# ANALYSIS OF $M^{[X]}/G(m)/1$ QUEUE WITH SERVICE INTERRUPTION AND GENERALIZED SERVER VACATIONS

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**Abstract:** In this paper we analyse a batch arrival queueing system with service interruption and m$^{th}$ server vacation based on Bernoulli schedule. A single server provides essential service to all arriving customers with service time following general distribution. After every service completion the server has the option to leave for phase one vacation of random of random length with probability P or to continue staying in the system with probability 1-P. The new assumption in this paper is that the server go on m$^{th}$ vacation, as soon as the completion of phase one vacation, the server undergoes phase "m" vacation on completion of m heterogeneous phase of vacation the server return back to the system. The vacation time are assumed to be general. The server is interrupted at random and the duration of attending interruption follows exponential distribution. Also we assume, the customer whose service is interrupted goes back to the head of the queue where the arrivals of are poisson. Using supplementary variable technique the Laplace transforms of time dependent probabilities of system, state are derived.

**Keywords:** Batch arrival, Transient State Solution, Extended vacation time, Average queue size, Average waiting time.

**Introduction:** Vacation queues have been extensively studied by numerous authors because of its day to day applications in real life. In particular the steady state and the transient solutions for the $M^{[X]}/G(m)/1$ model in various forms have been deal with by a wide range of authors. To quote a few, computer and communication network work where messages are processed by a single server with interruption, production system.

To mention a few references we would name Doshi[6], Kulkarni and Choi[7], Takagi[10], Takine and Sengupta[11Madan et al[8], Maraghi et al[9]. However, in this model, the server stops the original work in the vacation period and cannot come back to the regular busy period until the vacation period ends.

In queueing theory periods of temporary service unavailability are referred to as server vacations, server interruption or server breakdowns. Queueing models with service interruption have proved to be a useful abstraction in situations where a services facility is shared by multiple queues or where the facility is subject to failure.

Most of the recent studies have been devoted to batch arrival vacation models under different vacation policies because of its interdisciplinary character. In this paper, we consider a batch arrival queueing system $M^{[X]}/G(m)/1$ with service interruption, in which we assume that after every service completion, the server has the option to leave for a vacation of random length with probability 1-p. The vacation period has three heterogeneous phases. On completion of three vacation phases the server return back to the system. Also we assume, the customer whose service is interrupted goes back to the head of the queue where the arrivals are Poisson.

The rest of the paper is organized as follows:

Section 2: Mathematical description of the model.

Section 3: Definitions and equations governing the system.

Section 4: Generating functions of the queue length. The time – dependent solution.

Section 5: The steady state results.

Section 6: Conclusion

**Mathematical Description Of The Model**

We assume the following to describe the queueing model of our study.

a) Customers arrive at the system in batches of variable size in a compound Poisson process and they are provided one by one service on a first come – first served basis. Let $\lambda_{c_i} dt$ $(i = 1,2, \dots.)$ be the first order probability that a batch of $i$ customers arrives at the system during a short interval of time $(t, t + dt]$, where $0 \le c_i \le 1$ and $\sum_{i=1}^{\infty} c_i = 1$ and $\lambda > 0$ is the arrival rate of batches.

b) A single server provides services to all arriving customer, with the service time having general distribution. Let $B(v)$ and $b(v)$ be the distribution and the density function of the service time respectively.

c) We assume interruption arrive at random while serving the customers and assumed to occur according to a Poisson process with mean rate $\alpha > 0$. Let $\beta$ be the server rate of attending interruption. Further we assume that once the interruption arrives the customer whose service is interrupted comes back to the head of the queue. Let $\mu(x)dx$ be the conditional probability of completion of the service during the interval $(x, x + dx)$ given that the elapsed time is $x$, so that

$$\mu(x) = \frac{b(x)}{1 - B(x)} ,$$

and therefore,

$$b(s) = \mu(s)e^{-\int_0^x \mu(x)dx},$$

d) As soon as the service is over, the server may take a vacation with probability p or may continue staying in the system with probability 1-p. after phase one vacation completion the server undergoes phase m vacation. On completion m heterogeneous phase of vacation the server return back to the system.

e) The server's vacation time follows a general (arbitrary) distribution with distribution function $V_i(t)$ and density function $v_i(t)$. Let $\gamma_i(x)dx$ be the conditional probability of a completion of a vacation during the interval $(x, x+dx]$ given that the elapsed vacation time is $x$ so that

$$\gamma_i(x) = \frac{v_i(x)}{1 - V_i(x)} \quad , i = 1,2, \dots \dots .. m$$

and therefore,

$$v_i(t) = \gamma_i(t)e^{-\int_0^m \gamma_i(x)dx}, \qquad i = 1,2,3 \dots .. m$$

f) On returning from vacation the server instantly starts serving the customer at the head of the queue if any.

g) Various stochastic processes involved in the system are assumed to be independent of each other.

**Definitions And Equations Governing The System**

We define $P_n(x,t) =$ Probability that at time the server is active providing essential service and there are $n$ $(n \geq 0)$ customers in the queue excluding the one being served and the elapsed service time for this customer is $x$. Consequently $P_n(t) = \int_0^\infty P_n(x,t)\, dx$ denotes the probability that at time $t$ there are $n$ customers in the queue excluding one customer in the essential service irrespective of the value of $x$.

$V_n^{(i)}(x,t) =$ Probability that at time $t$, the server is under vacation with elapsed vacation time $x$ and there are $n$ $(n \geq 0)$ customers in the queue. Consequently $V_n^{(i)}(t) = \int_0^\infty V_n^{(i)}(x,t)dx$ denotes the probability that at time $t$ there are $n$ customers in the queue and the server is under vacation irrespective of the $x$ for $i = 1,2,3 \dots \dots m$

$R_n(t)$Probability that at time the server is inactive due to the arrival of interruption.

$Q(t) =$ Probability that at time $t$ the server there are no customers in the queue or in service and the server is idle but available in the system.

According to the mathematical model mentioned above, the system has the following set of differential-difference equations,

$$\frac{\partial}{\partial x}P_0(x,t) + \frac{\partial}{\partial t}P_0(x,t) + [\lambda + \alpha + \mu(x)]P_0(x,t) = 0 \dots \dots \dots \dots (1)$$

$$\frac{\partial}{\partial x}P_n(x,t) + \frac{\partial}{\partial t}P_n(x,t) + [\lambda + \alpha + \mu(x)]P_n(x,t) = \lambda \sum_{k=1}^{n} C_k P_{n-k}(x,t) \ , n \geq 1 \dots \dots \dots \dots (2)$$

$$\frac{\partial}{\partial x}\sum_{i=1}^{m} V_0^{(i)}(x,t) + \frac{\partial}{\partial t}\sum_{i=1}^{m} V_0^{(i)}(x,t) + [\lambda \sum_{i=1}^{m} \gamma_i(x)] \sum_{i=1}^{m} V_0^{(i)}(x,t) = 0 \dots \dots \dots \dots (3)$$

$$\frac{\partial}{\partial x}\sum_{i=1}^{m} V_n^{(i)}(x,t) + \frac{\partial}{\partial t}\sum_{i=1}^{m} V_n^{(i)}(x,t) + [\lambda \sum_{i=1}^{m} \gamma_i(x)] \sum_{i=1}^{m} V_n^{(i)}(x,t) \quad = \lambda \sum_{k=1}^{n}\sum_{i=1}^{m} C_k V_{n-k}(x,t) \ , n \geq 1 \dots \dots .(4)$$

$$\frac{d}{dt}R_0(t) = -(\lambda + \beta)R_0(t) \dots \dots \dots \dots (5)$$

$$\frac{d}{dt}R_n(t) = -(\lambda + \beta)R_n(t) + \lambda \sum_{k=1}^{n} C_k R_{n-k}(x,t) + \alpha \int_0^\infty P_{n-1}(x,t)dx \dots \dots \dots \dots (6)$$

$$\frac{d}{dt}Q(t) = -\lambda Q(t) + \beta R_0(t) + \int_0^\infty \sum_{i=1}^{m} \gamma_i(x) V_0^{(i)}(x,t) \quad + (1-P)\int_0^\infty \mu(x)P_0(x,t)dx \dots \dots \dots \dots (7)$$

Equations are to be solved subject to the following boundary conditions:

$$P_n(0,t) = \lambda C_{n+1}Q(t) + +(1-P)\int_0^\infty \mu(x)P_{n+1}(x,t)dx + \beta R_{n+1}(t) + \int_0^\infty \sum_{i=1}^{m} \gamma_i(x)V_0^{(i)}(x,t)\, dx \ , n \geq 0 \dots \dots \dots \dots (8)$$

$$V_0^{(1)}(0,t) = P\int_0^\infty \mu(x)P_n(x,t)dx, n \geq 0 \dots \dots ...(9)$$

$$\sum_{i=2}^{m} V_n^{(1)}(0,t) = \int_0^\infty \gamma_{i-1}(x)\, \bar{V}_n^{(i-1)}(x,t)dx, n \geq 0 \dots \dots \dots \dots (10)$$

We assume that initially three are no customers in the system and the server is idle. So the initial conditions are

$$V_0^{(i)} = 0 = V_n^{(i)}(0) = 0 \ , Q(0) = 1 \ , R_n(0) = 0$$
$$P_n(0) = 0 \ forn \geq 0 \ aand$$
$$i = 1,2,3, \dots \dots \dots .. m. \qquad \dots \dots \dots \dots (11)$$

**Generating Functions of the Queue length: The time Dependent Solution**

In this section we obtain the transient solution for the above set of differential-difference equations.

**Theorem 4.1:** The system of differential difference equations to describe $M^{[X]}/G/1$ an queue with essential service with service interruption and in phase of vacation are given by equations (1) to (10) with the initial conditions (11) and the generating functions of transient solutions are given by equation (44) to (46)

**Proof:** We define the probability generating functions.

$$P(x,z,t)=\sum_{n=0}^{\infty} Z^n P_n(x,t)\,,P(z,t)=\sum_{n=0}^{\infty} Z^n P_n(x,t)$$

$$R(z,t)=\sum_{n=0}^{\infty} Z^n R_n(t)\,;C(z)=\sum_{n=1}^{\infty} C_n Z^n \dots\dots(12)$$

$$V^{(i)}(x,z,t)=\sum_{n=0}^{\infty} Z^n {V_n}^{(i)}(x,t)\,,V^{(i)}(z,t)=\sum_{n=0}^{\infty} Z^n {V_n}^{(i)}(t)\;\;,i=1,2\dots\dots m\dots\dots(13)$$

Which are convergent inside the circle given by and define the Laplace transform of a function $f(t)$ as,

$$\overline{f}(s)=\int_0^{\infty} e^{-st} f(t)dt\,,R(s)>0\;\dots\dots(14)$$

We take the Laplace transform of equations (1) to (10) and define (11) , we obtain.

$$\frac{\partial}{\partial x}\overline{P_0}(x,s)+\big(s+\lambda+\alpha+\mu(x)\big)\overline{P_0}(x,s)=0\dots\dots\dots\dots(15)$$

$$\frac{\partial}{\partial x}\overline{P_n}(x,s)+\big(s+\lambda+\alpha+\mu(x)\big)\overline{P_n}(x,s)=\lambda\sum_{k=1}^{n} C_k\overline{P}_{n-1}(x,s),\qquad n>1\dots\dots\dots\dots(16)$$

$$\frac{\partial}{\partial x}\sum_{i=1}^{m}\overline{V}_0^{(i)}\,(x,s)++(s+\lambda\sum_{i=1}^{m}\gamma_i(x))\sum_{i=1}^{m}\overline{V}_0^{(i)}\,(x,s)=0\dots\dots\dots\dots(17)$$

$$\frac{\partial}{\partial x}\sum_{i=1}^{m}\overline{V}_0^{(i)}\,(x,s)++(s+\lambda\sum_{i=1}^{m}\gamma_i(x))\sum_{i=1}^{m}\overline{V}_0^{(i)}\,(x,s)=\lambda\sum_{k=1}^{n}\sum_{i=1}^{m} C_k\overline{V}_{n-k}^{(i)}\,(x,s)\,,\quad n\geq 1\dots\dots\dots\dots(18)$$

$$(s+\lambda+\beta)\overline{R}_0(s)=0\dots\dots\dots\dots(19)$$

$$(s+\lambda+\beta)\overline{R}_n(s)=\lambda\sum_{k=1}^{n} C_k\overline{R}_{n-k}^{(i)}\,(s)+\alpha\int_0^{\infty}\overline{P}_{n-1}(x,s)dx\,,n\geq 1\dots\dots\dots\dots(20)$$

$$(s+\lambda)\overline{Q}(s)=1+\beta\overline{R}_0(s)+\int_0^{\infty}\sum_{i=1}^{m}\gamma_i(x)\overline{V}_0^{(i)}(x,s)\,dx+(1-P)\int_0^{\infty}\mu(x)\overline{P}_0(x,s)dx\dots\dots\dots\dots(21)$$

$$P_n(0,s)=\lambda C_{n+1}\overline{Q}(s)+\overline{R}_{n+1}(s)+(1-P)$$
$$\int_0^{\infty}\mu(x)\overline{P}_{n+1}(x,s)dx+\int_0^{\infty}\sum_{i=1}^{m}\gamma_i(x)\overline{V}_0^{(i)}(x,s)\,dx\dots\dots\dots\dots(22)$$

$$\overline{V}_n^{(1)}(0,s)=P\int_0^{\infty}\overline{P}_n(x,s)\mu(x)dx\,,n\geq 0\dots\dots\dots\dots(23)$$

$$\sum_{i=2}^{m}V_n^{(i)}(0,s)=\int_0^{\infty}\sum_{i=2}^{m}V_n^{(i-1)}(x,s)\,\gamma_{i-1}(x)dx\,,\quad n\geq 0\quad\dots\dots\dots(24)$$

Now multiplying equations (16), (18), (20) by $Z^n$ and summing over $n$ from $1\ to\infty$ adding to equation (15), (17), (19) and using the generating functions defined in equations (12) and (13) we get,

$$\frac{\partial}{\partial x}\overline{P_n}(x,z,s)+\big(s+\lambda-\lambda c(z)+\alpha+\mu(x)\big)\overline{P_n}(x,z,s)=0\dots\dots\dots..(25)$$

$$\frac{\partial}{\partial x}\sum_{i=1}^{m}\overline{V}_n^{(i)}\,(x,z,s)++(s+\lambda c(z)+\sum_{i=1}^{m}\gamma_i(x))\sum_{i=1}^{m}\overline{V}_0^{(i)}\,(x,z,s)=0\dots\dots\dots\dots(26)$$

$$(s+\lambda-\lambda c(z)+\beta)\overline{R}(z,s)=\alpha z\int_0^{\infty}\overline{P}(x,z,s)dx\dots\dots\dots\dots(27)$$

For the boundary condition, we multiply both sides of equation (22) by $Z^n$ sum over $n$ from $1\ to\infty$ and use the equations (12) and (13) to get,

$$z\overline{P}(0,z,s) = \lambda c(z)\overline{Q}(s) + \beta\overline{R}(z,s) - \beta R_0(s) + (1-P)\int_0^\infty \mu(x)\overline{P}(x,z,s)dx$$
$$-(1-P)\int_0^\infty \mu(x)\overline{P}_0(x,s)dx + \int_0^\infty \sum_{i=1}^m \gamma_i(x)\overline{V}_0^{(i)}(x,z,s)\,dx$$
$$-\int_0^\infty \sum_{i=1}^m \gamma_i(x)\overline{V}_0^{(i)}(x,s)\,dx \ldots\ldots\ldots\ldots(28)$$

Using equation (21) and (28), equation becomes,

$$z\overline{P}(0,z,s) = \left[1 - S\overline{Q}(s)\right] + \lambda(c(z)-1)\overline{Q}(s) + \beta\overline{R}(z,s) + \int_0^\infty \sum_{i=1}^m \gamma_i(x)\overline{V}_0^{(i)}(x,z,s)\,dx$$
$$+(1-P)\int_0^\infty \mu(x)\overline{P}(x,z,s)dx \ldots\ldots\ldots\ldots(29)$$

Performing similar operation on equations (24)

$$\sum_{i=1}^m V^{(i)}(0,z,s) = \int_0^\infty \sum_{i=1}^m \gamma_{i-1}(x)\overline{V}_0^{(i-1)}(x,z,s)\,dx \ldots\ldots(30)$$

Integrating the equation (25) between 0 $tox$, we get,

$$\overline{P}(x,z,s) = \overline{P}(0,z,s)e^{-(s+\lambda-\lambda c(z)+\alpha)x} - \int_0^x \mu(t)dx \ldots\ldots\ldots\ldots(31)$$

Where $\overline{P}(0,z,s)$ is given by equation (29), Again

Integrating equation (31) by parts with respect to $x$ yields

$$\overline{P}(z,s) = \overline{P}(0,z,s)\left[\frac{1-\overline{B}(s+\lambda-\lambda c(z)+\alpha)}{s+\lambda-\lambda c(z)+\alpha}\right] \ldots(32)$$

Where

$$\overline{B}(s+\lambda-\lambda c(z)+\alpha) = \int_0^\infty e^{-(s+\lambda-\lambda c(z)+\alpha)x}dB(x) \ldots\ldots\ldots\ldots(33)$$

Is the Laplace – Stieltjes transform of the essential service time $B(x)$. Now multiplying both sides of equation service by $\mu(x)$ and integrating over $x$ we obtain.

$$\int_0^\infty \overline{P}(x,z,s)\mu(x)dx = \overline{P}(0,z,s)\,\overline{B}(s+\lambda-\lambda c(z)+\alpha) \ldots\ldots\ldots\ldots(34)$$

Similarly, one integrating equation (26) from 0 $tox$ , we get,

$$\sum_{i=1}^m \overline{V}^{(i)}(x,z,s) = \sum_{i=1}^m \overline{V}^{(i)}(0,z,s)e^{-[s+\lambda-\lambda c(z)]x} - \int_0^x \sum_{i=1}^m \gamma_i(t)dt \ldots\ldots\ldots\ldots(35)$$

Where $\overline{V}^{(i)}(0,z,s)$ are given by equations (30). Again integrating equations (35) by parts with respect to $x$ yields

$$\sum_{i=1}^m \overline{V}^{(i)}(z,s) = \sum_{i=1}^m \overline{V}^{(i)}(0,z,s)\left[\frac{1-\sum_{i=1}^m(s+\lambda-\lambda c(z))}{s+\lambda-\lambda c(z)}\right] \ldots\ldots(36)$$

Where

$$\sum_{i=1}^m \overline{V}_{(i)}\left(s+\lambda-\lambda c(z)\right) = \int_0^\infty e^{-[s+\lambda-\lambda c(z)]x}d\sum_{i=1}^m dV_i(x) \ldots\ldots\ldots\ldots(37)$$

Is the Laplace – Stieltjes transform of the m$^{th}$ phase of vacation time $V_m(x)$ respectively. Now multiplying both sides of equations (35) by $\gamma_i(x)$ integrating over $x$ we obtain,

$$\int_0^\infty \sum_{i=1}^m \overline{V}^{(i)}(x,z,s)\gamma_i(x)dx = \sum_{i=1}^m \overline{V}^{(i)}(0,z,s)\overline{V}_{(i)}[s+\lambda-\lambda c(z)] \ldots\ldots\ldots\ldots(38)$$

Using equation (34) in equation (30) we get,

$$\left[z - P\sum_{i=1}^m \overline{V}_i\overline{B}_i - (1-P)\overline{B}\right]\overline{P}(0,z,s) = \left[1 - s\overline{Q}(s)\right] + \lambda(c(z)-1)\overline{Q}(s) + \beta\overline{R}(z,s) \ldots\ldots\ldots\ldots(39)$$

From (27) and (31) we get,

$$\overline{R}(z,s) = \frac{\alpha z}{s+\lambda-\lambda c(z)+\beta}\overline{P}(0,z,s)$$

$$\left[\frac{1-\overline{B}(s+\lambda-\lambda c(z)+\alpha}{s+\lambda-\lambda c(z)+\alpha}\right] \dots\dots\dots (40)$$

Where$\overline{B} = \overline{B}(\lambda - \lambda c(z) + \alpha$ , $\sum_{i=1}^{m} V_i = \sum_{i=1}^{m} \overline{V}_i(\lambda - \lambda c(z))$
Now using equation (40) in (39) we have,

$$\overline{P}(0,z,s) = \frac{\sum_{i=1}^{m} f_{i-1}(z)\left[\left(1-s\overline{Q}(s)\right)+\lambda(c(z)-1)\overline{Q}(s)\right]}{dr} \dots\dots\dots\dots\dots..(41)$$

Similarly, using the equation (41) in equation (38) we get,

$$\sum_{i=1}^{m} V^{(i)}(0,z,s) = \frac{P\sum_{i=1}^{m}\overline{V}_{i-1}\overline{B}f_{i-1}(z)\left[\begin{matrix}\left(1-s\overline{Q}(s)\right)+\\ \lambda(c(z)-1)\overline{Q}(s)\end{matrix}\right]}{dr} \dots(42)$$

Where

$$dr = \sum_{i=1}^{m} f_{i-1}(z)\left[z - P\sum_{i=1}^{m}\overline{V}_i\,\overline{B}_i - (1-P)\overline{B}\right] - \alpha z\beta\left(1-\overline{B}\right) \dots\dots\dots\dots.(43)$$

Using equation (41), (42) in equations (32), (35) and (40)

$$\frac{\overline{P}(z,s) = f_1(z)\left[\begin{matrix}\left(1-s\overline{Q}(s)\right)+\\ \lambda(c(z)-1)\overline{Q}(s)\end{matrix}\right]\left[1-\overline{B}\right]}{dr} \dots\dots\dots (44)$$

$$\sum_{i=1}^{m} V^{(i)}(z,s) = \frac{\left[\left(1-s\overline{Q}(s)\right)+\lambda(c(z)-1)\overline{Q}(s)\right]}{dr}$$

$$P\overline{B}\sum_{i=1}^{m}\overline{V}_{i-1}f_{i-1}(z)\left[\frac{1-V_{i-1}(s+\lambda-\lambda c(z)}{s+\lambda-\lambda c(z)}\right] \dots.(45)$$

$$\overline{R}(z,s) = \frac{\alpha z(1-\overline{B})\left[(1-s\overline{Q}(s))+\lambda(c(z)-1)\overline{Q}(s)\right]}{dr} \dots.(46)$$

Where $dr$ is given by equation (43). Thus $\overline{P}(z,s)\sum_{i\neq 1}^{m}\overline{V}^{(i)}(z,s)$ and $\overline{R}(z,s)$ are completely determined from equations (44), (45) and (46) which is the proof of the theorem.

**The Steady State results**

In this section, we should derive the steady state probability distribution for our queueing model. To define the steady state probability we suppers the argument t whereres it appears in the time-dependent analysis. This can be obtained by applying the well-known Tauberian property.

$$\lim_{s\to 0} \overline{s} f(s) = \lim_{t\to\infty} f(t) \dots\dots\dots\dots.(47)$$

In order to determine $\overline{P}_{q,1}(z,s), \overline{V}(z,s)$ and $\overline{R}_q(z,s)$ completely, we have yet to determine the unknown $Q$ which appears in the numerators of the right hand sides of equations (44,45 & 46) for that purpose, we shall use the normalizing condition.

$$P(1) + \sum_{m=1}^{n} V^{(i)}(1) + R(1) + Q = 1 \dots\dots\dots\dots.(48)$$

**Theorem:** The steady state probability distribution for an $M/G/1$ queue with an essential service following general distribution subject to optional server vacation policy and random m interruption are given by

$$P(1) = \frac{\lambda E(I)\beta[1-\overline{B}(\alpha)]Q}{Dr}$$

$$V_i(1) = \frac{\sum_{i=1}^{m}\lambda P_i \quad \alpha\beta E(I)\overline{B}(\alpha)E(V_i)\,Q}{Dr}$$

$$R(1) = \frac{\lambda\alpha E(I)[1-\overline{B}(\alpha)]Q}{Dr}$$

Where$Dr = -\lambda E(I)(\alpha+\beta)\left[\left[1-\overline{B}(\alpha)\right]+\alpha\beta\overline{B}(\alpha)\right]\left[\sum_{i=1}^{m} P_1\,E(I)(E(V_1+P_2E(V_2+P_3E(V_3+\cdots+\sum_{i=1}^{m}P_iE(V_i)))\right]$

$P(1), V_m,\ R(1) and Q$are the steady state probabilities that the server is providing essential service, single m random number of vacation and server under idle respectively without regard to the number of customers in the queue.

Multiplying both sides of equation (44), (45) and (46) by S, taking limit as $S \to 0$ applying Tauberian property and simplifying, we obtain.

$$P(z) = \frac{f_1(z)(1-\overline{B})\lambda(c(z)-1)Q}{D(z)} \ldots\ldots\ldots\ldots(49)$$

$$\sum_{i=1}^{m} V_i(z) = \frac{\sum_{i=1}^{m} P_i f_{i-1} V_{i-1}\overline{B}(\overline{V}_i - 1)Q}{D(z)} \ldots\ldots\ldots\ldots(50)$$

$$R(z) = \frac{\lambda\alpha z(1-\overline{B})(c(z)-1)Q}{D(z)} \ldots\ldots\ldots\ldots(51)$$

Where,

$$D(z) = f_{i-1}\left[z - \overline{B}(1 - P_1 + P_1\overline{V}_1 f_{i-1}(z))\right] - \alpha z\beta(1-\overline{B}) \ldots\ldots\ldots\ldots(52)$$

Let $W_q(z)$ denote the probability generating function of the queue size irrespective of the state of the system. Then adding equations (49), (50) and (51) we obtain

$W_q(z) = P_{q,1}(z) + \sum_{i=1}^{m} V_i(z) + R_q(z)$

$$\frac{f_1(z)(1-E)\lambda(c(z)-1)Q}{Dr} + \frac{\sum_{i=1}^{m} P_i f_{i-1} V_{i-1}\overline{B}(\overline{V}_{i-1})Q}{Dr} + \frac{\lambda\alpha z(1-\overline{B})(c(z)-1)Q}{D(z)} \ldots\ldots\ldots\ldots(53)$$

It is easy to verify that for $z = 1$, $W_q(z)$ is indeterminate of the form $0/0$

Therefore apply L'Hospital rule and on simplifying we obtain the result (53) where,

$c(1) = 1$, $c''(1) = E(I)$ is mean batch size of the arriving customers$-\overline{B}'(0) = E(B) - \overline{V}_i{}' = E(V_i)$, $i = 1,2,3, \ldots\ldots\ldots m$

$$W_q(1) = \frac{\lambda E(I)\begin{bmatrix}(\alpha+\beta)\left(1-\overline{B}(\alpha)\right) + P_1\alpha\beta\overline{B}(\alpha E(V_1)) \\ +P_2(E(V_2) + P_3 \ldots\ldots\ldots\ldots + \sum_{i=1}^{m} P_i(V_i))\end{bmatrix}Q}{Dr} \ldots\ldots\ldots\ldots(54)$$

And $Dr$ is given by equation (52). Therefore adding $Q$ to equation (54) equating to 1 and simplifying, we get,

$$Q = 1 - P \ldots\ldots\ldots\ldots(55)$$

And hence the utilization factor $\rho$ of the system is given by

$$\rho = \lambda P_1 E(I)\left[E(V_1) + P_2(E(V_2)) + \cdots + \sum_{i=1}^{m} P_i(V_i)\right] - \frac{\lambda E(I)}{B(\alpha)}\left(\frac{1}{\beta} + \frac{1}{\alpha}\right)\left[1 - \overline{B}(\alpha)\right] < 1 \qquad \ldots\ldots(56)$$

Where is the stability condition under which the steady state exists. Equation (55) given the probability the server is idle substitute Q form (55) into (53) we have completely and explicitly determined $W_q(z)$.

**Conclusion:** In this paper clearly analysis the transient solution steady state results and the various performance measures of the queueing system with $m^{th}$ stages of service with several server vacation. Server provides essential service in all the stages to the arriving customers. This model can be utilized in large scale manufacturing industries and communication networks.

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