INTUITIONISTIC L- FUZZY β-FILTERS ON β-ALGEBRAS

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Abstract: In this paper, we define the notion of an Intuitionistic L- fuzzy filter on β-algebras and investigate some of their properties and results.

Keywords: β-algebra, β-filter, Fuzzy Filters, Intuitionistic L- Fuzzy β- Filters.

1.Introduction: In 2002, J.Neggers and H.S.Kim [3], introduced a new notion of algebra: namely β-algebra.The theory of fuzzy sets proposed by L.A.Zadeh [8] in 1965 is generalized in 1986 by K.T.Atanassov [1] to an Intuitionistic fuzzy sets. The study of fuzzy algebraic structures was initiated with the concept of the fuzzy subgroup by A.Rosenfeld[5].Then many researchers have been engaged in extending the concepts and results of abstract algebra.The notion of filters was introduced by Henri Cartan in 1937. In 1991, C.S. Hoo [2] introduced the concept of the filters in BCI-algebras. Also in 2013, A.Rezaei and A. Bourmand [4] introduced the notion of generalized fuzzy filters of BE-algebras. In our earlier papers [6],[7] we have introduced the notions of β-filter, Intuitionistic Fuzzy β-filter in β- algebras. In this paper, we discuss the concept of Intuitionistic L-fuzzy filters in β-algebras and prove some of their properties.

2.Preliminares: In this section we recall some basic definitions that are required in the sequel.

Definition 2.1.A β-algebra is a nonempty set X with a constant 0 and two binary operations + and – satisfying the following axioms:

1) x – 0 = x
2) (0 - x) + x = 0
3) (x –y) – z = x- (z + y)∀ x, y, z ∈ X.

Definition 2.2. A filter of X, is a non- empty subset S, such that$x\in S$and y ∈ S ⇒ xΔ y ∈ S, where x Δ y = x*(x*y) and 0 ∉ S.

Definition 2.3. Let X and Y be two β-algebras. A mapping f : X → Y is said to be a β- homomorphism, if f(x+y) = f(x)+f(y) and f(x-y) = f(x)-f(y) for all x, y ∈ X.

Definition 2.4. Let X be a β-algebra and A be β-subalgebra. A is said to be a β- filter on X, if for all x, y ∈ A,

1) xΔ y = x + (x +y)
2) x∇ y = x – (x –y) $\in A$.

Definition 2.5. Let X be a β-algebra and A be a fuzzy β-subalgebra. A is said to be a fuzzy β- filter on X,if it satisfies the following conditions. For all x, y ∈ A,

1) $\mu_A(x\Delta\, y) \geq \min\{\mu_A(x),\ \mu_A(x+y)\}$2) $\mu_A(x\nabla\, y) \geq \min\{\mu_A(x),\ \mu_A(x-y)\}$

3) $\mu_A(y) \geq \mu_A(x)$, if x ≤y.

Definition 2.6. Let X be a β-algebra and A be an Intuitionistic fuzzy β-subalgebra. A is said to be an Intuitionistic fuzzy β- filter on X, if it satisfies the following conditions. For all x, y ∈ A,

1) $\mu_A(x\Delta\, y) \geq \min\{\mu_A(x),\ \mu_A(x+y)\}$ and $\mu_A(x\nabla\, y) \geq \min\{\mu_A(x), \mu_A(x-y)\}$

2)$\vartheta_A(x\Delta\, y) \leq \max\{\vartheta_A(x), \vartheta_A(x+y)\}$ and $\vartheta_A(x\nabla\, y) \leq \max\{\vartheta_A(x), \vartheta_A(x-y)\}$

3)$\mu_A(y) \geq \mu_A(x)$ and $\vartheta_A(y) \leq \vartheta_A(x)$,
if x ≤y.

3. **Intuitionistic L- Fuzzy β-Filter:** In this section, we introduce the notion of Intuitionistic L- fuzzy β-filter on a β-algebra. We begin with the definition and examples.

Definition 3.1. Let X be a β-algebra and A be an Intuitionistic L-fuzzy β-subalgebra. A is said to be an Intuitionistic L-fuzzy β- filter on X, if it satisfies the following conditions.
For all x, y ∈ A,

1) $\mu_A(x\Delta\, y) \geq \{\mu_A(x) \wedge \mu_A(x+y)\}$ and $\mu_A(x\nabla\, y) \geq \{\mu_A(x) \wedge \mu_A(x-y)\}$

2) $\vartheta_A(x\Delta\, y) \leq \{\vartheta_A(x) \vee \vartheta_A(x+y)\}$ and $\vartheta_A(x\nabla\, y) \leq \{\vartheta_A(x) \vee \vartheta_A(x-y)\}$

3) $\mu_A(y) \geq \mu_A(x)$ and $\vartheta_A(y) \leq \vartheta_A(x)$,
if x ≤y.

Example 3.2. Let X= {0,1,2,3} be a β-algebra with constant 0 and two binary operations + and – defined on X with the cayley's table

+	0	1	2	3
0	0	0	0	0
1	1	1	1	1
2	0	0	2	3
3	3	3	3	3

-	0	1	2	3
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3

Now A = {2,3} is a β-filter on X. Choose$t_0, t_1 \in$ L where $t_0 \geq t_1$.A is an Intuitionistic L-fuzzy β-subalgebra, defined by the member-ship function and non membership function,

$\mu_A(x) = \begin{cases} 0, if x = 2 \\ t_0, if x = 3 \end{cases}$ and

$\vartheta_A(x) = \begin{cases} 1.0, if x = 2 \\ t_1\ , if x = 3 \end{cases}$

Then we can observe that, A is an Intuition-istic L-fuzzy β-filter on X.

Lemma 3.3. If A and B be any two Intuitionistic L- fuzzy β-filters on X, then A∩ B is also an Intuitionistic L- fuzzy β-filter of X.

Proof:$(\mu_A \cap \mu_B)(x\Delta y)$

$= (\mu_A \cap \mu_B)(x+(x+y))$

$\geq \{\{\mu_A(x) \wedge \mu_A(x+y)\} \wedge \{\mu_B(x) \wedge \mu_B(x+y)\}\}$

$\geq \{\{\mu_B(x) \wedge \mu_A(x)\} \wedge \{\mu_A(x+y) \wedge \mu_B(x+y)\}\}$

$= \{(\mu_A \cap \mu_B)(x) \wedge (\mu_A \cap \mu_B)(x+y)\}$

Similarly, we can prove that

$(\mu_A \cap \mu_B)(x\nabla y) \geq \{(\mu_A \cap \mu_B)(x) \wedge (\mu_A \cap \mu_B)(x-y)\}$

Also, we can prove that,

$(\vartheta_A \cap \vartheta_B)(x\Delta y) \leq \{(\vartheta_A \cap \vartheta_B)(x) \wedge (\vartheta_A \cap \vartheta_B)(x+y)\}$ and

$(\vartheta_A \cap \vartheta_B)(x\nabla y) \leq \{(\vartheta_A \cap \vartheta_B)(x) \wedge (\vartheta_A \cap \vartheta_B)(x-y)\}$

Hence A∩ B is also an Intuitionistic L- fuzzy β-filter of X.

Theorem 3.4. Every Intuitionistic L- fuzzy β-filter is also an Intuitionistic L- fuzzy β-sub-algebra.

The proof directly follows from our definition ofIntuitionistic L- fuzzy β- filter.

However every Intuitionistic L- fuzzy β- sub-algebra need not be an Intuitionistic L- fuzzy β-filter shown by the following example.

Example 3.5. Let X= {0,1,2,3} be a β-algebra with constant 0 and two binary operations + and – defined on X with the cayley's table

+	0	1	2	3
0	0	0	0	0
1	1	1	1	1
2	1	1	2	0
3	3	3	1	1

-	0	1	2	3
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3

Now, A = {1,3} is β-filter on X. Let $t_0, t_1 \in L$ where $t_0 \geq t_1$.A is fuzzy β-subalgebra, defined by the membership function and non membership function,

$\mu_A(x) = \begin{cases} 1.0, if x = 1 \\ t_0, if x = 3 \end{cases}$ and

$\vartheta_A(x) = \begin{cases} 0, if x = 1 \\ t_1, if x = 3 \end{cases}$

Then we can observe that, A is not an Intuitionistic L- fuzzy β-filter on X.

Since $\mu_A(3) \geq \mu_A(1) \Longrightarrow t_0 \ngeq 1$

Theorem 3.6. If μ is an IntuitionisticL-fuzzy β- filter of X, then

1. $\mu_A(x\Delta y) \geq \mu_A(x)$and $\mu_A(x\nabla y) \geq \mu_A(x)$, where x ≤ y.
2. $\vartheta_A(x\Delta y) \leq \vartheta_A(x)$and $\vartheta_A(x\nabla y) \leq \vartheta_A(x)$, where x ≤ y.

Proof: Assume that μ is an Intuitionistic L-fuzzy filter of X.Let x, y ∈ X. Then we get,

$\mu_A(x\Delta y) = \mu_A(x+(x+y))$

$\geq \{\mu_A(x) \wedge \mu_A(x+y)\}$

$\geq \{\mu_A(x) \wedge \{\mu_A(x) \wedge \mu_A(y)\}\}$

$= \{\mu_A(x) \wedge \mu_A(x)\}$

$= \mu_A(x)$

$since x \leq y \Longrightarrow \mu_A(y) \geq \mu_A(x)$

Similarly, we can prove that,

$\mu_A(x\nabla y) \geq \mu_A(x)$.

Also, we can prove for the nonmembership function, $\vartheta_A(x\Delta y) \leq \vartheta_A(x)$ and $\vartheta_A(x\nabla y) \leq \vartheta_A(x)$, where x ≤ y.

Definition 3.7. Let μ be an Intuitionistic L- fuzzy β-filter in a β-subalgebra X. For a s,t∈ [0,1], the set $\mu_s = \{x \in X/\mu(x) \geq s and \vartheta_A(x) \leq t\}$is called a level set of filter μ in X.

Theorem 3.8. An Intuitionistic L-fuzzy subset A of β-algebra X is an Intuitionistic L- fuzzy β-filter iff for any**s,**t∈ [0,1] the s,t - level subset$A_{s,t}$ either a β-filter or $A_{s,t} \neq \emptyset$.

Proof:Assume that the level subset of A in X, $A_t \neq \emptyset$.

Then x, y ∈ A_t, $\mu_A(x) \geq s, \mu_A(y) \geq s$

Now, $\mu_A(x\Delta y) = \mu_A(x+(x+y)$

$\geq \{\mu_A(x) \wedge \mu_A(x+y)\}$

$\geq \{\mu_A(x), \wedge\{\mu_A(x) \wedge \mu_A(y)\}\}$

$= \{\mu_A(x) \wedge \mu_A(x)\} = s$

which implies $x\Delta y \in \mu_{A_s}$.

Similarly, we can prove that $x\nabla y \in \mu_{A_s}$.

Also we can prove that, the non membership function, xΔ y and x∇ y ∈ ϑ_{A_t}.Hence $A_{s,t}$ is a β-filter of X .

Conversely, assume that A_t is a β-filter of X For all x, y ∈X, xΔ y and x∇ y ∈ A_t

$\Rightarrow \mu_A(x\Delta y) \geq t$ and $\mu_A(x\nabla y) \geq t$.

$\mu_A(x\Delta y) = \mu_A(x+(x+y)) \geq t$

$=\{\mu_A(x) \wedge \mu_A(x+y)\}$.

Similarly, we can prove that,

$\mu_A(x\nabla y) \geq t$.

Also we can prove for the non membership function,$\vartheta_A(x\Delta y) \leq t$ and$\vartheta_A(x\nabla y) \leq t$.

Thus proving that A is an Intuitionistic L- fuzzy β-filter.

Theorem 3.9. Let f be an onto β-algebra homomorphism from X to Y. If B is an Intuitionistic L- fuzzy β-filter of Y, then $f^{-1}(B)$ is also an Intuitionistic L- fuzzy β-filter on X.

Proof:Let B be an Intuitionistic L- fuzzy β-filter of Y.For x, y ∈ X, then

$f^{-1}(\mu_B(x\Delta y)) = f^{-1}(\mu_B(x+(x+y)))$

$= \mu_B(f(x+(x+y))$

$= \mu_B(f(x) + f(x+y))$

$\geq \{\mu_B(f(x)) \wedge \mu_B(f(x+y))\}$

$= \{f^{-1}(\mu_B(x) \wedge f^{-1}(\mu_B(x+y))\}$

Similarly, we can prove that, $f^{-1}(\mu_B(x\nabla y)) \geq \{f^{-1}(\mu_B(x)) \wedge f^{-1}(\mu_B(x-y))\}$

Let x, y ∈ X, such that x ≥ y. Since B is an Intuitionistic L- fuzzy β-filter, we have $\mu_B(f(y)) \geq$

$\mu_B(f(x)) = f^{-1}(\mu_B(x))$ such that $f^{-1}(\mu_B(y)) \geq f^{-1}(\mu_B(y))$.
Also we can prove for, the non member-ship function

Thus we can conclude that $f^{-1}(B)$ is an intuitionistic L-fuzzy β-filter on X.

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