k- SUPER GEOMETRIC MEAN GRAPHS

S.S.SANDHYA, E.EBIN RAJA MERLY, B.SHINY

Abstract: Let f: V(G) → {1,2,…,p+q} be an injective function. For a vertex labeling " f ", the induced edge labeling f*(e=uv) is defined by, f*(e) = $\lceil\sqrt{f(u)f(v)}\rceil$or $\lfloor\sqrt{f(u)f(v)}\rfloor$. Then " f " is called a "Super Geometric mean labeling" if {f(V(G))}∪{f*(e):e∈E(G)}={1,2,…,p+q}. A graph which admits Super Geometric mean labeling is called "Super Geometric mean graph". Let f: V(G) → {1,2,…,p+q+k-1} be an injective function. For a vertex labeling " f ", the induced edge labeling f*(e=uv) is defined by, f*(e) = $\lceil\sqrt{f(u)f(v)}\rceil$or $\lfloor\sqrt{f(u)f(v)}\rfloor$. Then "f" is called a "k-Super Geometric mean labeling" if {f(V(G))}∪{f*(e):e∈E(G)}={k,k+1,k+2,…,p+q+k-1}. A graph which admits k- Super Geometric mean labeling is called "k- Super Geometric mean graph".
In this paper we prove that "k- Super Geometric mean labeling" behavior for some standard graphs.

Key words: Super Geometric mean graph, k-super Geometric mean graph.

1. Introduction: All graphs in this paper are finite, simple and undirected graph G = (V,E) with p vertices and q edges. For a detailed survey of graph labeling we refer to Gallian[1]. For all other standard terminology and notations we follow Harary [2]. The concept of "Geometric mean labeling" has been introduced by S. Somasundaram, R. Ponraj and P.Vidhyarani in [4]. S.S.Sandhya, E. Ebin Raja Merly and B. Shiny introduced "Super Geometric mean labeling" in [5].

In this paper, we investigate some standard graphs are k-Super Geometric mean graphs.

Now we will provide a brief summary of definitions and other informations which are useful for our present investigation.

Definition: 1.1: A graph G=(V,E) with p vertices and q edges is called a "Geometric mean graph" if it is possible to label the vertices x∈V with distinct labels f(x) from 1,2,…,q+1 in such a way that when each edge e=uv is labeled with f(e=uv) = $\lceil\sqrt{f(u)f(v)}\rceil$ or $\lfloor\sqrt{f(u)f(v)}\rfloor$ then the edge labels are distinct. In this case, " f " is called a "Geometric mean labeling" of G.

Definition : 1.2: Let f : V(G) →{1,2,…,p+q} be an injective function. For a vertex labeling " f ", the induced edge labeling f*(e=uv) is defined by, f*(e) = $\lceil\sqrt{f(u)f(v)}\rceil$ or $\lfloor\sqrt{f(u)f(v)}\rfloor$. Then " f " is called a "Super Geometric mean labeling" if {f(V(G))}∪{f*(e):e∈E(G)}={1,2,…,p+q}.A graph which admits Super Geometric mean labeling is called "Super Geometric mean graph".

Definition: 1.3: Let f: V(G) → {1,2,…,p+q+k-1} be an injective function. For a vertex labeling " f ", the induced edge labeling f*(e=uv) is defined by, f*(e) = $\lceil\sqrt{f(u)f(v)}\rceil$or $\lfloor\sqrt{f(u)f(v)}\rfloor$. Then " f " is called a "k-Super Geometric mean labeling" if {f(V(G))}∪{f*(e):e∈E(G)}={k,k+1,k+2,…,p+q+k-1}. A graph which admits k- Super Geometric mean labeling is called "k- Super Geometric mean graph".

Definition : 1.4: A Path P_n is a walk in which all the vertices are distinct.

Definition :1.5: A graph obtained by joining a single pendant edge to each vertex of a path is called a Comb(P_nAK_1).

Definition :1.6: The Ladder L_n, n≥2 is the product graph $P_n x P_2$ and contains 2n vertices and 3n-2 edges.

Theorem 1.7[5]: Path, Comb, Ladder and ($P_nAK_{1,2}$) are Super Geometric mean graphs.

2. Main Results

Theorem: 2.1: Any Path is a k- super Geometric mean graph.

Proof:

Let $P_n = v_1v_2…v_n$ be a path,

Define a function f: V(P_n) →{1,2,3,…, p+q+k-1} by,

f(v_i) = 2i-1+k, 1≤i≤n.

Edges are labeled with,

f(v_iv_{i+1}) = 2i-1+k, 1≤i≤n-1

Then we get distinct edge labels.

Hence any path is a k- Super Geometric mean graph.

Example : 2.2: 10 – Super Geometric mean labeling of P_6 is displayed below.

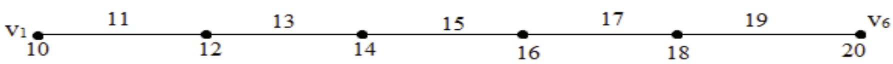

Figure: 1

Theorem : 2.3

Combs (P_nAK_1) are k – super Geometric mean graphs.

Proof:

Let G be a graph obtained from a path $P_n = v_1v_2…v_n$ by joining a vertex u_i to v_i, 1≤i≤n.

Define a function f: V(G) →{1,2,3,..., p+q+k-1} by,
$f(v_i) = 4i-2+k$, $1 \le i \le n$
$f(u_i) = 4i-4+k$, $1 \le i \le n$
Edge labels are given by,
$f(v_i v_{i+1}) = 4i-1+k$, $1 \le i \le n-1$
$f(u_i v_i) = 4i-3+k$, $1 \le i \le n$
Thus both vertices and edges together get distinct labels from
{k, k+1, k+2, ..., p+q+k-1}.
Hence G is a k- super Geometric mean graph.

Example : 2.4

8- Super Geometric mean labeling of P_4AK_1 is given below.

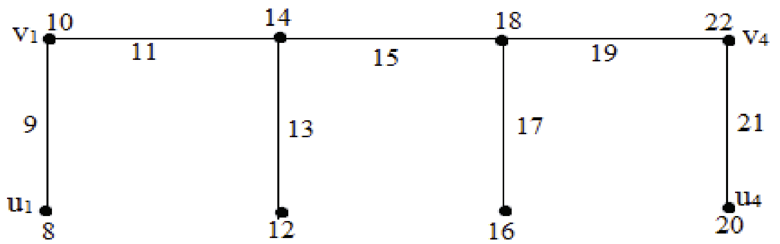

Figure: 2

Theorem: 2.5: A graph obtained by attaching $K_{1,2}$ at each pendant vertex of a Comb is a k- super Geometric mean graph.

Proof:

Let G_1 be a comb and G be the graph obtained by attaching $K_{1,2}$ at each pendant vertex of G_1.
Let its vertices be u_i, v_i, v_{i1}, v_{i2}, $1 \le i \le n$.
The graph $G = (P_nAK_1)AK_{1,2}$ given below.

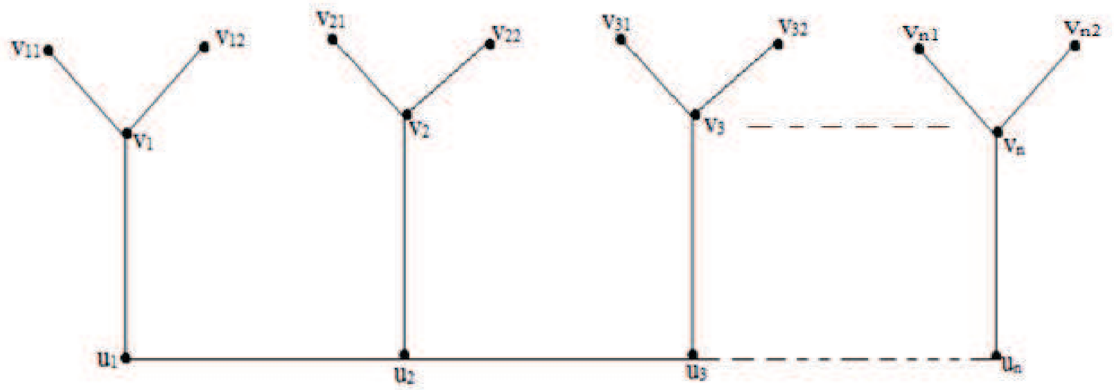

Figure: 3

Define a function f: V(G)→{1,2,3,...,p+q+k-1} by,
$f(v_{11}) = k$
$f(v_{i1}) = 8i-9+k$, $2 \le i \le n$
$f(v_{12}) = k+2$
$f(v_{i2}) = 8i-7+k$, $2 \le i \le n$
$f(v_1) = k+4$
$f(v_i) = 8i-2+k$, $2 \le i \le n$
$f(u_1) = k+6$
$f(u_i) = 8i-4+k$, $2 \le i \le n$
Edges are labeled with,
$f(u_i u_{i+1}) = 8i+k$, $1 \le i \le n-1$
$f(u_i v_i) = 8i-3+k$, $1 \le i \le n$
$f(v_1 v_{11}) = k+1$
$f(v_i v_{i1}) = 8i-6+k$, $2 \le i \le n$
$f(v_i v_{i2}) = 8i-5+k$, $1 \le i \le n$.
∴The edge labels are distinct.

Hence G admits a k-Super Geometric mean labeling.

Example: 2.6

15-Super Geometric mean labeling of $(P_4AK_{1,2})$ is shown below.

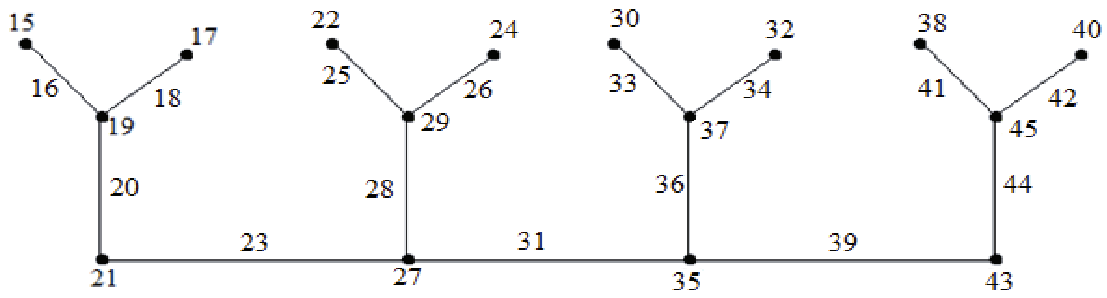

Figure: 4

Theorem : 2.7

Ladders are k-Super Geometric mean graphs.

Proof:

Let $L_n = P_n \times P_2$ be a Ladder.

Define a function f: $V(L_n) \rightarrow \{1,2,3,\ldots,p+q+k-1\}$ by,

$f(v_i) = 5i-5+k,\ 1 \leq i \leq n$

$f(u_1) = k+3$

$f(u_i) = 5i-3+k,\ 2 \leq i \leq n$

Edges are labeled with,

$f(v_1v_2) = k+2$

$f(v_iv_{i+1}) = 5i-2+k,\ 2 \leq i \leq n-1$

$f(u_iu_{+1}) = 5i-1+k,\ 1 \leq i \leq n-1$

$f(u_iv_i) = 5i-4+k,\ 1 \leq i \leq n$

$\therefore$ We get distinct edge labels.

Hence " f " provides a k- super Geometric mean labeling.

Example: 2.8

100 – Super Geometric mean labeling of L_5 is shown below.

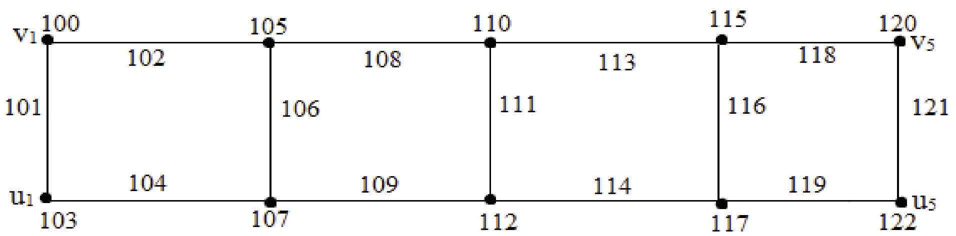

Figure: 5

Theorem: 2.9: Let G be a graph obtained by attaching each vertex of P_n to the central vertex of $K_{1,2}$. Then G is a k-Super Geometric mean graph.

Proof: Let P_n be path $u_1u_2 \ldots u_n$ and v_i, w_i be the vertices of $K_{1,2}$ which are attached with the vertex u_i of P_n.

Define a function f: $V(G) \rightarrow \{1,2,3,\ldots,p+q+k-1\}$ by,

$f(u_i) = 6i-4+k,\ 1 \leq i \leq n$

$f(v_i) = 6i-6+k,\ 1 \leq i \leq n$

$f(w_i) = 6i-2+k,\ 1 \leq i \leq n$

Edge labels are given by,

$f(u_iu_{i+1}) = 6i-1+k,\ 1 \leq i \leq n-1$

$f(u_iv_i) = 6i-5+k,\ 1 \leq i \leq n$

$f(u_iw_i) = 6i-3+k,\ 1 \leq i \leq n.$

From the above labeling pattern, we get

$\{f(V(G))\} \cup \{f^*(e): e \in E(G)\} = \{k, k+1, k+2, \ldots, p+q+k-1\}$.

Hence G is a Super Geometric mean graph.

Example : 2.10

1000- Super Geometric mean labeling of $(P_4AK_{1,2})$ is displayed below.

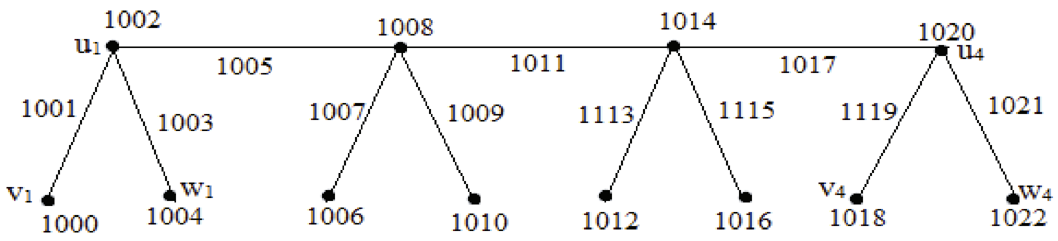

Figure: 6

Theorem : 2.11

Let G be a graph obtained by attaching pendant edges to both sides of each vertex of a path P_n. Then G is a k – Super Geometric mean graph.

Proof:

Consider a graph G which is obtained by attaching pendant edges to both sides of each vertex of a path P_n.

Let u_i, v_i, w_i, $1 \le i \le n$ be the vertices of G.

Define a function f: V(G) → {1,2,3,…,p+q+k-1} by,

$f(u_i) = 6i-2+k$, $1 \le i \le n$

$f(v_i) = 6i-4+k$, $1 \le i \le n$

$f(w_1) = k$

$f(w_i) = 6i-7+k$, $2 \le i \le n$

Edges are labeled with,

$f(u_iu_{i+1}) = 6i+k$, $1 \le i \le n$

$f(u_iv_i) = 6i-3+k$, $1 \le i \le n$

$f(u_iw_i) = 6i-5+k$, $1 \le i \le n$

∴ We get distinct edge labels.

Hence " f " provides a k- Super Geometric mean labeling.

Example: 2.12

27 – Super Geometric mean labeling of G when n=4 is displayed below.

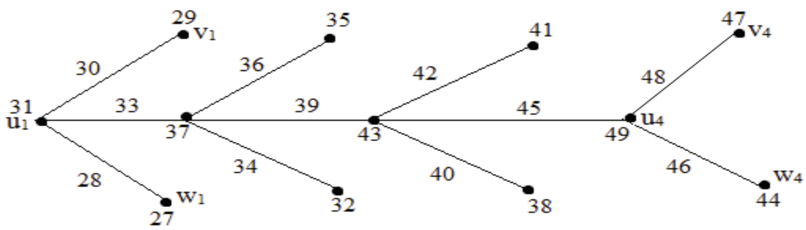

Figure: 7

Theorem: 2.13

Let G = P_nAC_3 be a graph obtained by attaching C_3 to each vertex of a path P_n. Then G is a k- super Geometric mean graph.

Proof:

Consider a graph G which is obtained by attaching C_3 to each vertex of a path P_n.

Let P_n be a path $u_1u_2 \ldots u_n$.

Let u_i, v_i, w_i, $1 \le i \le n$ be the vertices of C_3.

Define a function f: V(G) →{1,2,3,…,p+q+k-1}

$f(u_i) = 7i-2+k$, $1 \le i \le n$

$f(v_1) = k$

$f(v_i) = 7i-8+k$, $2 \le i \le n$

$f(w_i) = 7i-4+k$, $1 \le i \le n$

Edge labels are given by,

$f(u_iu_{i+1}) = 7i+1+k$, $1 \le i \le n-1$

$f(v_iu_i) = 7i-5+k$, $1 \le i \le n$

$f(w_iu_i) = 7i-3+k$, $1 \le i \le n$

$f(v_1w_1) = k+1$

$f(v_iw_i) = 7i-7+k$, $2 \le i \le n$.

In view of above labeling pattern, both vertices and edges together get distinct lables from {k, k+1, k+2,..., p+q+k-1}.
Hence G is a k-Super Geometric mean graph.

Example: 2.14
19- Super Geometric mean labeling of P_4AC_3 is shown below.

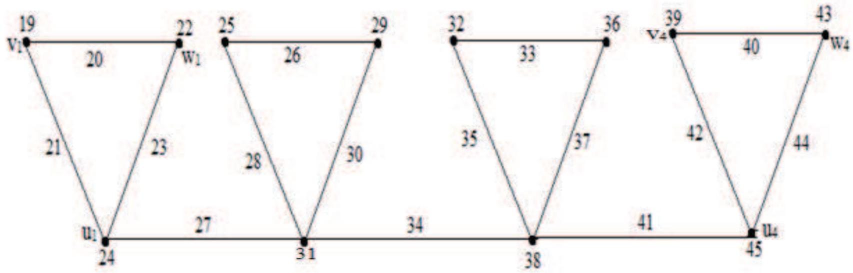

Figure: 8

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