QUASI α * OPEN AND QUASI α * CLOSED FUNCTIONS IN TOPOLOGICAL SPACES

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Abstract: The aim of this paper is to define a new class of functions namely quasi α * open functions and quasi α * closed functions and investigate some of its fundamental properties and its characterizations

Keywords: quasi α * open functions , quasi α * closed functions , α * -open , α * - closed

Introduction: Functions and of course open functions stand among the most important notions in the whole of mathematical science. Many different forms of open functions have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences.

Recently, S.Pious Missier and P.Anbarasi Rodrigo [9] have introduced the concept of ￼￼* -open sets and studied their properties. In this paper, we introduce quasi α * open functions and quasi α * closed functions and discuss some of its properties.

Preliminaries: Throughout this paper (X, τ), (Y, σ) and (Z, η) or X , Y , Z represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure and the interior of A respectively. The power set of X is denoted by P(X).

Definition 2.1: A subset A of a topological space X is said to be a α ***open** [1] if A $\subseteq$ int* (cl (int* (A))).

Definition 2.2: A be a subset of a topological space X. Then α * interior [1] of A is defined as the union of all α * - open subsets of A.

Definition 2.3: Let A be a subset of a space X. Then α *closure [1] of A is defined as the intersection of all α * - closed sets in X containing A.

Definition 2.4: A map f :(X, τ) ⟶(Y, σ) is said to be ***pre*** α ***open** [3] if the image of every α *open set of X is α *open in Y.

Definition 2.5: A map f: (X, τ)⟶ (Y, σ) is called a α * **closed** [2]if image of each closed set in X is α *closed in Y.

Definition 2.6: A function f: (X, τ) ⟶ (Y, σ) is called a α * continuous[4] if $f^{-1}(O)$ is a α *open set of (X, τ) for every open set O of (Y, σ).

Definition 2.7: A function f: (X, τ) ⟶(Y, σ) is said to be α *Irresolute [4] if $f^{1}(O)$ is a α *open in (X, τ) for every α *open set O in (Y, σ).

Quasi α * Open Functions

Definition 3.1: A function f: (X, τ) ⟶ (Y, σ) is said to be quasi α * open functionsif the image of every α *open set in X is open in Y.

Example 3.2: Let X = Y= {a, b, c}, τ = {φ, {a} , {ab}, X} and σ = {φ, {a}, {b} , {ab},Y} , α *O(X, τ) = {φ, {a} , {b} , {ab}, {ac} , X } and α *O(Y, σ) = {φ, {a} , {b} , {ab} , {ac} , Y} .Let f: (X, τ) ⟶(Y, σ) be defined by f(a) = f(c) = a, f(b) = b. Clearly, f is quasi α * open.

Theorem 3.3: A function f: (X, τ) ⟶ (Y, σ) is quasi α *open function iff for every subset U of X, f(α *Int (U)) ⊂ Int (f(U)) .

Proof: Let f be a quasi α *open function. We have, Int (U) ⊂ U and α *Int (U) is α *open set. Hence, we obtain that f(α *Int (U)) ⊂ f(U). As f(α *Int (U)) is open, f(α *Int (U)) ⊂ Int (f(U)).

Conversely, assume that U is α *open in X. Then, f(U) = f(α *Int (U)) ⊂ Int (f(U)). But Int (f(U)) ⊂ f(U) . Consequently, f(U) = Int (f(U)). Hence, f is quasi α *open function.

Theorem 3.4: If a function f: (X, τ) ⟶ (Y, σ) is quasi α *open function, then α *Int ($f^{-1}(G)$) ⊂ f^{-1}(Int (G)) for every subset G of Y.

Proof: Let G be any arbitrary subset of Y. Then α *Int ($f^{-1}(G)$) is a α *open in X and f is quasi α *open function, then f(α *Int ($f^{-1}(G)$)) ⊂ Int f ($f^{-1}(G)$) ⊂ Int (G). Thus, α *Int ($f^{-1}(G)$) ⊂ f^{-1}(Int (G)).

Definition 3.5: A subset S is called a α *neighbourhood of a point x of X if there exists a α *open set U such that x ∈ U ⊂ S.

Theorem 3.6: For a function f: (X, τ) ⟶ (Y, σ), the following are equivalent.

(i) f is quasi α *open function

(ii) for each subset U of X, f(α *Int (U)) ⊂ Int (f(U))

(iii) for each x ∈ X and each α *neighbourhood U of x in X, there exists a neighbourhood V of f(x) in Y such that V ⊂ f(U).

Proof:

(i) ⟹ (ii) It follows from the theorem 3.3

(ii) ⟹ (iii) Let x ∈ X and U be an arbitrary α *neighbourhood of x in X. Then there exists a α *open V in X such that x ∈ V ⊂ U. Then by (ii) we have f(V) = f(α *Int (V)) ⊂ Int (f(V)) and hence f(V) = Int f(V). Therefore, it follows that f(V) is open in Y such that f(x) ∈ f(V) ⊂ f(U)

(ii) ⟹ (iii) Let U be an arbitrary α *open set in X. Then for each y ∈ f(U), by (iii) there exists a neighbourhood V_y of y in Y such that V_y ⊂ f(U). As V_y is a neighbourhood of y , there exists an open set W_y in Y such that y ∈ W_y ⊂ V_y. Thus, f(U) = ∪ { W_y: y ∈ f(U) } which is a open set in Y which implies , f is a quasi α *open function.

Theorem 3.7: A function f: (X, τ) ⟶ (Y, σ) is quasi α *open function iff for any subset B of Y and for any α *closed set F of X containing $f^{-1}(B)$, there exists a closed set G of Y containing B such that $f^{-1}(G) \subset F$

Proof : Suppose f is quasi α *open function. Let B ⊂ Y and F be a α *closed set of X containing $f^{-1}(B)$. Put G = Y – (X – F). It is clear that $f^{-1}(B) \subset F$ implies B ⊂ G. Since, f is quasi α *open. We obtain, G as a closed set of Y. Moreover, we have $f^{-1}(G) \subset F$.

Conversely, let U be an α *open set of X and put B = Y – f(U). Then X – U is a α *closed set in X containing $f^{-1}(B)$. By hypothesis, there exists a closed set F of Y such that B ⊂ F and $f^{-1}(F) \subset (X - U)$. Hence, we obtain f(U) ⊂ (Y – F). On the other hand, it follows that B ⊂ F , Y - F ⊂ Y – B = f(U). Thus, we obtain f(U) = Y – F which is open and hence, f is quasi α *open function.

Theorem 3.8: A function f: (X, τ) ⟶ (Y, σ) is quasi α *open function iff $f^{-1}(cl (B)) \subset \alpha$ *cl $(f^{-1}(B))$ for every subset B of Y.

Proof: Suppose that f is quasi α *open. For any subset B of Y , $f^{-1}(B) \subset \alpha$ *cl $(f^{-1}(B))$. Therefore, by thm (3.7) there exists a closed set F in Y such that B ⊂ F and $f^{-1}(F) \subset \alpha$ *cl $(f^{-1}(B))$. Therefore, we obtain $f^{-1}(cl (B)) \subset f^{-1}(F) \subset \alpha$ *cl $(f^{-1}(B))$.

Conversely, let B ⊂ Y and F be α *closed set of X containing $f^{-1}(B)$. Put W = $cl_Y(B)$, then we have B ⊂ W and W is a closed set and $f^{-1}(W) \subset \alpha$ *cl $(f^{-1}(B))$ ⊂ F. Then, by thm (3.7) , f is quasi α *open.

Theorem 3.9: Let f: X ⟶ Y and g: Y ⟶ Z be two functions and g∘f : X ⟶ Z is a quasi α *open. If g is continuous injective, then f is quasi α *open.

Proof: Let U be a α *open set in X, then g∘f (U) is open in Z, since (g∘f) is quasi α *open. Again g is injective continuous function, f(U) = g^{-1}(g∘f (U)) is open in Y. This shows that f is quasi α *open.

Theorem 3.10: If f: X ⟶ Y and g: Y ⟶ Z are two quasi α *open functions, then g∘f: X ⟶ Z is quasi α *open.

Proof: Let F be any α *open set in X. Since, f is quasi α *open function, f(F) is open in Y. We know that every open set is α *open, f(F) is α *open in Y. Since, again g is quasi α *open , g(f(F)) is open in Z, that is g∘f (F) is open in Z and g∘f is quasi α *open.

Theorem 3.11: Let f: X ⟶ Y and g: Y ⟶ Z be any two function. Then

(i) If f is α *open and g is quasi α *open function, then g∘f is open function.

(ii) If f is quasi α *open and g is α *open, then g∘f is pre α *open function.

(iii) If f is pre α *open function and g is quasi α *open , then g∘f is quasi α *open.

Proof:

(i) Let F be any open set in X. Since, f is α *open function, f(F) is a α *open in Y. Since, g is a quasi α *open, g(f(F)) is open set in Z. That is , g∘f (F) = g(f(F)) is open in Z and hence g∘f is open function.

(ii) Let F be any α *open set in X. Since, f is quasi α *open, f(F) is open in Y. Since, g is α *open function, g(f(F)) is α *open in Z. That is g∘f (F) = g(f(F)) is α *open in Z and hence g∘f is pre α *open.

(iii) Let F be any α *open in X. Since, f is pre α *open, f (F) is a α *open in Y. Since, g is quasi α *open, g(f(F)) is open set in Z. That is , g∘f (F) = g(f(F)) is open in Z and hence g∘f is quasi α *open function.

Quasi α *closed function

Definition 4.1: A function f: (X, τ) ⟶ (Y, σ) is said to be quasi α * closed functionsif the image of every α *closed set in X is closed in Y.

Example 4.2: Let X = Y= {a, b, c, d}, τ^c = {φ, {d},{ad},{cd},{acd},{bcd}, X} and σ^c ={ φ , {d} , {ad} , {bd} , {cd} , {abd} , {acd} , {bcd} , Y}, α *C(X, τ) = {φ, {c} , {d} , {ad} , {bd} , {cd} , {abd} , {acd} , {bcd} , X } and α *C(Y, σ) = {φ , {d} , {ad} , {cd} , {bd} , {acd} , {abd} , {bcd} , Y } .Let f: (X, τ) ⟶(Y, σ) be a map defined by f(a) = a, f(b) = b, f(c) = f(d) = d. Clearly , f is quasi α * closed.

Clearly, every quasi α * closed function is closed as well as α *closed

Remark 4.3: Every α * closed functions (resp closed) need not be quasi α * closed as shown by the following examples.

Example 4.4: Let X = Y= {a, b, c, d}, τ^c = {φ, {d},{cd},{bcd}, X} and σ^c = {φ, {d},{cd},{acd},{bcd}Y}, α *C(X, τ) = {φ, {b} , {c} , {d} , {ad} , {bc} , {bd} , {cd} , {abd} , {acd} , {bcd} , X } and α *C(Y, σ) = {φ , {c} , {d} , {ad} , {cd} , {bd} , {acd} , {abd} , {bcd} , Y } .Let f: (X, τ) ⟶(Y, σ) be a map defined by f(a) = a, f(b) = b, f(c) = d , f(d) = c. Clearly , f is α * closed. But for the α * closed set {b} in X, the image f{b} = {b} is not closed in Y. Therefore, f is not quasi α * closed.

Example 4.5: Let X = Y= {a, b, c, d}, τ^c = {φ, {d},{cd},{bcd}, X} and σ^c = {φ, {d},{ad} , {bd} , {cd}, {abd} , {acd} , {bcd} , Y }, α *C(X, τ) = {φ, {b} , {c} , {d} , {ad} , {bc} , {bd} , {cd} , {abd} , {acd} , {bcd} , X } and α *C(Y, σ) = {φ , {d} , {ad} , {cd} , {bd} , {acd} , {abd} , {bcd} , Y } .Let f: (X, τ) ⟶(Y, σ) be a map defined by f(a) = c, f(b) = a, f(c) = b , f(d) = d. Clearly , f is closed. But for the α * closed set {b} in X, the image f{b} = {a} is not closed in Y. Therefore, f is not quasi α * closed.

Theorem 4.6: If a function is quasi α * closed, then $f^{-1}(int(B)) \subset \alpha$ *Int ($f^{-1}(B)$) for every subset B of Y.

Proof: This proof is similar to the proof of theorem 3.4

Theorem 4.7: A function f: (X, τ) ⟶ (Y, σ) is quasi α * closed function iff for any subset B of Y and for any α *open set G of X containing $f^{-1}(B)$, there exists a closed set U of Y containing B such that $f^{-1}(U) \subset G$.

Proof: This proof is similar to the theorem 3.7

Theorem 4.8: If f: X ⟶ Y and g: Y ⟶ Z are two quasi α *closed functions, then $g \circ f$: X ⟶ Z is quasi α *closed.

Proof: Obvious

Theorem 4.9: Let f: X ⟶ Y and g: Y ⟶ Z be any two function. Then

(i) If f is α *closed and g is quasi α *closed function, then $g \circ f$ is closed function.

(ii) If f is quasi α *closed and g is α *closed, then $g \circ f$ is pre α *closed function.

(iii) If f is pre α *closed function and g is quasi α *closed , then $g \circ f$ is quasi α *closed.

Proof: Obvious

Theorem 4.10: Let f: X⟶ Y and g: Y ⟶ Z be any two function such that $g \circ f$ is quasi α *closed. Then

(i) If f is α *irresolute surjective, then g is closed.

(ii) If g is α *continuous injective, then f is pre α *closed function.

Proof:

(i) Suppose that F is an arbitrary closed set in Y. As f is α *irresolute, $f^{-1}(F)$ is α *closed in X. Since, $g \circ f$ is quasi α *closed and f is surjective , $(g \circ f(f^{-1}(F))) = g(F)$, which is closed in Z. This implies that g is a closed function.

(ii) Suppose F is any α *closed in X. Since, $g \circ f$ is quasi α *closed, $g \circ f(F)$ is closed in Z. Again, g is a α *continuous injective, $g^{-1}(g \circ f(F)) = f(F)$, which is α *closed in Y. This shows that f is pre α *closed function.

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