\phi+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)\{(n-1)f\theta'-2mf'\theta\}+\frac{Br}{(1-\phi)^{2.5}}(f'')^2\right]=0 \qquad (12)$$

where prime denotes the differentiation with respect to $\eta$.

The corresponding boundary conditions are

$$f'(0)=1,\quad f(0)=0,\quad \theta(0)=1$$
$$f'(\infty)\to 0,\quad \theta(\infty)\to 0 \qquad (13)$$

The quantities of practical interest in this study are the skin friction or the shear stress coefficient $C_f$ and the local Nusselt number $Nu_x$, which are defined as

$$C_f\sqrt{\mathrm{Re}_x}=\frac{\sqrt{2}}{(1-\phi)^{2.5}}f''(0),$$
$$\frac{Nu_x}{\sqrt{\mathrm{Re}_x}}=-\frac{k_{nf}}{\sqrt{2}\,k_f}\theta'(0) \qquad (14)$$

**3.Resultsand Discussion:** The coupled non linear differential equations (11) and (12) along with the boundary conditions (13) are solved numerically using Runge - Kutta four order method along with shooting technique. In order to bring out the salient features of the flow and the heat transfer characteristics in the *Cu-EG* based nanofluid, the numerical values for different values of the governing parameters $\phi$, $M$, $m, n$, $Nr, Br$ are portrayed in Figs. 2 -9. Here in these

Table.3: Comparison values of $f''(0)$ for different $n$with $M$=0.

| $n$ | Kuiken[9] (Series Method) | Ishak*et al.* [10] (Keller-Box) | Present Results (R-K Method) |
|---|---|---|---|
| 0.2 | −0.38191349 | −0.3819 | −0.38243 |
| 0.4 | −0.6389882 | −0.6390 | −0.63915 |
| 0.8 | −1.00779210 | −1.0078 | −1.00796 |
| 1.5 | −1.19485513 | −1.1949 | −1.2019 |
| 100 | −1.28047587 | −1.2817 | −1.29223 |

figures, the line graphs corresponds to spherical nanoparticles, whereas the dashed line corresponds nanotubes. We can observe that the usage of cylindrical nanoparticles improvises the heat transfer. Table. 3 presents a comparison of our numerical results with that of Kuiken [9] and Ishak*et al.* [10] for the reduced cases and they are in a good agreement.

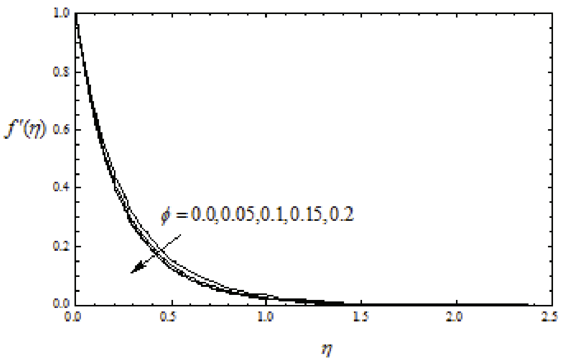


**Fig.2** Velocityprofiles for different values of $\phi$

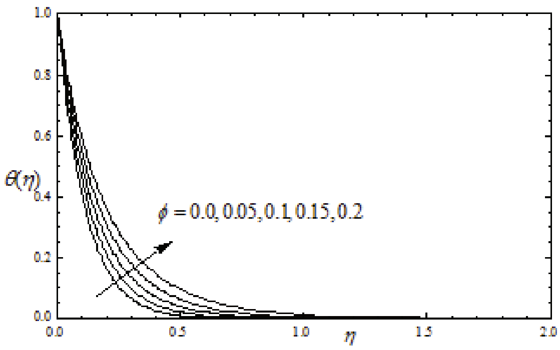


**Fig.3** Temperature profiles for different values of $\phi$

Velocity profiles with respect to Volume fraction $\phi$ is portrayed in Fig.2. It is observed that velocity descends as $\phi$ ascends. The temperature of the fluid is found to be increasing for increasing $\phi$ (Fig.3).Physically copper nanoparticles in the fluid have high thermal conductivity and in the presence of magnetic field, it retards the motion of the flow and increases the temperature.

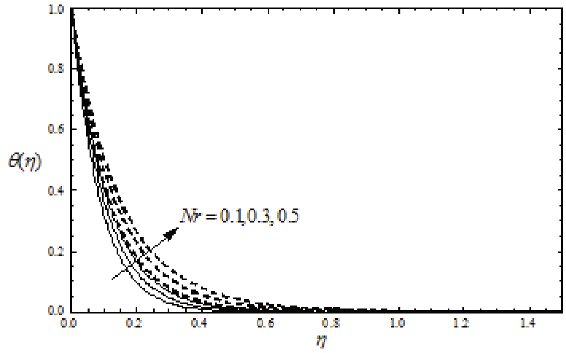


**Fig.4** Temperature profiles for different values of $Nr$

Effect of *Nr* on the temperature is depicted in Fig.4.It is noticed that for increasing *Nr*, the temperature increases. Larger values of *Nr* are indicative of larger amount of radiative heat energy being poured into the system, causing a rise in $\theta(\eta)$.

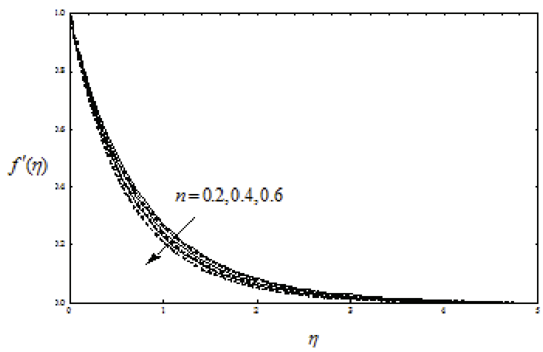


**Fig.5** Velocity profiles for different values of n<1

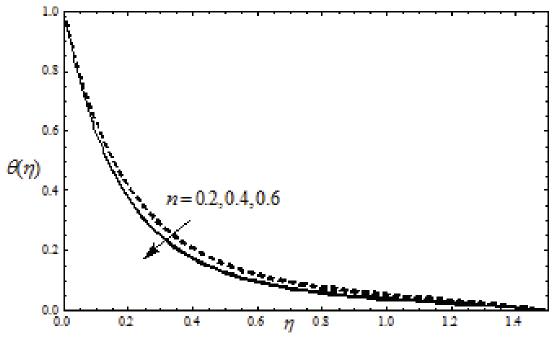


**Fig.6** Temperature profiles for different values of n<1

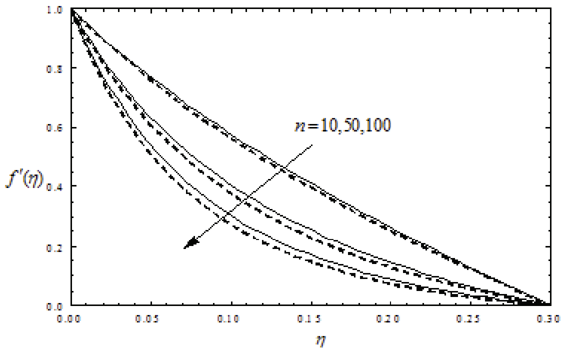


Fig.7 Velocity profiles for different values of n>1

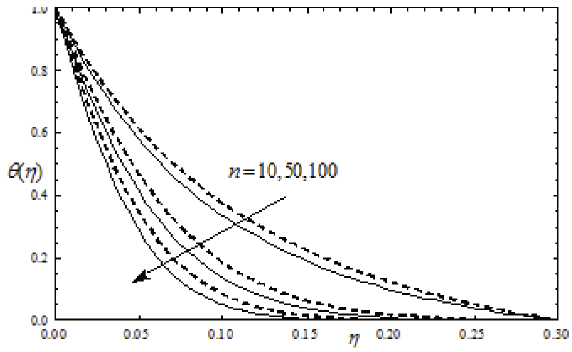


Fig.8 Temperature profiles for different values of n>1

Velocity and temperature profiles for $n$ <1 and $n$ >1 is illustrated in Figs.5-8. For increasing velocity power-law index for less than 1, the velocity and temperature of the fluid decreases (Figs.5-6). The velocity descends because of the sheet stretching and moving continuously towards the x-axis (i.e., continuous stretching).For n=10, almost the profile is linear and for the remaining values the convergence is smooth and quite clear (Fig.7).The temperature profiles also shows an decreasing trend as the wall(sheet) stretches continuously, the temperature also fluctuates significantly(Fig.8).

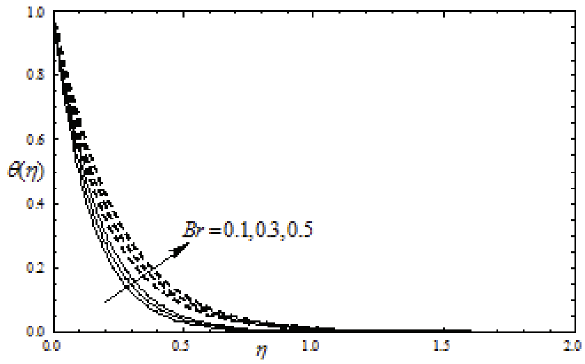


Fig.9 Temperature profiles for different values of $Br$

The influence of $Br$ on the temperature is depicted inFig.9. It is noteworthy that the nanofluid temperature increases with an increase in the Brinkmann number ($Br$) due to the action of viscous heating. This is due to the fact that in a viscous fluid flow the viscosity of the fluid will take energy from the motion of the fluid and transform it into internal energy of the fluid. That is heating and up the fluid. In all the figures, it is clear that usage of nanotubes instead spherical shaped nanoparticles gives a better result.

Table.2. displays the influence of various physical quantities on friction, heat transfer rate. Skin-friction coefficient values are of negative sign due to the sheet stretching. The shear stress near the sheet accelerates for the increase in particle volume fraction and the magnetic parameter, whereas, it decelerates for Brinkmann number or velocity power law index. No significant difference is there even changing the shape of particles for shear stress. But a quite significant impact is seen for rate of heat transfer. For increasing Br, the Nusselt number decreases. Heat is getting transferred more from the sheet to the fluid hence at the sheet, Nu decreases. The heat transfer rate is pronounced more for ϕ or radiation parameter, while decreasing for M or n.

**4. Conclusions:** The aim of the paper is to examine the effect of viscous dissipation on MHD radiating Cu-EG nanotubes due to an extensible stretching sheet. The following results can be summarized basing on the investigation.The heat transfer rate is pronounced more for $\phi$or radiation parameter, while decreasing for $M$ or $Br$ or $n$. Significant impact is observed for the nanotubes and nanoparticles.Shape of the nanoparticle also plays an important role in using nanofluids.Velocity descends for velocity power-law index.Temperature increases for an increase in $Nr$ or $Br$,while it decreases for velocity power-law index.

**Table.2** Value of $f''(0)$ and $\theta'(0)$ for various values of $\phi$, *M, Ec, R* and *n*.

| $\phi$ | *M* | *Br* | *Nr* | *n* | $C_f$ (Sperical) | $C_f$ (Nanotubes) | *Nu* (Sperical) | *Nu* (Nanotubes) |
|---|---|---|---|---|---|---|---|---|
| 0 | 0.1 | 0.1 | 1.0 | 10 | −5.71323 | −5.71323 | 5.09593 | 5.09593 |
| 0.05 | 0.1 | 0.1 | 1.0 | 10 | −7.07225 | −7.07225 | 5.3439 | 5.1828 |
| 0.1 | 0.1 | 0.1 | 1.0 | 10 | −8.4856 | −8.4856 | 5.61037 | 5.29927 |
| 0.1 | 0.5 | 0.1 | 1.0 | 10 | −8.6469 | −8.6469 | 5.5751 | 5.26736 |
| 0.1 | 1.0 | 0.1 | 1.0 | 10 | −8.84428 | −8.84428 | 5.53182 | 5.2282 |
| 0.1 | 0.1 | 0.3 | 1.0 | 10 | −8.4856 | −8.4856 | 4.87301 | 4.58203 |
| 0.1 | 0.1 | 0.5 | 1.0 | 10 | −8.4856 | −8.4856 | 4.13565 | 3.86479 |
| 0.1 | 0.1 | 0.1 | 2.0 | 10 | −8.4856 | −8.4856 | 4.30225 | 4.07702 |
| 0.1 | 0.1 | 0.1 | 2.0 | 10 | −8.4856 | −8.4856 | 3.56349 | 3.38385 |
| 0.1 | 0.1 | 0.1 | 1.0 | 50 | −3.62395 | −3.53594 | 1.03201 | 0.939825 |
| 0.1 | 0.1 | 0.1 | 1.0 | 100 | −1.06445 | −1.06445 | 0.399824 | 0.365659 |

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