NEW STATIC SPHERICALLY SYMMETRIC ISOTROPIC SOLUTIONS FOR PERFECT FLUID DISTRIBUTIONS

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Abstract: In this paper we have investigated the isotropic models through spherically symmetric perfect fluid distributions on the basis of reality conditions. All the three solutions satisfy a differential equation of perfect fluid distribution for three different values of parameter 'a', $(i.e.\ a=1,\ a>1,\ a<1)$ present in the equations. All the solutions satisfy the reality conditions $\rho \geq p \geq 0,\ p_r < 0,\ \rho_r < 0,\ 0 \leq \frac{p}{\rho} \leq 1$ as well as causality conditions $0 < \frac{dp}{d\rho} < 1$ for some range of constants in the corresponding solutions.

Keywords: Perfect fluid distributions, Einstein field equations, Spherical symmetry and Isotropic solutions.

Introduction: It is by no means obvious to transform a solution from canonical to isotropic form and vice versa. So, there exist many solutions in either system which do not appear in the other. A large number of workers have solved the Einstein-Field equations by considering the spherically symmetric space-time in the canonical form. Therefore, it would be worth solving the field equations in isotropic form to get new solutions describing the perfect fluid spheres. Tolman (1939), Narlikar et al. (1943), Wyman (1946, 1978), Nariai (1950, 1951), Buchdahl (1964), Kuchowicz (1971a, 1971b, 1972a, 1972b), Chakravarty et al. (1976), Bayain (1978), Glass and Goldman (1978), Goldman (1978), Stewart (1982), Lake (1983), Stephani (1983), Tikekar (1984), Pant and Sah (1982,1985), Burlankov (1993), Salah Hagg et al. (1994), Pant (2010), Pant et al. (2010a, 2010b) and Kumar et al. (2010) have tried the problem by considering static and non-static spherically symmetric line element in the isotropic form.

Interestingly, the field equations for static spherically symmetries are reducible to a simpler form involving two unknowns L and G. If one of these is assumed on mathematical or physical grounds, then the other can be found out.

However, it is not always certain to obtain the solution explicitly in the closed form for a given expression of one of the unknowns. So, we will have to choose only those expressions which enable us to solve the equation in the closed form. We have opted some forms and derived a class of new perfect fluid spheres. In this article, all the three solutions are obtained and subjected to following conditions which are essential for meaningful solution and regular solution.

(I) The solution should be free from physical and geometrical singularities i.e. finite and positive values of central pressure, central density and non zero positive values of $g_{11} = g_{22} = g_{33} = A$ and $g_{44} = C$ i.e. $p_0 > 0$ and $\rho_0 > 0$.

(II) The solution should have positive and monotonically decreasing expressions for fluid parameters (p and ρ) with the increase of r i.e.

(a) $p' = 0 \Rightarrow r = 0$ and $\left(p''\right)_{r=0} < 0$ and p' is negative valued function for $r > 0$.

(b) $\rho' = 0 \Rightarrow r = 0$ and $\left(\rho''\right)_{r=0} < 0$ and ρ' is negative valued function for $r > 0$.

(III) The solution should have positive and monotonically decreasing expression for fluid parameter $\frac{p}{\rho c^2}$ with the increase of r i.e $\left(\frac{p}{\rho c^2}\right)' = 0 \Rightarrow r = 0$ and $\left(\frac{p}{\rho c^2}\right)''_{r=0} < 0$ and $\left(\frac{p}{\rho c^2}\right)'$ is negative valued function of $r > 0$.

(IV) The solution should have positive and monotonically decreasing expression for fluid parameter $\left(\frac{dp}{d\rho}\right)$ with the increase of r. Thus in view of (II) we infer that $\left(\frac{dp}{d\rho}\right)$ should be monotonically increasing with the increase of density ρ i.e. $\left(\frac{d^2 p}{d\rho^2}\right) > 0$.

(V) The solution should have causality condition within the ball i.e. $0<\frac{1}{c^2}\left(\frac{dp}{d\rho}\right)\le 1$.

(VI) The solution should have positive value of ratio of pressure-density and less than 1 within the ball i.e. $0<\left(\frac{p}{\rho c^2}\right)\le 1$.

1. For meaningful solution, it should satisfy (II), (V) and (VI).
2. For regular solution, it should satisfy (I), (II), (III), (V) and (VI).

2.A. Isotropic Fluid Distribution in Isotropic Coordinate:

The Spherically symmetric metric in the canonical coordinate system is expressed as

$$ds^2=-A(r,t)dr^2-B(r,t)\left(d\theta^2+\sin^2\theta\, d\phi^2\right)+C(r,t)dt^2 \tag{2.1}$$

Where A, B and C are functions of r and t only.

An appropriate transformation in the static case, i.e. $A=A(r)$, $B=B(r)$ and $C=C(r)$.

$$B=\bar{r}^2 A\left(\frac{dr}{d\bar{r}}\right)^2, \qquad r=r(\bar{r}) \tag{2.2}$$

Consequently (2.1) with reference (2.2), we have

$$ds^2=-A(\bar{r})\left[d\bar{r}^2+\bar{r}^2\left(d\theta^2+\sin^2\theta\, d\phi^2\right)\right]+C(\bar{r})dt^2 \tag{2.3}$$

then the metric (2.1) is said to be in isotropic coordinate system.

2.B. Field Equations and Isotropic Fluid Spheres:

The Einstein's field equation for perfect fluid distributions can be written as below

$$-8\pi T_{ij}=R_{ij}-\frac{1}{2}R\,g_{ij}=-8\pi\left[(p+\rho)v_i v_j-p\,g_{ij}\right] \tag{2.4}$$

where ρ being the density, p the pressure and v_i the flow vector.

The Einstein's field equations (2.4) for the metric (2.3) can be expressed as

$$8\pi T_1^1=-\frac{1}{A}\left[\frac{1}{r}\left(\frac{C'}{C}+\frac{A'}{A}\right)+\frac{A'C'}{2AC}+\frac{A'^2}{4A^2}\right]=-8\pi p, \tag{2.5}$$

$$8\pi T_2^2=8\pi T_3^3=-\frac{1}{A}\left[\frac{1}{2}\left(\frac{A'}{A}\right)'+\frac{1}{2}\left(\frac{C'}{C}\right)'+\frac{C'^2}{4C^2}+\frac{1}{2r}\left(\frac{C'}{C}+\frac{A'}{A}\right)\right]=-8\pi p, \tag{2.6}$$

$$8\pi T_4^4=-\frac{1}{A}\left[\left(\frac{A'}{A}\right)'+\frac{A'^2}{4A^2}+\frac{2A'}{rA}\right]=8\pi\rho, \tag{2.7}$$

The consistency of (2.5) and (2.6) reveals

$$\frac{A''}{A}-\frac{3A'^2}{2A^2}+\frac{C''}{C}-\frac{C'^2}{2C^2}-\frac{1}{r}\left(\frac{A'}{A}+\frac{C'}{C}\right)-\frac{A'C'}{AC}=0 \tag{2.8}$$

which can be reduced to a differential equation

$$L\bar{\bar{G}}=2\bar{\bar{L}}G \tag{2.9}$$

where $L=A^{-\frac{1}{2}},\ G=\left(\frac{C}{A}\right)^{\frac{1}{2}}$

and the overhead bars denotes the differentiation with respect to $x\left(=r^2\right)$.

Kustaanheimo and Qvist (1948), If $A=e^{2\lambda}$ and $C=e^{2\nu}$ then the equation (2.8) assumes the following form

$$\lambda''+\nu''+\nu'^2-\lambda'^2-2\lambda'\nu'-\frac{(\lambda'+\nu')}{r}=0 \tag{2.10}$$

which is invariant under the transformations

$\nu^*=-\nu$ and $\lambda^*=\lambda+2\nu$ Buchdahl (1956)

The transformation can be used to generate new static perfect fluid solutions with λ^* and ν^*, the later can be obtained by means of λ and ν of the seed solution.

The equation (2.9) involves two unknowns L and G. Therefore, we shall discuss some cases corresponding to the conditions adopted for L and G, respectively.

3. Solutions of the Einstein Field Equations:

Solution (I): Let us assume

$$\frac{2\bar{\bar{L}}}{L} = -\frac{a}{\left(1-x^2\right)^2} \quad \text{and} \quad \text{hence} \quad \frac{\bar{\bar{G}}}{G} = -\frac{a}{\left(1-x^2\right)^2}$$

when $a=1$, then which integrates to

$$A = \frac{1}{\left(r^2+1\right)^2\left(a_1 P^{\left(\frac{1}{4}\sqrt{2}-\frac{1}{2}\right)} + a_2 P^{\left(-\frac{1}{4}\sqrt{2}-\frac{1}{2}\right)}\right)^2},$$

$$C = \frac{\left(r^4-1\right)\left(b_1+b_2 Q\right)^2}{\left(r^2+1\right)^2\left(a_1 P^{\left(\frac{1}{4}\sqrt{2}-\frac{1}{2}\right)} + a_2 P^{\left(-\frac{1}{4}\sqrt{2}-\frac{1}{2}\right)}\right)^2},$$

, ,

where $P = \left(\frac{1+r^2}{1-r^2}\right)$, $Q = \log\left(\frac{1+r^2}{1-r^2}\right)$, a_1, a_2, b_1, b_2 being constants.

The pressure, density, pressure gradient and density gradient of the fluid can then be computed as

$$p = -\frac{2}{\left(r^4-1\right)\left(b_1+b_2 Q\right)}\alpha_1,$$

where

$$\alpha_1 = \begin{pmatrix} a_1^2 b_2 P^{\left(\frac{1}{2}\sqrt{2}\right)}\left[\left(r^4+1\right)\left(4-2\sqrt{2}\,Q\right) - r^2\left(4\sqrt{2}-5Q\right)\right] \\ + a_2^2 b_2 P^{\left(-\frac{1}{2}\sqrt{2}\right)}\left[\left(r^4+1\right)\left(4+2\sqrt{2}\,Q\right) + r^2\left(4\sqrt{2}+5Q\right)\right] \\ + a_1^2 b_1 P^{\left(\frac{1}{2}\sqrt{2}\right)}\left[-2\sqrt{2}\left(r^4+1\right)+5r^2\right] + a_1 a_2 b_2\left[8\left(r^4+1\right)-2r^2 Q\right] \\ + a_2^2 b_1 P^{\left(-\frac{1}{2}\sqrt{2}\right)}\left[2\sqrt{2}\left(r^4+1\right)+5r^2\right] - 2r^2 a_1 a_2 b_1 \end{pmatrix},$$

$$\rho = \frac{2}{\left(r^4-1\right)}\left(a_1^2 P^{\left(\frac{1}{2}\sqrt{2}\right)}\left[-3\sqrt{2}\left(r^4+1\right)+11r^2\right] + a_2^2 P^{\left(-\frac{1}{2}\sqrt{2}\right)}\left[3\sqrt{2}\left(r^4+1\right)+11r^2\right] + 10r^2 a_1 a_2\right),$$

$$\rho - p = \frac{2}{\left(r^4-1\right)\left(b_1+b_2 Q\right)}\alpha_2,$$

where

$$\alpha_2 = \begin{pmatrix} a_1^2 b_2 P^{\left(\frac{1}{2}\sqrt{2}\right)}\left[\left(r^4+1\right)\left(4-5\sqrt{2}\,Q\right)-r^2\left(4\sqrt{2}-16Q\right)\right] \\ +\ a_2^2 b_2 P^{\left(-\frac{1}{2}\sqrt{2}\right)}\left[\left(r^4+1\right)\left(4+5\sqrt{2}\,Q\right)+r^2\left(4\sqrt{2}+16Q\right)\right] \\ +\ a_1 a_2 b_2\left[8\left(r^4+1\right)+8r^2 Q\right]+a_1^2 b_1 P^{\left(\frac{1}{2}\sqrt{2}\right)}\left[-5\sqrt{2}\left(r^4+1\right)+16r^2\right] \\ +\ a_2^2 b_1 P^{\left(-\frac{1}{2}\sqrt{2}\right)}\left[5\sqrt{2}\left(r^4+1\right)+16r^2\right]+8r^2 a_1 a_2 b_1 \end{pmatrix},$$

$$\frac{d\rho}{dr} = -\frac{4r}{\left(r^4-1\right)^2}\left(10a_1a_2\left(r^4+1\right)+a_2^2 P^{\left(-\frac{1}{2}\sqrt{2}\right)}\left[5\left(r^4+1\right)+\sqrt{2}r^2\right]+a_1^2 P^{\left(\frac{1}{2}\sqrt{2}\right)}\left[5\left(r^4+1\right)-\sqrt{2}r^2\right]\right),$$

$$\frac{dp}{dr} = \frac{4r}{\left(r^4-1\right)^2\left(b_1+b_2Q\right)^2}\alpha_3,$$

where

$$\alpha_3 = \begin{pmatrix} a_2^2 b_2^2 P^{\left(-\frac{1}{2}\sqrt{2}\right)}\left[\left(r^4+1\right)\left(-8+Q^2\right)+r^2\left(-8\sqrt{2}+3\sqrt{2}Q^2+8Q\right)\right] \\ -2\left(r^4+1\right)a_1a_2b_1^2+a_1a_2b_2^2\left[\left(r^4+1\right)\left(-16-2Q^2\right)+32r^2Q\right] \\ +a_2^2 b_1^2 P^{\left(-\frac{1}{2}\sqrt{2}\right)}\left[\left(r^4+1\right)+3\sqrt{2}r^2\right]+\ a_1^2 b_1^2 P^{\left(\frac{1}{2}\sqrt{2}\right)}\left[\left(r^4+1\right)-3\sqrt{2}r^2\right] \\ +a_1^2 b_2^2 P^{\left(\frac{1}{2}\sqrt{2}\right)}\left[\left(r^4+1\right)\left(-8+Q^2\right)+r^2\left(8\sqrt{2}-3\sqrt{2}Q^2+8Q\right)\right] \\ +a_2^2 b_1 b_2 P^{\left(-\frac{1}{2}\sqrt{2}\right)}\left[2Q\left(r^4+1\right)+r^2\left(8+6\sqrt{2}Q\right)\right] \\ +\ a_1^2 b_1 b_2 P^{\left(\frac{1}{2}\sqrt{2}\right)}\left[2Q\left(r^4+1\right)+r^2\left(8-6\sqrt{2}Q\right)\right] \\ +\ a_1a_2b_1b_2\left[-4Q\left(r^4+1\right)+32r^2\right] \end{pmatrix},$$

The reality conditions, i.e. $p>0$, $\rho>0$ and $\rho-p\geq 0$ at $r=0$ imply

$$\left(p\right)_{r=0} = \frac{4}{b_1}\left(2b_2\left(a_1^2+a_2^2\right)+4a_1a_2b_2-\sqrt{2}b_1\left(a_1^2-a_2^2\right)\right)>0,$$

$$\left(\rho\right)_{r=0} = 6\sqrt{2}\left(a_1^2-a_2^2\right)>0,$$

$$\left(\rho-p\right)_{r=0} = \frac{2}{b_1}\left(-4b_2\left(a_1^2+a_2^2\right)-8a_1a_2b_2+5\sqrt{2}b_1\left(a_1^2-a_2^2\right)\right)\geq 0,$$

$$\left(\frac{p}{\rho}\right)_{r=0} = \frac{\sqrt{2}}{3b_1\left(a_1-a_2\right)}\left(2b_2\left(a_1+a_2\right)-\sqrt{2}b_1\left(a_1-a_2\right)\right),$$

provided $0<\frac{\sqrt{2}}{3b_1\left(a_1-a_2\right)}\left(2b_2\left(a_1+a_2\right)-\sqrt{2}b_1\left(a_1-a_2\right)\right)\leq 1,$ (3.1)

Also, we have

$$\left(\frac{dp}{d\rho}\right)_{r=0}=\frac{8b_2^{\,2}}{5b_1^{\,2}}-\frac{\left(a_1-a_2\right)^2}{5\left(a_1+a_2\right)^2}, \qquad \left(\frac{d}{dr}\left(\frac{p}{\rho}\right)\right)_{r=0}=0,$$

which further demands: $a_1>0,\;\; a_2>0,\;\; b_1>0,\;\; b_2>0$

$$0<\frac{8b_2^{\,2}}{5b_1^{\,2}}-\frac{\left(a_1-a_2\right)^2}{5\left(a_1+a_2\right)^2}<1, \qquad (3.2)$$

The solution is physical valid subject to the given inequality satisfied. Also the involved constants are so restricted that $\left(\frac{d^2p}{dr^2}\right)_{r=0}<0,\;\; \left(\frac{d^2\rho}{dr^2}\right)_{r=0}<0$. That's why the pressure and density may be maximum at the centre.

Solution (II): Let us assume

$$\frac{2\bar{\bar{L}}}{L}=-\frac{a}{\left(1-x^2\right)^2} \qquad \text{and} \qquad \text{hence} \qquad \frac{\bar{\bar{G}}}{G}=-\frac{a}{\left(1-x^2\right)^2}$$

when $a<1$, which integrates to

$$A=\frac{1}{\left(r^2+1\right)^2\left(c_1M^{\left(\frac{1}{4}\sqrt{(4-2a)}-\frac{1}{2}\right)}+c_2M^{\left(-\frac{1}{4}\sqrt{(4-2a)}-\frac{1}{2}\right)}\right)^2},$$

$$C=\frac{\left(d_1M^{\left(\frac{1}{2}\sqrt{(1-a)}-\frac{1}{2}\right)}+d_2M^{\left(-\frac{1}{2}\sqrt{(1-a)}-\frac{1}{2}\right)}\right)^2}{\left(c_1M^{\left(\frac{1}{4}\sqrt{(4-2a)}-\frac{1}{2}\right)}+c_2M^{\left(-\frac{1}{4}\sqrt{(4-2a)}-\frac{1}{2}\right)}\right)^2},$$

where $M=\left(\frac{1+r^2}{1-r^2}\right)$, $a,\, c_1,\, c_2,\, d_1,\, d_2$ being arbitrary constants.

The reality conditions for pressure, density, pressure gradient and density gradient of the fluid $p>0,\;\; \rho>0,\;\; \rho-p\geq 0$ at $r=0$ yield

$$\left(p\right)_{r=0}=\frac{4}{\left(d_1+d_2\right)}\left(\sqrt{(1-a)}\left(d_1-d_2\right)\left(c_1+c_2\right)^2-\sqrt{(4-2a)}\left(d_1+d_2\right)\left(c_1^{\,2}-c_2^{\,2}\right)\right)>0,$$

$$\left(\rho\right)_{r=0}=6\sqrt{(4-2a)}\left(c_1^{\,2}-c_2^{\,2}\right)>0,$$

$$\left(\rho-p\right)_{r=0}=\frac{2}{\left(d_1+d_2\right)}\left(5\sqrt{(4-2a)}\left(d_1+d_2\right)\left(c_1^{\,2}-c_2^{\,2}\right)-2\sqrt{(1-a)}\left(d_1-d_2\right)\left(c_1+c_2\right)^2\right)\geq 0,$$

$$\left(\frac{p}{\rho}\right)_{r=0}=\frac{2}{3}\left(\frac{\sqrt{(1-a)}\left(d_1-d_2\right)\left(c_1+c_2\right)}{\sqrt{(4-2a)}\left(d_1+d_2\right)\left(c_1-c_2\right)}-1\right),$$

provided $0<\frac{2}{3}\left(\frac{\sqrt{(1-a)}\left(d_1-d_2\right)\left(c_1+c_2\right)}{\sqrt{(4-2a)}\left(d_1+d_2\right)\left(c_1-c_2\right)}-1\right)\leq 1,$ (3.3)

Also, we obtain

$$\left(\frac{dp}{dr}\right)_{r=0}=0, \qquad \left(\frac{d\rho}{dr}\right)_{r=0}=0 \qquad \text{and} \qquad \left(\frac{d}{dr}\left(\frac{p}{\rho}\right)\right)_{r=0}=0,$$

$$\left(\frac{dp}{d\rho}\right)_{r=0}=-\frac{\begin{bmatrix} ac_2^2d_1^2+6ac_1c_2d_1^2-4ac_1c_2d_1d_2-6ac_2^2d_1d_2+ac_1^2d_2^2+ac_2^2d_2^2 \\ -8c_1c_2d_1^2+8c_1^2d_1d_2+ac_1^2d_1^2+8c_2^2d_1d_2-6ac_1^2d_1d_2+6ac_1c_2d_2^2-8c_1c_2d_2^2 \end{bmatrix}}{5a(d_1+d_2)^2(c_1+c_2)^2},$$

or $$\left(\frac{dp}{d\rho}\right)_{r=0}=-\frac{\begin{bmatrix} a(c_1+c_2)^2(d_1^2+d_2^2)+4ac_1c_2(d_1+d_2)^2-6ad_1d_2(c_1+c_2)^2 \\ -8c_1c_2(d_1^2+d_2^2)+8d_1d_2(c_1^2+c_2^2) \end{bmatrix}}{5a(d_1+d_2)^2(c_1+c_2)^2},$$

which further demands: $c_1>0,\ c_2>0,\ d_1>0,\ d_2>0$

$$0<-\left[\frac{\begin{bmatrix} a(c_1+c_2)^2(d_1^2+d_2^2)+4ac_1c_2(d_1+d_2)^2-6ad_1d_2(c_1+c_2)^2 \\ -8c_1c_2(d_1^2+d_2^2)+8d_1d_2(c_1^2+c_2^2) \end{bmatrix}}{5a(d_1+d_2)^2(c_1+c_2)^2}\right]<1, \quad (3.4)$$

The solution is physical valid subject to the given inequality satisfied. Also the involved constants are so restricted that $\left(\frac{d^2p}{dr^2}\right)_{r=0}<0,\quad \left(\frac{d^2\rho}{dr^2}\right)_{r=0}<0$. That's why the pressure and density may be maximum at the centre.

Solution (III): Let us assume

$$\frac{2\bar{\bar{L}}}{L}=-\frac{1}{(1-x^2)^2} \quad \text{and} \quad \text{hence} \quad \frac{\bar{\bar{G}}}{G}=-\frac{1}{(1-x^2)^2}$$

when $a>1$, which integrates to

$$A=\frac{1}{(1-r^4)[m_1\cos Y+m_2\sin Y]^2},$$

$$C=\frac{[n_1\cos X+n_2\sin X]^2}{[m_1\cos Y+m_2\sin Y]^2},$$

where $X=\left[\frac{1}{2}\sqrt{(a-1)}\log\left(\frac{1+r^2}{1-r^2}\right)\right]$, $Y=\left[\frac{1}{4}\sqrt{(2a-4)}\log\left(\frac{1+r^2}{1-r^2}\right)\right]$, $m_1,\ m_2,\ n_1,\ n_2$ being constants.

The reality conditions for pressure, density, pressure-gradient and density-gradient of the fluid $p,\ \rho,\ \rho-p\geq 0$ at $r=0$ can also be written as

$$(p)_{r=0}=\frac{4m_1}{n_1}\left(\sqrt{(a-1)}m_1n_2-\sqrt{(2a-4)}m_2n_1\right)>0,,$$

$$(\rho)_{r=0}=6\sqrt{(2a-4)}m_1m_2>0,$$

$$(\rho-p)_{r=0}=\frac{2m_1}{n_1}\left(5\sqrt{(2a-4)}m_2n_1-2\sqrt{(a-1)}m_1n_2\right),$$

$$\left(\frac{p}{\rho}\right)_{r=0}=\frac{2}{3}\left(\frac{\sqrt{(a-1)}\,m_1n_2}{\sqrt{(2a-4)}\,m_2n_1}-1\right),$$

provided $0 < \frac{2}{3}\left(\frac{\sqrt{(a-1)}\, m_1 n_2}{\sqrt{(2a-4)}\, m_2 n_1} - 1 \right) \le 1,$ (3.5)

Also, we have

$$\left(\frac{dp}{dr}\right)_{r=0} = 0, \quad \left(\frac{d\rho}{dr}\right)_{r=0} = 0, \quad \text{and} \quad \left(\frac{d}{dr}\left(\frac{p}{\rho}\right)\right)_{r=0} = 0,$$

$$\left(\frac{dp}{d\rho}\right)_{r=0} = \frac{(2-a)m_2^2}{5am_1^2} - \frac{(2-2a)n_2^2}{5an_1^2},$$

which further demands: $m_1 > 0,\ m_2 > 0,\ n_1 > 0,\ n_2 > 0$

$$0 < \frac{(2-a)m_2^2}{5am_1^2} - \frac{(2-2a)n_2^2}{5an_1^2} < 1, \tag{3.6}$$

The solution is physical valid subject to the given inequality satisfied. Also the involved constants are so restricted that $\left(\frac{d^2 p}{dr^2}\right)_{r=0} < 0, \left(\frac{d^2 \rho}{dr^2}\right)_{r=0} < 0$. So that, the pressure and density may be maximum at the centre.

4. Conclusion and Summary of the Results:

In this paper we could obtain three perfect fluid solutions. All the solutions are new and analyzed physically.

Solution (I), Solution (II) and Solution (III) these solutions satisfy a differential equation of perfect fluid distribution for three different values of parameter 'a', $(i.e.\ a = 1,\ a > 1,\ a < 1)$ present in the equations. All the three solutions satisfy the reality conditions $\rho \ge p > 0$, $p_r < 0,\ \rho_r < 0,\ 0 < \frac{p}{\rho} \le 1$ as well as causality conditions $0 < \frac{dp}{d\rho} < 1$ for some range of constants in the corresponding solutions.

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