A NOTE ON s-IDEALS OF SEMI NEAR-RINGS

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Abstract: In this paper, an algebraic system Semi near-ring is considered, which is a generalization of both a near-ring and a semiring. A Near-ring is an algebraic system with binary operations of addition and multiplication satisfying all the ring axioms except possibly one of the distributive laws and commutativity of addition. A semiring is an algebraic system which is closed and associative under two operations, usual addition, multiplication, and satisfies both distributive laws.
A Semi near-ring S is an algebraic system with two binary operations: usual addition and usual multiplication such that S forms a semigroup with respect to both the operations, and satisfies the right distributive law. In this paper, Normal fuzzy s-ideals of Semi near-rings are considered and some related results are proved. We denote throughout S for a semi near-ring.

Keywords: Semi near-ring, Normal fuzzy s-ideal

Definition 1.1 : A subset I of a seminearring Sis a right (respectively, left) s-ideal if
(i) $x + y \in I$, for all $x, y \in I$.
(ii) $xr \in I$ (right s-ideal), $rx \in I$ (left s-ideal) for all $x, y \in I$ and $r \in S$.

Definition 1.2 : A non-empty fuzzy subset μ (that is, $\mu(x) \neq 0$ for some $x \in S$) of seminearring S
is called a fuzzy s-ideal if it satisfies
(i) $\mu(x + y) \geq \min\{(\mu(x), \mu(y)\}$
(ii) $\mu(xy) \geq \max\{\mu(x), \mu(y)\}$

Definition 1.3.A fuzzy left (resp. right) s-ideal $\mu$of a Semi near-ring S is said to be normal if $\mu(0) = 1$.

Theorem 1.4 : For any s-ideal I of S, the characteristic function λ_I of I is a normal fuzzy s-ideal of
S and $S_{\lambda_I} = I$.

Proof : Suppose I is an s-ideal of S. Define λ_I : $S \to [0, 1]$ as $\lambda_I = \begin{cases} 1 \text{ if} & x \in I \\ 0 & \text{otherwise} \end{cases}$

It is clear that λ_I is an s-ideal of S.
Since $0 \in I$, $\lambda_I(0) = 1$. Therefore, λ_I is normal
Further,

$$S_{\lambda_I} = \{x \in S \mid \lambda_I(x) = \lambda_I(0)\}.$$

Now we will prove that $S_{\lambda_I} = I$.

Let $x \in I$. Then $\lambda_I(x) = 1 = \lambda_I(0)$ (since λ_I is normal).
Therefore, $x \in S_{\lambda_I}$ and hence $I \subseteq S_{\lambda_I}$.

On the other hand,

$$x \in S_{\lambda_I} \Rightarrow \lambda_I(x) = \lambda_I(0) = 1.$$

This implies $x \in I$. Therefore,
$S_{\lambda_I} \subseteq I$. Thus $S_{\lambda_I} = I$.

Lemma 1.5 : Let μ and σ be fuzzy s-ideals of S. If $\mu \subseteq \sigma$ and $\mu(0) = \sigma(0)$ then $S_\mu \subseteq S_\sigma$.

Proof : Let μ and σ be fuzzy s-ideals of S.
Suppose $\mu \subseteq \sigma$ and $\mu(0) = \sigma(0)$.
Let $x \in S_\mu$. Then $\mu(x) = \mu(0)$ (by definition)
Now $\sigma(x) \geq \mu(x)$
$= \mu(0)$ (since $\mu(x) = \mu(0)$)
$= \sigma(0)$ (by supposition).
Also $\sigma(x) \leq \sigma(0)$ (since σ is a fuzzy s-ideal of S) for all $x \in S$.
Therefore, $\sigma(x) = \sigma(0)$ and hence $x \in S_\sigma$.

Lemma 1.6 : If μ and σ are normal fuzzy s-ideals of S and $\mu \subseteq \sigma$, then $S_\mu \subseteq S_\sigma$.

Proof : Take $x \in S_\mu$. This implies $\mu(x) = \mu(0)$ (by definition).
Now $\sigma(x) \geq \mu(x)$
$= \mu(0)$ (since $\mu(x) = \mu(0)$)
$= 1$ (since μ is normal)
$= \sigma(0)$ (since σ is normal)
Also ,$\sigma(x) \leq \sigma(0)$ for all $x \in S$.
Therefore, $\sigma(x) = \sigma(0)$ and hence $x \in S_\sigma$. Thus $S_\mu \subseteq S_\sigma$.

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