CONVECTIVE HEAT TRANSFER OF A MICRO POLAR FLUID THROUGH A POROUS MEDIUM IN A CYLINDRICAL ANNULUS WITH VISCOUS DISSIPATION

SRINIVASA REDDY B, TULASI LAKSHMI DEVI B, ALFUNSA PRATHIBA

Abstract: In this paper we make an attempt to investigate combined influence of magnetic field and dissipation on connective heat transfer flow of a viscous fluid in the concentric cylindrical annulus with inner cylinder maintained constant temperature on the other cylinder maintained constant heat flux. The equations governing the flow heat and micro rotation are solved by employing Galerkin finite element analysis with quadratic approximation functions. The temperature and micro concentration are analysed for different values. The rate of heat transfer and couple stress is numerically evaluated for different variations of the governing parameters.

Keywords: Heat Transfer, Micro polar Fluid, Porous medium, Viscous Dissipation, FEM.

Introduction: There has been widespread interest in the study of effect of magnetic field on natural convection in fluid saturated cylindrical porous annulus / annuli. Most of the studies found in literature on the effect of magnetic field on natural convection are mainly confined to rectangular enclosures or single cylindrical annulus in the presence and absence of porous median. Shankar and Venkatachalappa [5] have showed that the radial magnetic field is more effective in suppressing the convection in tall cavities, while the axial magnetic field is more effective in suppressing the convection in tall cavities.

B.I. Olajuwon [1] reported that the porosity and the heat are inversely proportional. Also reported that the Nusselt number decreases with increase magnetic field intensity. Ramachandran *et al.*[4] investigated the heat transfer in the stagnation point flow of a micro polar fluid. Vidyanidhi *et al.*[7] examined the dispersion of a chemically non-reacting and chemically reacting solute in a micro polar fluid, for a circular pipe geometry. Soundagekar *et al.* [6] analysed numerically the micro-polar thermo – convection past a wedge, showing that micro-polarity reduces drag (skin-friction) and also heat transfer rates. Viscous dissipation effects in micro-polar thermo fluid dynamics have also attracted some attention. Important studies in this regard were reported by Migun *et al* [3] who studied the Coutts channel convection flow of a micro-polar liquid with viscous energy dissipation. The effects of viscous heating on micro-polar lubrication flows was considered by Khonsari *et al.*[2]who demonstrated that the heat generation due to viscous dissipation exerts a marked effect on the load carrying of a journal bearing lubricated with micro polar fluid.

Many studies are based on the hypothesis that the effect of dissipation is neglected. This is possible in case of ordinary fluid flow like air and water under gravitational force. But this effect is expected to the relevant for fluids with high values of the dynamic viscosity.

Formulation of the Problem: We consider the steady flow of an incompressible, viscous, electrically conducting micro polar fluid through a porous medium in an annulus region between the concentric porous cylinders r = a and r = b (b > a). Under the influence of a radial magnetic field.

The fluid is injected through the inner cylinder with radial velocity u_b and flows outward through the moving cylinder with a radial velocity u_a. We also take the viscous, Darcy and Ohmic dissipation into account.

The velocity and micro rotation are taken in the form

$v_r = u(r)$, $v_\theta = v = 0$, $v_z = w(r)$ and $N_r = 0$,

$N_\theta = N(r)$, $N_z = 0$ 2.1

The equations governing the flow and heat transfer are:

$$\frac{\partial u}{\partial r}+\frac{u}{r}=0 \quad \rho u\frac{\partial u}{\partial r}=-\frac{\partial p}{\partial r}+(\mu+k)\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right)-\left(\frac{\mu}{k_1}\right)u \tag{2.2}$$

$$\rho u\frac{\partial w}{\partial r}=-\frac{\partial p}{\partial z}+(\mu+k)\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)-\rho\bar{g}-\left(\frac{\mu}{k_1}\right)w-k\frac{\partial w}{\partial r}(rN)-\frac{\sigma\mu_e^2 H_0^2}{r^2}w \tag{2.3}$$

$$\rho j u \frac{\partial N}{\partial r} = r\left(\frac{\partial^2 N}{\partial r^2} + \frac{1}{r}\frac{\partial N}{\partial r} - \frac{N}{r^2}\right) - k\frac{\partial w}{\partial r} - 2kN \qquad 2.4$$

$$\rho_0 C_p w \frac{\partial T}{\partial z} = k_f\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + (2\mu + k)\left[\left(\frac{\partial u}{\partial r}\right)^2 + \frac{\mu^2}{r^2} + \frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2\right] + 2k\left(\frac{1}{2}\frac{\partial w}{\partial r} + N\right)^2 - 2\frac{\beta}{r}N\frac{\partial N}{\partial r} + r\left(\left(\frac{\partial N}{\partial r}\right)^2 + \frac{N^2}{r^2}\right) \qquad 2.5$$

where u, w are the velocity components along o(r, z) directions, T is the temperature, N is the micro rotation, p is the pressure, ρ is the density, μ is the dynamic viscosity, c_p is the specific heat at constant pressure, k_f is the thermal conductivity, k_1 is the permeability of the porous permeability, σ is the electrical conductivity μ is the magnetic permeability and k, r, β are the material constants.

The boundary conditions are

at $u = u_b$, $w = 0$, $N = 0$, $T = T_o + A_o z$ on $r = a$

at $u = u_a$, $w = 0$, $N = 0$, $T = T_1 + A_o z$ on $r = b$ 2.6

From the equation of continuity we obtain

$r\,u = c$, constant $\Rightarrow r\,u = a\,u_a = b\,u_b$

In view of the boundary condition on temperature, we may write

$T = T_o + A_o(z) + \theta(r)$

On introducing the non-dimensional variables r′, w′, θ′, p′ and N′ as

$$r' = \frac{r}{a},\ w' = \frac{w}{\left(\frac{v}{a}\right)},\ \theta = \frac{T - T_0}{T_i - T_0},\ N' = \frac{(\mu + k)N}{\rho\left(\frac{u^2}{a^2}\right)},\ p' = \frac{p}{\rho\left(\frac{v^2}{a^2}\right)}$$

The equations (2.2) – (2.4) reduces to (on dropping the dashes).

$$N_{rr} + \left(1 - \frac{\lambda}{1+\Delta}\right)\frac{1}{r}w_r = -\pi_1 + \frac{\mu^2}{r^2}w + D_1^{-1}w - G_1\theta - \frac{\Delta}{r}\frac{\partial}{\partial r}(rN) \qquad 2.7$$

$$w_{rr} + (1 - \lambda A)\frac{1}{r}N_r - \left(\frac{1}{r^2} - \frac{2\Delta}{A}\right)N = \frac{\Delta}{A}\frac{1}{r}\frac{dw}{dr} \qquad 2.8$$

$$\Pr N_T\, w = \theta_{rr} + \frac{1}{r}\theta_r + Ec\Pr\left\{(2+\Delta)\left(\frac{\lambda^2}{r^4} + w_r^2\right) + 2\Delta\left(\frac{1}{2}w_r + N\right)^2 - 2\Delta N\frac{\partial N}{\partial r} + A_1\left(w_r^2 + \frac{N^2}{r^4}\right)\right\} \qquad 2.9$$

Where $G = \frac{\beta g a^3 \Delta T}{\nu^2}$ (Grashof number),

$D^{-1} = \frac{a^2}{k_1}$ (Darcy parameter),

$\Pr = \frac{\mu C_p}{k_f}$ (Prandtl number),

$Ec = \frac{a^2}{C_p \Delta T\, v^2}$ (Eckert number)

$M^2 = \frac{\sigma\mu_e^2 H_0^2 a^2}{v^2}$ (Hartmann number),

$\lambda = \frac{a u_a}{\nu}$ (Suction parameter)

$\Delta = \frac{\mu}{k}$ (Micro polar parameter),

$A = \frac{r}{\mu a^2}$ (Micro polar parameter)

$\Delta_1 = \frac{\Delta}{1+\Delta},\ D_1^{-1} = \frac{D^{-1}}{1+\Delta},\ G_1 = \frac{G}{1+\Delta},\ M_1^2 = \frac{M^2}{1+\Delta},\ r = \frac{b}{a}$ The non-dimensional boundary conditions are

$w = 0$, $\theta = 1$, $N = 0$ on $r = 1$; $w = 0$, $\theta = 0$, $N = 0$ on $r = s$ 2.10

Finite Element Analysis: The finite element analysis with quadratic polynomial approximation functions is carried out along the radial distance across the circular cylindrical annulus. The behavior of the velocity, temperature and concentration profiles has been discussed computationally for different variations in governing parameters. The Gelarkin method has been adopted in the variational formulation in each element to obtain the global coupled matrices for the velocity, temperature and concentration in course of the finite element analysis. Choose an arbitrary element e_k and let w^k, θ^k and N^k be the values of w, θ and N in the element e_k. We define the error residuals as

$$E_w^k = \frac{d}{dr}\left(r\frac{dw^k}{dr}\right) - \frac{\lambda}{1+\Delta}\frac{dw^k}{dr} + \Delta_1(rN^k) + \pi_1 r - \frac{M^2}{r}w^k + D^{-1} r w^k - G r\,\theta^k \qquad 3.1$$

$$E_\theta^k = \frac{d}{dr}\left(r\frac{d\theta^k}{dr}\right) - \Pr N_t w_r + Ec\Pr\begin{pmatrix}(2+\Delta)\dfrac{\lambda^2}{r^3} + (2+\Delta) r\left(\dfrac{dw}{dr}\right)^2 \\ +A\left(\dfrac{dw}{dr}\right)^2 + 2\Delta r\left(\dfrac{1}{2}\dfrac{dw}{dr}+N\right)^2 \\ -2AN\dfrac{dN}{dr} + A_1\dfrac{N^2}{r}\end{pmatrix} \qquad 3.2$$

$$E_N^k = \frac{d}{dr}\left(r\frac{dN^k}{dr}\right) - \lambda A r\frac{dN^k}{dr} - \left(\frac{1}{r} - \frac{2\Delta r}{A}\right)N^k - \frac{\Delta}{A}\frac{dw^k}{dr} \qquad 3.3$$

Where w^k, θ^k and N^k are values of w, θ and N in the arbitrary element e_k. These are expressed as linear combinations in terms of respective local nodal values.

$$w^k = \sum_{i=1}^{3} w_i^k\,\psi_i^k\ ,\ \theta^k = \sum_{i=1}^{3}\theta_i^k\,\psi_i^k\quad N^k = \sum_{i=1}^{3} N_i^k\,\psi_i^k$$

Where ψ_1^k, ψ_2^k,..... etc are Lagrange's quadratic polynomials.

The shape function corresponding to

$$\psi_{k,1} = \frac{\left(r - \frac{2*k-1}{n} * s - 1\right) * \left(r - s * \frac{2*k}{n} - 1\right)}{s * \left(\frac{2*k-2}{n} - \frac{2*k-1}{n}\right) * s * \left(\frac{2*k-2}{n} - \frac{2*k}{n}\right)}$$

$$\psi_{k,2} = \frac{\left(r - s * \frac{2*k-2}{n} - 1\right) * \left(r - s * \frac{2*k}{n} - 1\right)}{s * \left(\frac{2*k-1}{n} - \frac{2*k-2}{n}\right) * s * \left(\frac{2*k-1}{n} - \frac{2*k}{n}\right)}$$

$$\psi_{k,3} = \frac{(r - s*\frac{2*k-1}{n} - 1)*(r - s*\frac{2*k-2}{n} - 1)}{s*(\frac{2*k}{n} - \frac{2*k-2}{n})*(\frac{2*k}{n} - \frac{2*k-1}{n})*s},$$

The rate of heat transfer (Nusselt number) is evaluated using the formula $Nu = -\left(\frac{d\theta}{dr}\right)_{r,S}$

Discussion: We assume Pr = 0.71(water).The velocity distribution (u) is exhibited in figs 1-6 for different values of G, D^{-1}, M, Δ, Ec and S. Fig. 1 illustrates u with Grashof numbers u < 0 is the actual flow and therefore u > 0 represents a reversal flow. It is found that the axial velocity exhibits a reversal flow for G < 0 and the region of reversal flow enlarges with |G| (<0). Also the magnitude of u experiences an enhancement with increase in G with maximum occurring r = 1.4. The variation of u with Darcy parameter D^{-1} and Hartman number M is shown in figs. 2 & 4 respectively. It is found that lesser the permeability of the porous medium or higher the Lorentz force smaller |u| everywhere in the flow region. The intensity of the magnetic field or porous medium thus can be used for decreasing the velocity. Fig. 3 illustrates that an increase in the micro polar parameter Δ leads to a depreciation in |u| in the entire flow field, the maximum occurring at r = 1.5. It is really interesting to observe this behavior of micro polar flow. The variation of u with dissipation is shown in fig. 5. it is observed that |w| depreciates in the flow region with increase in Ec ≤ 0.3 and for higher Ec ≥ 0.5 the magnitude of u enhances continuously in the entire flow region. Thus the inclusion of dissipative heat reduces |w| for smaller values of Ec and enhances for higher Ec. The effect of annular width S on u is shown in fig. 6. It is evident that |u| enhances with increase in the width of the annular region.

The micro rotation (N) is depicted in figs 7-11 for different values of G, D^{-1}, M, Δ and S. It is found that the micro concentration is positive for G > 0 and negative for G < 0(fig.7). The magnitude of N enhances continuously with increase in |G| in the entire flow region with maximum occurring at r = 1.5. The variation of N with D^{-1} shows that lesser the permeability of the porous medium smaller N in the flow region (fig. 8). The variation of N with magnetic parameter M shows that the values of N for M = 2 are positive and for higher M ≥ 5, the values of micro rotation are negative the first half and positive in the second half of the flow region, thus showing reverse rotation near the two boundaries. Also higher the Lorentz force smaller |N| and for further enhancing the Lorentz force larger |N| in the region (fig. 9). Fig. 10 illustrates that the micro rotation continuously decrease with an increase in the micro polar parameter Δ for a fixed values of D^{-1} and M. As expected the depreciation in the micro concentration reduces the angular rotation. The effect of width of the annular region on N is illustrated in fig. 11. An increase in the width S of the annular space results in an enhancement in the micro concentration everywhere in the flow region.

The non-dimensional temperature distribution (θ) is exhibited in figs 12-14 for the different values of Δ, Ec, S. The effect of micro polar parameter Δ on θ is shown in fig. 12. It is noticed that the actual temperature enhances with increase in Δ≤1 and depreciates with higher Δ≥ 3. The effect of dissipation on θ shows that the actual temperature experiences a depreciation everywhere in the flow exact in a narrow region abutting inner cylinder r = 1 for smaller values of Ec ≤ 0.3 but for higher Ec≥ 0.5 we observe an enhancement in θ in the entire region (fig. 13). The variation of θ with S exhibits an increasing tendency in the flow region. Thus the non-dimensional temperature increases with increase in the width of the annular region. in fig.14.

The rate of heat transfer (Nu) at the inner and outer cylinders is exhibited in table for different parametric values. It is forced that an increase in G reduces |Nu| at both the cylinders. The variation of Nu with D^{-1} illustrates that lesser the permeability of the porous medium smaller |Nu| and for further lowering of the permeability larger |Nu| at r = 1 while at r = 2 smaller |Nu|. Also the rate of heat transfer at r = 1 enhances with increase in M ≤ 5 and reduces with higher u ≥ 10 while at r = 2, the Nusselt number reduces with M ≤ 3 and enhances with M ≥ 5. An increase in Ec enhances |Nu| at both r = 1 & 2. With respect to the width of the annular region enhances |Nu| at r = 1 for all G while at r = 2, |Nu| enhances with S ≤ 0.6 and depreciates with higher S ≥ 0.8 for all G. The variation of Nu with micro polar parameter Δ shows that an increase Δ≤ 3 enhances Nu and reduces with higher Δ≥ 5 at the inner cylinder r = 1 whole at the outer cylinder the rate of heat transfer depreciates with Δ for all G (table1). This implies that the magnetic field as well as micro concentration are the important parameters in reducing heating effects and can be used for controlling the rate of heat transfer which is desired in many MHD devices.

G	Ec=0.01, S =0.06 Δ=1	Ec=0.03, S =0.06 Δ=1	Ec=0.05, S =0.04 Δ=1	Ec=0.05, S =0.08 Δ=1	Ec=0.05, S =0.06 Δ=3	Ec=0.05 S =0.06 Δ=5
10^3	-67.602	-74.933	47.690	-19.028	-19.951	-16.063
$3x10^3$	-26.521	-59.215	19.129	-75.501	-79.235	-10.362
-10^3	67.602	74.933	-47.690	19.028	19.951	16.063
$-3x10^3$	26.521	59.215	-19.129	75.501	79.235	10.362

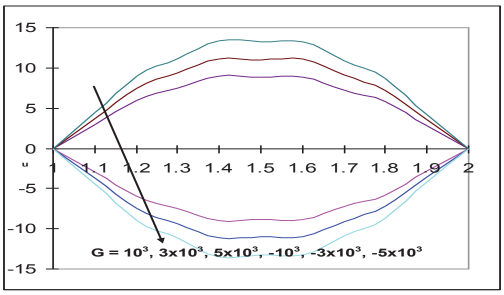

Fig. 1 : Variation of u with G

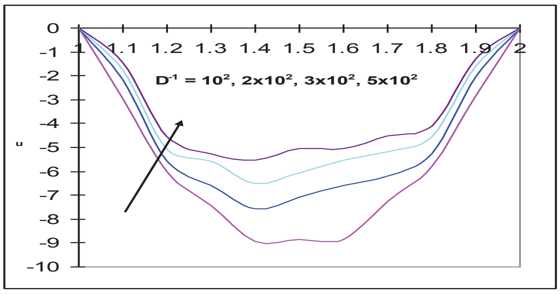

Fig. 2 : Variation of u with D^{-1}

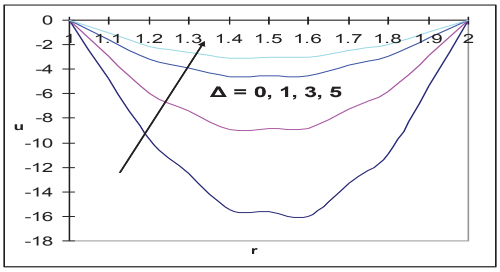

Fig. 3: Variation of u with Δ

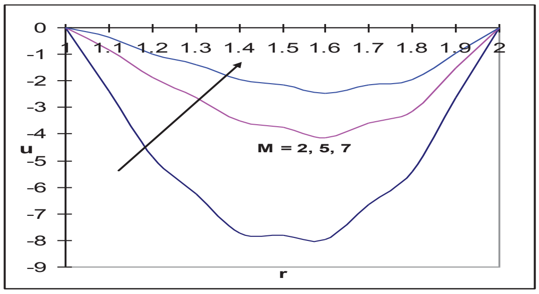

Fig. 4: Variation of u with M

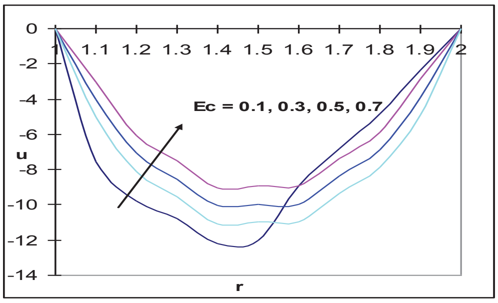

Fig. 5: Variation of u with Ec

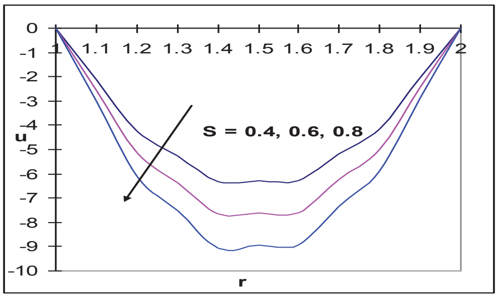

Fig. 6: Variation of u with S

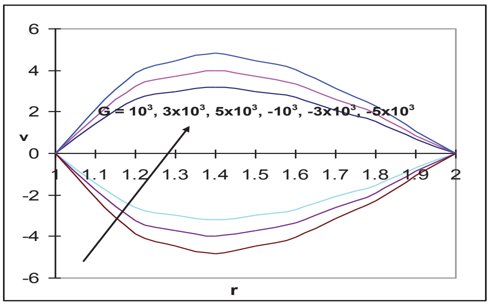

Fig. 7: Variation of Micro rotation (N) with G

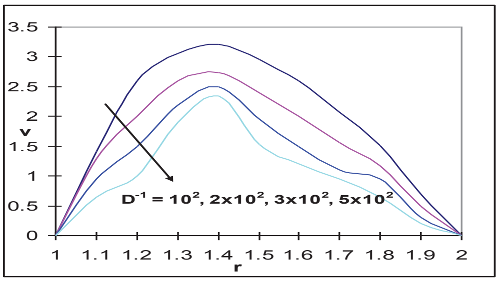

Fig. 8: Variation of Micro rotation (N) with D^{-1}

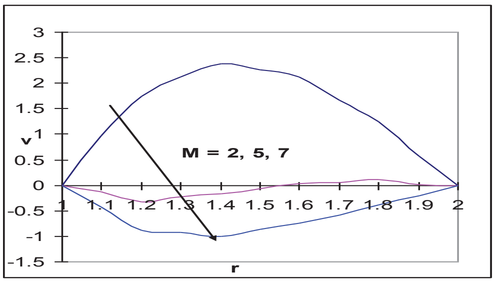

Fig. 9: Variation of Micro rotation (N)with M

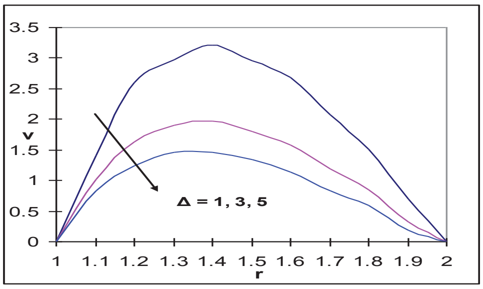

Fig. 10 Variation of Micro rotation (N) with Δ

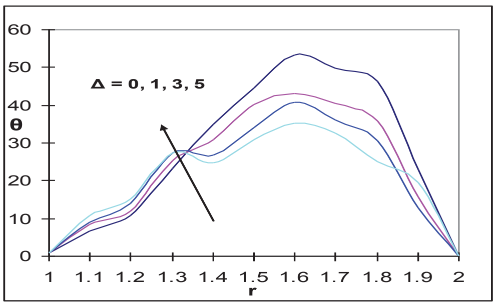

Fig. 11: Variation of Micro rotation (N) with S

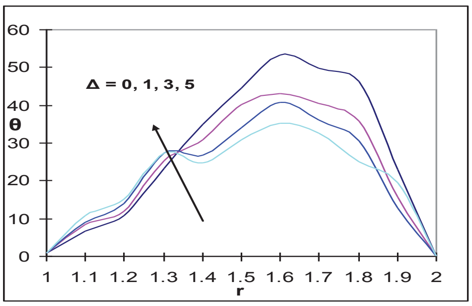

Fig. 12: Variation of θ with Δ

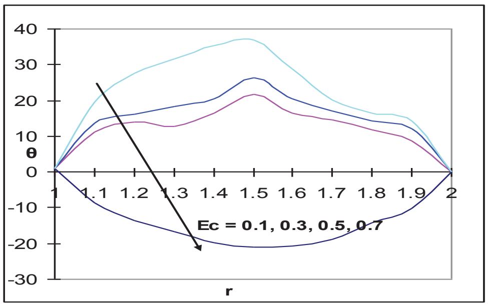

Fig. 13: Variation of θ with Ec

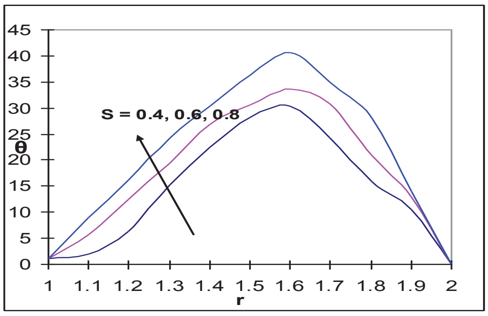

Fig. 14: Variation of θ with S

Conclusion: The effect of dissipation and magnetic field on the fully developed mixed convective flow of a micro-polar fluid in a cylindrical annulus. The coupled equations governing the velocity, temperature and micro-rotation are solved by finite element analysis and iteration procedure. The effect of a dissipative heat, magnetic field, heat source parameter and micro-polar parameter on the flow characteristics are discussed graphically.

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Srinivasa Reddy B/Assistant Professor/Yogi Vemana University/
Department of Mathematics/Kadapa – 516003/ India.
Tulasi Lakshmi Devi B/ Associate Professor/Guru Nanak Institute of Technology/
Department of Mathematics/Hyderabad – 501506/India.
Alfunsa Prathiba/Assistant Professor/Guru Nanak Institute of Technology/
Department of Mathematics/Hyderabad – 501506/India.