IDEAL SEMI REGULAR SPACES

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Abstract: The aim of this paper is to introduce and study new class of spaces called Is-regular spaces. Is-regularity is separation property obtained by using semi open sets. Furthermore, some characterization of Is-regular spaces have been given.

Keywords: Ideal, Is-regular, semi open set.

Introduction: The concept of ideals in a topological space (X, τ) is treated in the classic text by K. Kuratowski [4, 5]. The topic has won its importance by the wealth of research work produced on ideals in topology by various authors since then, notably by Vaidyanathaswamy [11], Newcomb [8], Samules [10], Hamlet and Jankovic [1, 2].

By using semi-open sets introduced by N. Levine [6], Maheshwari and Prasad [7] introduced notion of s-regular spaces. By considering the idea of ideals Renukadevi and Sivaraj [9] introduced I-regular spaces and I-normal spaces and studied some properties of I-normal spaces.

The aim of this paper is to introduce and study new class of spaces called Is-regular spaces. Is-regularity is separation property obtained by using semi open sets. Furthermore, some characterization of Is-regular spaces have been given.

Preliminaries: In this section, we present the basic definitions of ideal, I-regular, semi-open sets, s-regular found in earlier studies.

Throughout this paper (X, τ) and (Y, σ) are topological spaces with no separation axioms assumed. For a subset A of topological space, cl(A) and int(A) are denoted by closure and interior of A respectively.

Definition 2. 1: A non empty collection I of subsets on a topological space (X, τ) is called a topological ideal [1] if it satisfies following two conditions:

(i) $A \in I$ and $B \subseteq A$ implies $B \in I$. (heredity).

(ii) $A \in I$ and $B \in I$ implies $A \bigcup B \in I$ (finite additivity).

We denote a topological space (X, τ) with an ideal I defined on X by (X, τ, I). If (X, τ, I) is an ideal space, (Y, σ) is a topological space and $f:(X, \tau, I) \to (Y, \sigma)$ is a map, then $f(I) = \{f(I_1): I_1 \in I\}$ is an ideal of Y [8]. If I is ideal of subsets of X and Y is subset of X, then $I_Y = \{Y \bigcap I_1 : I_1 \in I\}$ is an ideal of subsets of Y [8]. If $f:(X, \tau) \to (Y, \sigma, I)$ is an injection then $f^{-1}(I) = \{f^{-1}(B): B \in I\}$ is an ideal on X [8].

Definition 2.2: Let (X, τ) be a topological space. A subset A of X is said to be semi-open set [6] if $A \subseteq cl(\text{int } A)$ and a semi-closed set if $\text{int}(cl\ A) \subseteq A$.

Remark 2. 3: Intersection of semi open set and open set is semi open [6].

Definition 2.4: The intersection of all semi closed sets containing a subset A of a space X is called semi closure [6] of A and is denoted by scl(A). Also scl(A)=A$\bigcup$int(cl(A)).

Definition 2.5: A space X is said to be s-regular [7] if for each closed set F and a point $x \notin F$, there exist disjoint semi open sets U and V such that $x \in U$ and $F \subset V$.

Definition 2.6: An ideal space (X, τ, I) is said to be I-regular [9] if for each closed sets F and a point $p \notin F$, there exist disjoint open sets U and V such that $p \in U$ and $F - V \in I$.

Definition 2.7: A function $f: X \to Y$ is called s-continuous [3] if $f^{-1}(G)$ is open in X for every semi open set G of Y.

Lemma 2.8: [1] For subsets A and B of X, the following assertions are valid:

1. sint(X-A) = X-scl(A)
2. scl(X-A) = X-sint(A)
3. scl(A)$\subset$ cl(A)
4. scl (scl(A))=scl(A)

3. Is-regular spaces

Definition 3.1: An ideal space X is said to be Is-regular if for each closed set F and a point $x \notin F$, there exist disjoint semi open sets U and V such that $x \in U$ and $F - V \in I$.

We have s-regular implies Is-regular, but converse is not true. For converse we have following example.

Example 3.1: Let X= {a, b, c, d} with topology $\tau = \{\phi$, X, {c}, {a, b}, {a, b, c}} and Ideal I= { ϕ, {a}, {d}, {a, d}}. Then (X, τ, I) is Is-regular but not s-regular.

The following theorem gives characterizations of Is-regular spaces.

Theorem 3.1:

Let (X, τ, I) be an ideal space. Then the following are equivalent.

(a) X is Is-regular.
(b) For each $x \in X$ and open set U containing x, there is a semi-open set V containing x such that scl(V)-U$\in$I.
(c) For each $x \in X$ and closed set A not containing x, there is a semi-open set V containing x such that scl(V)$\bigcap$A$\in$I.

Proof: (a)$\Rightarrow$(b) Let $x \in X$ be arbitrary and U be open set containing x. Since X is Is-regular, there exist disjoint semi open sets V and W such that $x \in V$ and $(X-U)-W \in I$. Let $(X-U)-W = I_1 \in I$, then $(X-U) \subset W \bigcup I_1$.

Now $V \bigcap W = \phi$, implies that V$\subset$(X-W) and scl(V) $\subset$ scl(X-W) = X-W. Hence scl(V)-U $\subset$ (X-W)$\bigcap$(W $\bigcup I_1$) = (X-W)$\bigcap I_1$. Here (X-W)$\bigcap I_1 \subset I_1 \in$I. Therefore scl(V)-U$\in$ I.

(b)$\Rightarrow$(c) Let A be any closed set in X such that $x \notin A$. Then by (b), there exist a semi open set V containing x such that scl(V)-(X-A)$\in$I,which implies scl(V)$\bigcap$A$\in$I. Therefore (c) holds.

(c)$\Rightarrow$(a) Let A be any closed set in X such that $x \notin A$. Then, there exist a semi open set V containing x such that scl(V)$\bigcap$A$\in$I. Therefore A-(X-scl(V))$\in$I. Here V and (X-scl(V)) are disjoint semi open sets such that $x \in V$ and A-(X-scl(V))$\in$I. Hence X is Is-regular.

Theorem 3.2:
Let (X,τ,I) be Is-regular space and Y be open subset of X, then Y is $I_Y s$ -regular.

Proof: Let (X,τ,I) be Is-regular space and Y be open subset of X. Let $x \in Y$ and F be a closed subset of Y such that $x \notin F$. Then there is a closed set A in X with F=Y$\bigcap$A and $x \notin A$. Since X is Is-regular, there exist disjoint semi open sets G and H such that $x \in G$ and $A-H \in I$. Here Y$\bigcap$G and Y$\bigcap$H are semi open sets in Y by remark 2.3. Also $x \in G$ and $x \in Y$ which implies $x \in Y \bigcap G$ and A-H $= I_1 \in I$ implies $A \subseteq I_1 \bigcup H$. By this we have $Y \bigcap A \subseteq Y \bigcap (I_1 \bigcup H) = (Y \bigcap I_1) \bigcup (Y \bigcap H)$ which implies $F-(Y \bigcap H) \in I_Y$. Here $x \in Y \bigcap G$ and $F-(Y \bigcap H) \in I_Y$, also (Y$\bigcap$G)$\bigcap$(Y$\bigcap$H)= ϕ. Hence Y is $I_Y s$ regular.

Theorem 3.3:
Let (X,τ,I) be an ideal topological space. Let X be Is-regular and p, q be two distinct points in X. Then either scl({p})=scl({q}) or scl({p}) $\bigcap$scl({q})$\in$ I.

Proof: Let p$\in$scl({q}) and q$\in$scl({p}). Then scl({p}) $\subseteq$ scl(scl({q}))= scl({q}) and scl({q}) $\subseteq$ scl(scl({p}))= scl({p}). So by this we have scl({p})=scl({q}). Now assume that q$\notin$scl({p}). Given (X,τ,I) is Is-regular, by theorem 3.1 (c), there exists a semi open subset V containing q such that scl(V)$\bigcap$scl({p})$\in$I. Also q$\in$V, scl({p})$\bigcap$scl({q}) = scl(V)$\bigcap$scl({p})$\in$I. This implies scl({p})$\bigcap$ scl({q})$\in$I.

Theorem 3.4:
If $f:(X,\tau)\rightarrow(Y,\sigma,I)$ is a s-continuous closed injection and Y is Is-regular, then X is $f^{-1}(I)$-regular.

Proof: Let $x \in X$ and F be closed subset of X such that $x \notin F$, since f is closed injection, f(x) and f(F) are disjoint in Y. Since Y is Is-regular, there exist disjoint semi-open sets U and V such that f(x)$\in$U and f(F)-V$\in$I. Since f is s-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are disjoint open sets in X.
Here $x\in f^{-1}(U)$ and $f^{-1}(f(F)-V)\in f^{-1}(I)$ imply $F-f^{-1}(V)\in f^{-1}(I)$. Therefore, X is $f^{-1}(I)$-regular.

Theorem 3.5:
Let (X,τ,I) be Is-regular space. Then for every nonempty set G and a closed set H in X with G$\bigcap$H= ϕ, there exists disjoint semi open subsets U and V of X such that G$\bigcap$U$\neq\phi$ and H-V $\in$I.

Proof: Given X is Is-regular. Let H be closed in X and G is nonempty set with G$\bigcap$H=ϕ. For $x \in G$, there exists disjoint semi open subsets U and V such that $x \in U$ and H-V $\in$I. We have $x \in G$, so clearly G$\bigcap$U $\neq \phi$.

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