CONNECTED S-VALUED GRAPHS

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Abstract : In our earlier paper [RJC], we have introduced the notion of Semiring valued graphs. In this paper, we discuss the notion of Connected S– valued graphs and prove some simple results.

Keywords: S– valued graphs, Operations on S– valued graphs, connected S– valued graphs.

1.Introduction: H.S.Vandiver[V], in the year 1934, introduced the notion of semirings, while studying the algebraic structure of ideals in rings. In [JG], Jonathan Golan introduced the notion of S- valued graph where S is a semiring. However, nothing more have been said. This motivated us to study the notion of semiring valued graphs in our paper [RJC1]. In [RJC2], we introduce the notion of Operations on S – valued graphs. In this paper, we introduce the notion of Connected S- valued graphs and discuss some simple but elegant results.

2. Preliminaries: In this section, we recall some basic definitions that are needed for our work.

Definition 2.1 [6] : Let G = (V, E⊂V × V) be the underlying graph with both V, E $\neq \emptyset$. For any semiring (S,+,·), a Semiring – valued graph (or a S- valued graph) $G^S$is defined to be the graph G^S= (V,E,σ, ψ) where σ : V ⟶ Sandψ : E ⟶ S is defined by

$$\psi(x,y)=\begin{cases}\min(\sigma(x),\sigma(y)) & if \sigma(x) \precsim \sigma(y) or \sigma(y) \precsim \sigma(x)\\ 0 & otherwise\end{cases}$$

for every unordered pair (x , y) of E⊂V x V. We call σ, a S − vertex set andψ, aS − edge set of S– valued graph G^S.

Definition 2.2 : Let G^S = (V , E , σ , ψ) be the S – valued graph corresponding to a given underlying graph G = (V , E). An S –valued graph $H^S = (P,L,r,\gamma)$ is called a S – subgraph of G^S if H = (P , L) is a subgraph of G with P⊂V, L⊂ E, r⊂ σ and $\gamma \subset \psi$.

That is, r⊂ σ⟹r(x) ≾ σ(x), x∈P and $\gamma \subset \psi \Longrightarrow \gamma(x,y) \precsim \psi(x,y)$, (x , y) ∈ L ⊂ P × P.

Definition 2.3 : Let G^S be a S –valued graph. G^S is said to have a S – Path if there is a path in its underlying graph G along with S – values.

Definition 2.4 : Let G^S be a S–valued graph corresponding to an underlying graph G, and let a ∈ S. G^S is said to be (a,k)– regular if the following conditions are true.

(1) The crisp graph G is k–regular.

(2) σ(v) = a for every v ∈ V.

Definition 2.5: Let G_1^S = (V_1 , E_1 , σ_1 , ψ_1) and G_2^S = (V_2 , E_2 , σ_2 , ψ_2) be two S – connected graphs with $V_1 \cap V_2 = \emptyset$.

Now $G_1^S \cup G_2^S$ = (V , E , σ , ψ) where V = $V_1 \cup V_2$ and $V_1 \cap V_2 = \emptyset$and E = $E_1 \cup E_2$,

For v ∈ V, $\sigma(v) = \begin{cases}\sigma_1(v), & if v \in V_1\\ \sigma_2(v), & if v \in V_2,\end{cases}$

For every $(v_i, v_j) \epsilon E$,

$$\psi(v_i,v_j) = \begin{cases}\psi_1(v_i,v_j), & if (v_i,v_j)\epsilon E_1\\ \psi_2(v_i,v_j), & if (v_i,v_j)\epsilon E_2\end{cases}$$

Definition 2.6: Let G_1^S = (V_1 , E_1 , σ_1 , ψ_1) and G_2^S = (V_2 , E_2 , σ_2 , ψ_2) be two S – connected graphs with $V_1 \cap V_2 = \emptyset$.

Now their sum $G_1^S+G_2^S$= (V , E , σ , ψ) where V = $V_1 \cup V_2$ and $V_1 \cap V_2 = \emptyset$and

E=$E_1 \cup E_2 \cup$ {(u , v) / u$\epsilon V_1$and vϵ V_2 }

$$\sigma(v) = \begin{cases}\sigma_1(v), & if v \in V_1\\ \sigma_2(v), & if v \in V_2,\end{cases}$$

And for $(v_i,v_j) \in E$,

$$\psi(v_i,v_j) = \begin{cases}\psi_1(v_i,v_j), & if (v_i,v_j)\epsilon E_1\\ \psi_2(v_i,v_j), & if (v_i,v_j)\epsilon E_2\\ \min\{\sigma_1(v_i),\sigma_2(v_j)\}, & if v_i \epsilon V_1 and v_j \epsilon V_2\end{cases}$$

3.CONNECTED S – GRAPHS

In this section we introduce the notion of connected S-valued graphs and study some of its properties.

Definition 3.1 : The neighbourhood of v_i in G^S is defined as a subgraph of G^S such that

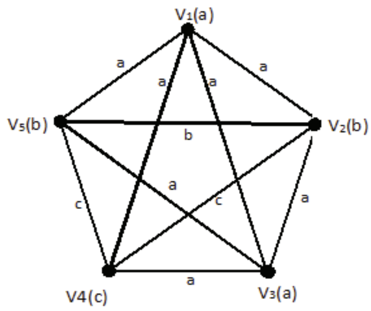
$N_S(v_i)$ = ({v_j , σ(v_j) / v_j's are adjacent to v_i },{($(v_i, v_j) \in E, \psi(v_i,v_j) \epsilon$ S)}).

Example 3.2 : Let (S = {0,a,b,c}, + , ·) be a semiring with the following Cayley Tables.

+	0	a	b	c
0	0	b	b	c
a	a	a	b	c
b	b	b	b	b
c	c	c	b	c

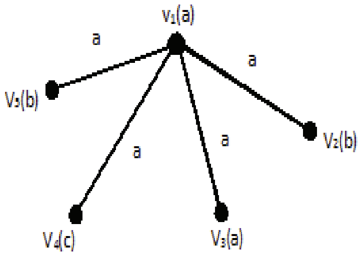
.	0	a	b	c
0	0	0	0	0
a	0	0	a	0
b	0	a	b	c
c	0	0	c	c

Let ≾be a canonical pre-order in S, given by 0≾0, 0≾a, 0≾b , 0≾c, a≾a, a≾b, a≾c, b≾b, c≾c,c≾b.Consider the graph $\mathbf{G^S}$ **:**



Here $N_S(V_1)$ = ({(v_5 , b), (v_4 , c), (v_3 , a), (v_2 , b)},{((v_1,v_5),a),((v_1,v_4),a),((v_1,v_3),a),((v_1,v_2),a)})

And the subgraph $N_S(v_1)$ is,



Remark 3.3 : To every S – graph, except for a vertex graph or an isolated vertex in its underlying graph,it has atleast one neighbourhood.

Theorem 3.4 : In every (a,k)-regular S-valued graph, the neighbourhood of any vertex is S-regular whose underlying graph is a complete bipartite graph K(1,k).

Proof : Let G^S = (V,E,σ,ψ) be a (a , k) – regular S-valued graph.

Then $\sigma(v_i) = a$, $\forall v_i \epsilon V$, for some $a \epsilon S$.

$\Longrightarrow \psi(v_i,v_j) = a$, $\forall (v_i,v_j) \epsilon E$, and

$deg(v_i) = k$, ∀ i.Let $v \epsilon V$ be arbitrary

$\Longrightarrow deg(v) = k$.

Let the adjacent vertices to v be $v_1, v_2, v_3, \ldots v_k$

Now

$N_S(v)$=({(v_1,a),(v_2,a),(v_3,a),...(v_k,a)},{((v,v_1),a),((v,v_2),a),...., ((v,v_k), a)})

such that $\sigma(v_j) = a$, $\forall j$= 1,2 ,... k and

$\sigma(v) = a \Longrightarrow N_S(v)$ is S-vertex regular.

∴ $N_S(v)$ is S-regular.Since $deg(v_j) = 1, \forall j$= 1,2 ,... k and deg(v) = k.That is the crisp graph of $N_S(v)$ is a complete bipartite graph K(1, k).

Remark 3.5 : The above proof states that, in general any neighbourhood $N_S(v)$ is not a (a, k)-regular S-valued graph.

In $N_S(v)$, v is a cut-vertex of a subgraph.

Definition 3.6 : Two vertices u and v are said to be S – connected in a S – valued graph G^S if there is a S – path between them. If every pair of vertices in G^S is S - connected, then the S – graph G^S said to be S – connected.

Example 3.7 : Every pair of vertices in example 3.2 is S - connected.

Therefore the S – valued graph G^S is S-connected.

Theorem 3.8 : Union of two S – connected graph is not S – connected.

Proof: Let $G_1^S = (V_1, E_1, \sigma_1, \psi_1)$ and $G_2^S = (V_2, E_2, \sigma_2, \psi_2)$ be two S – connected graphs with $V_1 \cap V_2 = \emptyset$.

∴ we can find a S – path between every pair of vertices (u , v) in V_1 or inV_2.

Now $G_1^S \cup G_2^S$ = (V , E , σ , ψ) where $V = V_1 \cup V_2$ and $V_1 \cap V_2 = \emptyset$and $E = E_1 \cup E_2$,

For $v \in V$, $\sigma(v) = \begin{cases}\sigma_1(v), & if v \in V_1\\ \sigma_2(v), & if v \in V_2,\end{cases}$

For every $(v_i, v_j) \epsilon E$,

$$\psi(v_i, v_j) = \begin{cases}\psi_1(v_i, v_j), & if (v_i, v_j) \epsilon E_1\\ \psi_2(v_i, v_j), & if (v_i, v_j) \epsilon E_2\end{cases}$$

Suppose $u \epsilon V_1$and $v \epsilon V_2$ then there is no path between u and v in $G_1^S \cup G_2^S$ (∵ there is no uv edge)$\Longrightarrow G_1^S \cup G_2^S$is not S-connected.

Remark 3.9 : The above results implies union of two S-connected S-regular and S-connected (a,k)-regular are not S-connected. Further, union of any S-valued graph is not S-Connected.

Theorem 3.10 : Sum of two S-connected graph is S-connected.

Proof : Let $G_1^S = (V_1, E_1, \sigma_1, \psi_1)$ and $G_2^S = (V_2, E_2, \sigma_2, \psi_2)$ be two S – connected graphs with $V_1 \cap V_2 = \emptyset$.

We can find a S-path between every pair of (u , v) of vertices both in G_1^Sor in G_2^S.

Now their sum $G_1^S + G_2^S$= (V , E , σ , ψ) where $V = V_1 \cup V_2$ and $V_1 \cap V_2 = \emptyset$and

$E = E_1 \cup E_2 \cup \{(u , v) / u \epsilon V_1 and\ v \epsilon V_2\}$

$$\sigma(v) = \begin{cases}\sigma_1(v), & if v \in V_1\\ \sigma_2(v), & if v \in V_2,\end{cases}$$

And for $(v_i, v_j) \in E$,

$$\psi(v_i, v_j) = \begin{cases}\psi_1(v_i, v_j), & if (v_i, v_j) \epsilon E_1\\ \psi_2(v_i, v_j), & if (v_i, v_j) \epsilon E_2\\ \min\{\sigma_1(v_i), \sigma_2(v_j)\}, & if v_i \epsilon V_1 and v_j \epsilon V_2\end{cases}$$

Claim : $G_1^S + G_2^S$ is S-connected.

Let $u,v \in V = V_1 \cup V_2$

Case (i) : Both $u,v \in V_1$

Since G_1^S is S-connected, there exist a S-path between u and v in G_1^S.

Hence $G_1^S + G_2^S$ is S-connected.

Case (ii) : Both $u,v \in V_2$.

Since G_2^S is S-connected. There exist a S-path between u and v in G_2^S.

Hence $G_1^S + G_2^S$ is S-connected.

Case (iii) : Suppose $u \in V_1$ and $v \in V_2$. Choose u_1 in V_1 and v_1 in V_2.

Since $(u,u_1) \in V_1$and $(v,v_1) \in V_2$ and G_1^S and G_2^S are S-connected,we find a S-path uu_1 in G_1^S and vv_1 in G_2^S.

Then $uu_1 + u_1v_1 + v_1v$ becomes a S-path in $G_1^S + G_2^S$.

∴ To every pair (u , v) in $G_1^S + G_2^S$ we can find a S-path between them.

$\Longrightarrow G_1^S + G_2^S$ is S-connected graphs.

Theorem 3.11 : Sum of two S-connected S-vertex regular graphs is a S-connected S-vertex graph iff their corresponding S-vertex sets are constant and assigns the same value in S.

Proof : Let $G_1^S + G_2^S = (V, E, \sigma, \psi)$ S-connected S-vertex regular graph, where G_1^S, G_2^S are S-connected S-vertex regular graph.

Then $\sigma(v) = a$, $for every v \epsilon V$ and for some a ϵ S. For v ϵ $V_1 \subset V \Rightarrow \sigma_1(v) = \sigma(v) = a$ and for v ϵ $V_2 \subset V \Rightarrow \sigma_2(v) = \sigma(v) = a$.

∴ Their corresponding S-vertex sets are constant and assigns the same value in S

Conversely, let the S-vertex sets $\sigma_1(v) = \sigma_2(v) = a$; and G_1^S, G_2^S be S-connected S-vertex regular graph.

By the above theorem we know that $G_1^S + G_2^S$ be S-connected S-vertex regular graph.

Remark 3.12 : The above theorem holds good for S-connected S-regular graphs.

Corollory 3.13 : Sum of two S – connected (a , k) – regular S – valued graph is S – connected S – regular.

Proof: Since sum of two (a , k) – regular S – valued graph is S – regular, and S – connected graph is S – connected, by above theorem, sum of two S – connected (a , k) – regular S – valued graph is S – regular.

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