ON CERTAIN SUBCLASS OF MEROMORPHIC STARLIKE FUNCTIONS ASSOCIATED WITH CERTAIN INTEGRAL OPERATORS

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Abstract: The aim of the present paper is to introduce a new class $\sum_p^*(\alpha,\beta,\sigma,\mu)$ of meromorphically star like functions defined by certain integral operator in the unit disc $E=\{ z: 0<|z|<1\}$ and investigate coefficients, distortion properties and radius of convexity for the class. Furthermore it is shown that the class $\sum_p^*(\alpha,\beta,\sigma,\mu)$ is closed under convex linear combinations.

Keywords: Meromorphic, Starlike, Coefficient estimates, Distortion properties, integral operators.

1.Introduction: Let $\sum$ denote the class of functions of the form

$$f(z) = \frac{1}{z}+\sum_{n=1}^{\infty} a_n z^n \qquad (1.1)$$

which are regular in E = {z : 0 < |z| < 1}, having a simple pole at the origin. Let $\sum_s$, $\sum^*(\alpha)$ and $\sum_k(\alpha)$ $(0 \le \alpha < 1)$ denote the subclasses of $\sum$ that are univalent, meromorphically starlike of order α and meromorphically convex of order α respectively. Analytically $f(z)$ of the form (1.1) is in $\sum^*(\alpha)$ if and only if

$$\operatorname{Re}\left\{-\frac{zf'(z)}{f(z)}\right\} > \alpha,\ z \in E. \qquad (1.2)$$

Similarly, $f \in \sum_k(\alpha)$ if and only if, $f(z)$ is of the form (1.1) and satisfies

$$\operatorname{Re}\left\{-\left(1+\frac{zf''(z)}{f'(z)}\right)\right\} > \alpha,\ z \in E. \qquad (1.3)$$

It being understood that if α = 1 then $f(z)=\frac{1}{z}$ is the only function which is $\Sigma^*(1)$ and $\Sigma_k(1)$. The classes $\Sigma^*(\alpha)$ and $\Sigma_k(\alpha)$ have been extensively studied by Pommerenke [6], Clunie [2], Royster [7] and others.

Recently the integral operator of $f(z)$ in $\sum_s$ for $\sigma > 0$ is denoted by I^σ and defined as following

$$\mathrm{I}^\sigma f(z) = \frac{1}{z^2\Gamma(\sigma)}\int_0^z \left\{\log\frac{z}{t}\right\}^{\sigma-1} tf(t)dt \qquad (1.4)$$

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That is defined by Jung, Kim and Srivastava [4]. It is easy to verify that if $f(z)$ is of the form (1.1), then

$$I_\mu^\sigma f(z) = \frac{1}{z}+\sum_{n=1}^{\infty}\left\{\frac{1}{n+2}\right\}^\sigma a_n z^n \qquad (1.5)$$

Analogous to the operators defined by Jung, Kim and Srivastava [4] on the analytic functions, Lashin [5] defined the following integral operator

$$I_\mu^\sigma f(z) = \frac{\mu^\sigma}{z^{\mu+1}\Gamma(\sigma)}\int_0^z t^\mu \left\{\log\frac{z}{t}\right\}^{\sigma-1} f(t)dt\ ,\ \sigma>0\ ,\ \mu>0\ , z\in E \qquad (1.6)$$

Where $\Gamma(\sigma)$ is the familiar Gamma function. Using the integral representation of the Gamma function and Beta function given by (1.6) it can be shown that

$$I_\mu^\sigma f(z) = \frac{1}{z}+\sum_{n=1}^{\infty}\left\{\frac{\mu}{n+\mu+1}\right\}^\sigma a_n z^n\ ,\ \sigma>0\ ,\ \mu>0\ , z\in E \qquad (1.7)$$

The aim of the present paper is to introduce the class of meromorphically starlike functions, which we denote by $\sum^{*}(\alpha,\beta,\sigma,\mu)$ for some $\alpha(0\le\alpha<1)$, $\beta(0<\beta\le1)$ and $\mu>0$, $\sigma>0$.

We then consider the class $\sum_{p}^{*}(\alpha,\beta,\sigma,\mu)=\sum_{p}\cap\sum^{*}(\alpha,\beta,\sigma,\mu)$ and extend some of the results of Juneja and Reddy[3] and Atshan and Kulakarni [1] to this class. We obtain coefficients estimates, distortion properties and radius of convexity for the class. Furthermore it is shown that the class $\sum_{p}^{*}(\alpha,\beta,\sigma,\mu)$ is closed under convex linear combinations and integral transforms.

Definition 1: Let the function $f(z)$ defined by (1.1).Then $f(z)\in\sum^{*}(\alpha,\beta,\sigma,\mu)$ if and only if

$$\left|\frac{z(I_{\mu}^{\sigma}f(z))'}{(I_{\mu}^{\sigma}f(z))}+1\right|<\beta\left|\frac{z(I_{\mu}^{\sigma}f(z))'}{(I_{\mu}^{\sigma}f(z))}+2\alpha-1\right|$$

For some $\alpha(0\le\alpha<1)$, $\beta(0<\beta\le1)$, $\mu>0$ and $\sigma>0$ and for all $z\in E$.

2. Coefficient Estimates

The following theorem give a sufficient condition for a function to be in $\sum^{*}(\alpha,\beta,\sigma,\mu)$.

Theorem 2.1: Let $f(z)=\frac{1}{z}+\sum_{n=1}^{\infty}a_n z^n$ be regular in *E*. If

$$\sum_{n=1}^{\infty}\left[(1+\beta)n+(2\alpha-1)\beta+1\right]\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}|a_n|\le 2\beta(1-\alpha). \qquad (2.1)$$

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$0\le\alpha<1$, $0<\beta\le1$, and $\sigma>0$ then $f(z)\in\sum^{*}(\alpha,\beta,\sigma,\mu)$.

Proof. Suppose (2.1) holds for all admissible values of α and β. Consider the expression

$$H(f,f')=\left|z(I_{\mu}^{\sigma}f(z))'+(I_{\mu}^{\sigma}f(z)\right|-\beta\left|z(I_{\mu}^{\sigma}f(z))'+(2\alpha-1)(I_{\mu}^{\sigma}f(z)\right| \qquad (2.2)$$

Then we have

$$H(f,f')=\left|\sum_{n=1}^{\infty}(n+1)\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}a_n z^n\right|-\beta\left|2(\alpha-1)\frac{1}{z}+\sum_{n=1}^{\infty}(n+2\alpha-1)\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}a_n z^n\right|$$

or

$$rH(f,f')\le\sum_{n=1}^{\infty}(n+1)\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}|a_n|r^{n+1}-\beta\left\{2(1-\alpha)-\sum_{n=1}^{\infty}(n+2\alpha-1)\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}|a_n|r^{n+1}\right\}$$

$$=\sum_{n=1}^{\infty}[1+\beta)n+(2\alpha-1)\beta+1]\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}|a_n|r^{n+1}-2\beta(1-\alpha).$$

Since the above inequality holds for all *r*, $0<r<1$, letting $r\to1$, we have

$$H(f,f')\le\sum_{n=1}^{\infty}\left[(1+\beta)n+(2\alpha-1)\beta+1\right]\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}|a_n|-2\beta(1-\alpha)$$

≤ 0, by (2.1)

Hence it follows that $\left|\frac{z(I_\mu^\sigma f(z))'}{(I_\mu^\sigma f(z))}+1\right| < \beta\left|\frac{z(I_\mu^\sigma f(z))'}{(I_\mu^\sigma f(z))}+2\alpha-1\right|$

So that $f(z)\in \sum^*(\alpha,\beta,\sigma,\mu)$.

Hence the theorem.

Theorem 2.2: Let $f(z)=\frac{1}{z}+\sum_{n=1}^{\infty}a_n z^n, a_n\geq 0,$ be regular in *E*. Then $f(z)\in\sum_p^*(\alpha,\beta,\sigma,\mu)$ if only if (2.1) is satisfied.

Proof: In view of theorem 2.1 it is sufficient to show that 'only if' part. Let us assume that $f(z)=\frac{1}{z}+\sum_{n=1}^{\infty} a_n z^n,\ a_n\geq 0$, is in $\sum_p^*(\alpha,\beta,\sigma,\mu)$. Then

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$$\left|\frac{\frac{z(I_\mu^\sigma f(z))'}{(I_\mu^\sigma f(z))}+1}{\frac{z(I_\mu^\sigma f(z))'}{(I_\mu^\sigma f(z))}+2\alpha-1}\right| = \left|\frac{\sum_{n=1}^{\infty}(n+1)\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}a_n z^n}{2(1-\alpha)\frac{1}{z}-\sum_{n=1}^{\infty}(n+2\alpha-1)\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}a_n z^n}\right| < \beta$$

for all $z\in E$. Using the fact that $Re(z)\leq|z|$ for all z, it follows that

$$\mathrm{Re}\left\{\frac{\sum_{n=1}^{\infty}(n+1)\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}a_n z^n}{2(1-\alpha)\frac{1}{z}-\sum_{n=1}^{\infty}(n+2\alpha-1)\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}a_n z^n}\right\}<\beta,\ z\in E. \tag{2.3}$$

Now choose the values of *z* on the real axis so that $\frac{z(I_\mu^\sigma f(z))'}{(I_\mu^\sigma f(z))}$ is real. Upon clearing the denominator in (2.3) and letting z→ 1 through positive values, we have

$$\sum_{n=1}^{\infty}(n+1)\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}a_n\leq\beta\left\{2(1-\alpha)-\sum_{n=1}^{\infty}(n+2\alpha-1)\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}a_n\right\}\ \text{Or}$$

$$\sum_{n=1}^{\infty}\left[(1+\beta)n+(2\alpha-1)\beta+1\right]\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}|a_n|\leq 2\beta(1-\alpha).$$

Hence the result.

Corollary 1. If $f(z)=\frac{1}{z}+\sum_{n=1}^{\infty} a_n z^n,\ a_n\geq 0$ is in $\sum_p^*(\alpha,\beta,\sigma,\mu)$ then

$$a_n\leq\frac{2\beta(1-\alpha)}{[(1+\beta)n+(2\alpha-1)\beta+1]\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}},\qquad n=1,\ 2,\ldots\ldots \tag{2.4}$$

with equality for each n, for function of the form

$$f_n(z)=\frac{1}{z}+\frac{2\beta(1-\alpha)}{[(1+\beta)n+(2\alpha-1)\beta+1]\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}}z^n,\qquad n=1,\ 2,\ldots$$

If $\beta = 1$ and $\mu = 1$ in the above theorem we get the following result of Atshan and Kulakarni[1].

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Corollary 2. If $f(z) \in \sum_p^* (\alpha, \sigma)$ then $a_n \le \frac{(1-\alpha)(n+2)^\sigma}{n+\alpha}$, $n = 1, 2, \ldots$ The result is sharp for the functions $f_n(z)$ is given by

$$f_n(z) = \frac{1}{z} + \frac{(1-\alpha)(n+2)^\sigma}{(n+\alpha)} z^n, \quad n = 1, 2, \ldots \tag{2.5}$$

3.Distortion Properties and Radius of Convexity Estimates

Theorem 3. If $f(z) \in \sum_p^* (\alpha, \beta, \mu, \sigma)$, then for $0 < |z| = r < 1$,

$$\frac{1}{r} - \frac{\beta(1-\alpha)}{[1+\alpha\beta]\left(\frac{\mu}{n+\mu+1}\right)^\sigma} r \le |f(z)| \le \frac{1}{r} + \frac{\beta(1-\alpha)}{[1+\alpha\beta]\left(\frac{\mu}{n+\mu+1}\right)^\sigma} \tag{3.1}$$

where equality holds for the function

$$f_1(z) = \frac{1}{z} + \frac{\beta(1-\alpha)}{[1+\alpha\beta]\left(\frac{\mu}{n+\mu+1}\right)^\sigma} z \quad \text{at } z = ir, r \tag{3.2}$$

Proof. Suppose $f(z) \in \sum_p^* (\alpha, \beta, \sigma, \mu)$. In view of Theorem 2, we have

$$\sum_{n=1}^{\infty} a_n \le \frac{\beta(1-\alpha)}{[1+\alpha\beta]\left(\frac{\mu}{n+\mu+1}\right)^\sigma} \tag{3.3}$$

Thus, for $0 < |z| = r < 1$,

$$|f(z)| = \left|\frac{1}{z} + \sum_{n=1}^{\infty} a_n z^n\right| \le \frac{1}{|z|} + \sum_{n=1}^{\infty} a_n |z|^n \le \frac{1}{r} + r\sum_{n=1}^{\infty} a_n$$

$$\le \frac{1}{r} + \frac{\beta(1-\alpha)}{[1+\alpha\beta]\left(\frac{\mu}{n+\mu+1}\right)^\sigma} r, \quad \text{by (3.3)}$$

This gives the right hand side of (3.1). Also,

$$|f(z)| = \left|\frac{1}{z} - \sum_{n=1}^{\infty} a_n z^n\right| \ge \frac{1}{|z|} - \sum_{n=1}^{\infty} a_n |z|^n \ge \frac{1}{r} - r\sum_{n=1}^{\infty} a_n$$

$$\ge \frac{1}{r} - \frac{\beta(1-\alpha)}{[1+\alpha\beta]\left(\frac{\mu}{n+\mu+1}\right)^\sigma} r$$

which gives the left hand of (3.1).

Theorem 4. If *f*(z) is in $\sum_{p}^{*}(\alpha,\beta,\sigma,\mu)$, then *f*(z) is meromorphically convex of order $\delta\left(0\le\delta<1\right)$ in $|z|<r=r\left(\alpha,\beta,\sigma,\mu,\delta\right)$, where

$$r\left(\alpha,\beta,\sigma,\delta\right)=\inf_{n}\left\{\frac{(1-\delta)\left[(1+\beta)n+(2\alpha-1)\beta+1\right]\left(\frac{\mu}{n+\mu+1}\right)^{\sigma}}{2\beta(1-\alpha)n(n+2-\delta)}\right\}^{(1/n+1)}, \quad \text{n=1,2 ....} \qquad (3.4)$$

The bound for $|z|$ is sharp for each *n*, with the extremal function being of the form (2.5)

Proof. Let $f(z)\in\sum_{p}^{*}(\alpha,\beta,\sigma,\mu)$. Then, by Theorem 2

$$\sum_{n=1}^{\infty}\frac{\left[(1+\beta)n+(2\alpha-1)\beta+1\right]\left(\frac{\mu}{n+\mu+1}\right)^{\sigma}}{2\beta(1-\alpha)}a_n\le 1. \qquad (3.5)$$

In view of (1.3), it is sufficient to show that $\left|2+\frac{zf''(z)}{f'(z)}\right|\le 1\text{-}\delta$ for $|z|<r\left(\alpha,\beta,\sigma,\mu,\delta\right)$

or equivalently, to show that

$$\left|\frac{f'(z)+\left(zf'(z)\right)'}{f'(z)}\right|\le 1-\delta \qquad (3.6)$$

for $|z|<r\left(\alpha,\beta,\sigma,\mu,\delta\right)$

where $r\left(\alpha,\beta,\sigma,\mu,\delta\right)$ is as specified in the statement of the theorem substituting the series expansions for $f'(z)$ and $\left(zf'(z)\right)'$ in the left side of (3.6) we have

$$\left|\frac{\sum_{n=1}^{\infty}n(n+1)a_n z^{n-1}}{-\frac{1}{z^2}+\sum_{n=1}^{\infty}na_n z^{n-1}}\right|\le\frac{\sum_{n=1}^{\infty}n(n+1)a_n|z|^{n+1}}{1-\sum_{n=1}^{\infty}na_n|z|^{n+1}}$$

This will be bounded by (1-δ) if

$$\sum_{n=1}^{\infty}\frac{n(n+2-\delta)}{1-\delta}a_n|z|^{n+1}\le 1. \qquad (3.7)$$

In view of (3.5), it follows that (3.7) is true if

$$\frac{n(n+2-\delta)}{1-\delta}|z|^{n+1}\le\frac{[(1+\beta)n+(2\alpha-1)\beta+1]\left(\frac{\mu}{n+\mu+1}\right)^{\sigma}}{2\beta(1-\alpha)}, \quad n=1,\ 2,\ldots \quad \text{Or}$$

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$$|z| \le \left\{ \frac{(1-\delta)[(1+\beta)n+(2\alpha-1)\beta+1]\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}}{2\beta(1-\alpha)n(n+2-\delta)} \right\}^{(1/n+1)}, \quad n=1,\ 2,\ldots \qquad (3.8)$$

setting $|z| = r(\alpha,\beta,\sigma,\mu,\delta)$ in (3.8), the result follows.

The result is sharp, the extremal function being of the form

$$f_n(z) = \frac{1}{z} + \frac{2\beta(1-\alpha)}{[(1+\beta)n+(2\alpha-1)\beta+1]\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}} z^n, \quad n=1,\ 2,\ldots$$

Convex Linear Combinations:

In this section we shall prove that the class $\sum_p^*(\alpha,\beta,\sigma,\mu)$ is closed under convex linear combinations.

Theorem 5: Let $f_0(z) = \frac{1}{z}$ and

$$f_n(z) = \frac{1}{z} + \frac{2\beta(1-\alpha)}{[(1+\beta)n+(2\alpha-1)\beta+1]\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}} z^n, \quad n=1,\ 2,\ldots$$

Then $f(z) \in \sum_p^*(\alpha,\beta,\sigma,\mu)$ if and only if it can be expressed in the form

$f(z) = \sum_{n=0}^{\infty} \lambda_n f_n(z)$ where $\lambda_n \ge 0$ and $\sum_{n=0}^{\infty} \lambda_n = 1$.

Proof: Let $f(z) = \sum_{n=0}^{\infty} \lambda_n f_n(z)$ with $\lambda_n \ge 0$ and $\sum_{n=0}^{\infty} \lambda_n = 1$. Then

$$f(z) = \sum_{n=0}^{\infty} \lambda_n f_n(z) = \lambda_0 f_0(z) + \sum_{n=1}^{\infty} \lambda_n f_n(z)$$

$$= \left[1 - \sum_{n=1}^{\infty} \lambda_n\right] f_0(z) + \sum_{n=1}^{\infty} \lambda_n f_n(z)$$

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$$= \left[1 - \sum_{n=1}^{\infty} \lambda_n\right] \frac{1}{z} + \sum_{n=1}^{\infty} \lambda_n \left[\frac{1}{z} + \frac{2\beta(1-\alpha)}{[(1+\beta)n+(2\alpha-1)\beta+1\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}]} z^n \right]$$

$$= \frac{1}{z} + \sum_{n=1}^{\infty} \lambda_n \frac{2\beta(1-\alpha)}{[(1+\beta)n+(2\alpha-1)\beta+1\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}]} z^n \text{, since}$$

$$\sum_{n=1}^{\infty} \frac{(1+\beta)n+(2\alpha-1)\beta+1]\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}}{2\beta(1-\alpha)} \lambda_n \frac{2\beta(1-\alpha)}{(1+\beta)n+(2\alpha-1)\beta+1\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}]}$$

$$= \sum_{n=1}^{\infty} \lambda_n = 1-\lambda_0 \le 1.$$

Therefore $f(z) \in \sum_p^* (\alpha,\beta,\sigma,\mu)$

Conversely, suppose $f(z) \in \sum_p^* (\alpha,\beta,\sigma,\mu)$. Since

$$a_n \le \frac{2\beta(1-\alpha)}{[(1+\beta)n+(2\alpha-1)\beta+1]\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}}, \quad n=1,\ 2,...$$

Setting

$$\lambda_n = \frac{[(1+\beta)n+(2\alpha-1)\beta+1]\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}}{2\beta(1-\alpha)} a_n, n=1,2,..., \text{ and } \lambda_0 = 1-\sum_{n=0}^{\infty} \lambda_n.$$

it follows that $f(z) = \sum_{n=0}^{\infty} \lambda_n f_n(z)$.

This completes the proof of the theorem.

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Theorem 6. The class $\sum_p^* (\alpha,\beta,\sigma,\mu)$ is closed under convex linear combination.

Proof. Let the function $F_k(z)$ be given by

$$F_k(z) = \frac{1}{z} + \sum_{n=1}^{\infty} f_{n,k} z^n, k = 1,2,......m$$

be in the class $\sum_p^* (\alpha,\beta,\sigma)$**.** Then it is enough to show that the function

$H(z) = \lambda F_1(z) + (1-\lambda) F_2(z)$, $(0 \le \lambda \le 1)$ is also in the class $\sum_p^* (\alpha,\beta,\sigma,\mu)$.

Since for $0 \le \lambda \le 1$,

$$H(z)=\frac{1}{z}+\sum_{n=1}^{\infty}[\lambda f_{n,1}+(1-\lambda)f_{n,2}]z^n,$$

We observe that

$$\sum_{n=1}^{\infty}[(1+\beta)n+(2\alpha-1)\beta+1]\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}[\lambda f_{n,1}+(1-\lambda)f_{n,2}]$$

$$\lambda\sum_{n=1}^{\infty}[(1+\beta)n+(2\alpha-1)\beta+1]\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}f_{n,1}+(1-\lambda)\sum_{n=1}^{\infty}[(1+\beta)n+(2\alpha-1)\beta+1]\left\{\frac{\mu}{n+\mu+1}\right\}^{\sigma}f_{n,2}$$
$$\le 2\beta\lambda(1-\alpha)+(1-\lambda)2\beta(1-\alpha)=2\beta(1-\alpha)$$

By Theorem 2, we have $H(z)\in\sum_{p}^{*}(\alpha,\beta,\sigma,\mu)$.

References:

1. W.G.Atshan and S.R.Kulakarni, Meromorphic p -valent functions with positive coefficients defined by convolution and integral operator, Journal of Indian Acad.Math. 29 (2007), 409-423.
2. J.Clunie, On meromorphic schlicht functions, J. London Math.Soc.34 (1959), 215-216.
3. *N. Nithyadevi, A. Shamadhani Begum,* Buoyancy and thermocapillary Driven Flows in An Open Cavity With ; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 2 Issue 2 (2013), Pg 268-272
4. O.P. Juneja and T. R. Reddy, Meromorphic starlike univalent functions with positive coefficients, Ann.Univ. Mariae Curie Sklodowska. Sect A 39, (1985), 65-76.
5. I.B.Jung, Y.C.Kim and H.M.Srivastava, The Hardy space of analytic functions associated with certain one-parameter families of integral operators, J.Math.Anal.Appl.176(1993), 138-147.
6. S. S. Handibag, B.D. Karande, Solution of Partial Differential Equations involving; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 3 Issue 1 (2014), Pg 142-146
7. A.Y.Lashin, On certain subclass of meromorphic function associated with certain integral operators, Computer Math.Appl. 59 (1) (2010), 524-531.
8. Ch.Pommarenke, On meromorphic starlike functions, Pacfic J.Math. 13(1963), 221-235.
9. R. Subba Rao, A. Naga Durgamamba, R.R.L. Kantam, Group Acceptance Sampling Plan – Size Biased Lomax Model; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 3 Issue 2 (2014), Pg 552-557
10. W.C.Royster, Meromorphic starlike multivalent functions, Trans. Amer. Math. Soc.107 (1963). 300-308.

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