STABILITY ANALYSIS OF A MAXWELL FLUID IN ANISOTROPIC POROUS MEDIUM HEATED FROM BELOW

RUCHI GOEL, SUDHIR KUMAR YADAV

Abstract: Based on a modified-Darcy–Brinkman–Maxwell model, stability analysis of a horizontal layer of Maxwell fluid in anisotropic porous medium heated from below is performed. By solving the eigenvalue problems, the critical Rayleigh number, wave number and frequency for overstability are determined. It is found that the critical Rayleigh number for overstability decreases as the relaxation time increases and the elasticity of a Maxwell fluid has a destabilizing effect on the fluid layer in anisotropic porous media. On the other hand, the critical Rayleigh number for overstability increases by increasing the anisotropic porous parameter which acts to stabilize the system. In limiting cases, some previous results for viscoelastic fluids in nonporous media are recovered from our results.

Keywords: Maxwell fluid, Anisotropic Porous medium, Rivlin-Ericksen fluid.

Introduction: The problem of onset of thermal instability in a horizontal layer of viscous fluid heated from below has its origin in the experimental observation of **Benard** [1]. The theory of this problem was founded by **Lord Rayleigh** [24]. This phenomenon of buoyancy-induced instability for Newtonian viscous fluids has been widely studied as a basic stability problem involving heat transfer and fluid mechanics [**5,7,8,9,12,23**]. An overview of this work was given by **Bejan** [2]. Although the problem of Rayleigh-Benard convection has been extensively investigated for Newtonian fluids, relatively little attention has been devoted to the thermal convection of viscoelastic fluids. The objective of this paper is to extend the Rayleigh-Benard convection to a viscoelastic fluid in a porous medium.

Thermal convection in porous media is a subject of considerable interest in contemporary fluid flow and heat transfer research. Its importance stems from a wide range of occurrences in industrial applications and geological systems. From a purely scientific point of view, porous convection is also of great interest because it is one of the simplest systems exhibiting nonlinear instability. The Rayleigh-Benard instability for a Newtonian fluid in a porous medium was first investigated by **Horton and Rogers** [10] and later by **Lapwood** [20]. They found the critical Rayleigh number to be $4\pi^2$ for onset of convection in an infinitely wide horizontal porous layer. Katto and Masouka showed experimentally the effect of Darcy number on the onset condition of buoyancy-driven convection [14]. Otero et. al. investigated high Rayleigh number convection in a fluid saturated porous layer by using numerical method [13]. Bejan also presented two simplest methods (scale analysis and the intersection of asymptotes) for convection in porous media [3]. The thermal convection of Newtonian fluids in porous media has been widely investigated up to now. Recently, interest in viscoelastic flows through porous media has also grown considerably, due largely to the demands of such diverse areas as biorheology, geophysics, chemical and petroleum industries [**6,15,16,21,22,25,26,33**]. As compared with Newtonian fluid flows in a porous medium, only a few mathematical macroscopic models have been proposed concerning viscoelastic fluid flows in anisotropic porous media. By analogy with the constitutive equation of the Maxwell fluid, the following phenomenological model has been introduced [**15, 16**].

$$\left(1+t_m\frac{\partial}{\partial t}\right)\nabla p=-\frac{\mu}{\left(k_x,k_y,k_z\right)}\mathbf{V} \qquad \dots (1)$$

which is called the modified Darcy-Maxwell model. On the basis of the constitutive equation of the Oldroyd-B fluid, the modified-Darcy-Oldroyd model for describing both relaxation and retardation phenomenon was suggested [**15, 16**].

$$\left(1+t_m\frac{\partial}{\partial t}\right)\nabla p=-\frac{\mu}{\left(k_x,k_y,k_z\right)}\left(1+t_p\frac{\partial}{\partial t}\right)\mathbf{V} \qquad \dots (2)$$

where t_m and t_p are respectively the stress and strain relaxation characteristic time constants, $\left(k_x,k_y,k_z\right)$ is anisotropic effect, μ the effective fluid viscosity in the anisotropic porous medium, p the pressure and **V** the Darcian velocity. When $t_m=t_p=0$, equations (1) and (2) can be simplified to Darcy's law. Furthermore, published work on thermal convection of viscoelastic fluids in porous media is fairly limited. Rudraiah et al. studied the Rayleigh-Benard convection of viscoelastic fluids through porous media using the **Darcy-Brinkman-Jeffrey model** [17]. But there was a shortcoming in their research work that the flow resistance of the viscoelastic fluid in the porous medium was estimated by using Darcy's law. It is well known that

Darcy's law is not valid for the non-Newtonian fluid in porous media. Recently, Kim et al. investigated the thermal instability of viscoelastic fluids in porous media using the modified **Darcy-Oldroyd model** [**29**]. But the modified Darcy-Oldroyd model is independent of shear rate due to the neglect of viscous shear effect. Thus, it cannot entertain the full set of boundary conditions and cannot predict the boundary layer region near the boundaries of the porous layer [**17, 21**].

In the present paper, the convection stability of a Maxwell fluid in anisotropic porous medium heated from below is investigated by using a modified **Darcy-Brinkman-Maxwell model** [**30,31**]. The modified Darcy-Brinkman-Maxwell model has been developed on the base of the local volume averaging technique and the balance of the forces acting on a volume element of viscoelastic fluids in porous media [**11,27,28**]. This model not only overcomes the shortcomings encountered in the modified Darcy-Brinkman-Oldroyd model, but also overcomes the disadvantage encounted in the Darcy-Brinkman-Jeffrey model. Since we are interested in finding the effects of the elastic and porous parameters on the onset of convection, our efforts are mainly based on the linear theory analysis [**29**]. A normal mode analysis method is then used. The critical Rayleigh number, wave number and frequency for overstability are determined. The effects of the porous parameter and the relaxation time on the exchange of stabilities and overstability are also investigated.

Formulation:

For a Maxwell fluid, the constitutive equation of stress is given by [**26**]

$$\boldsymbol{\tau}+t_m\left[\frac{\partial\boldsymbol{\tau}}{\partial t}+\left(\mathbf{V}.\nabla\right)\boldsymbol{\tau}-\left(\nabla\mathbf{V}\right)^{T}\boldsymbol{\tau}-\boldsymbol{\tau}\left(\nabla\mathbf{V}\right)\right]$$

$$=\mu\left[\nabla\mathbf{V}+\left(\nabla\mathbf{V}\right)^{T}\right] \qquad ...(3)$$

where temperature difference $\nabla T=T_1-T_2$,heat capacity $\left(\rho c\right)_s$,constant relaxation time for Maxwell fluid t_m,heat capacity for Maxwell fluid $\left(\rho c\right)_f$,and $\mathbf{V}=\left(u,v,w\right)$ is the volume average velocity obtained by the local volume averaging technique [**11,27,28**], τ the volume average stress tensor. We assume that at quiescent state the temperature varies linearly across the layer thickness. When the magnitude of ΔT becomes larger than the critical one, thermal convection will set in due to buoyancy forces. In the present paper, the fluid is assumed to obey the following equation of state:

$$\rho=\rho_0\left\{1-\alpha\left(T-T_0\right)\right\}, \qquad ...(4)$$

where ρ and ρ_0 are the densities at the temperature T and T_0 respectively, and α is the coefficient of volumetric expansion. If the Boussinesq approximation, which states that the effect of compressibility is negligible everywhere in the conversations except in the buoyancy term, is assumed to hold, then the equations for conservation of mass, momentum and energy read, respectively

$$\nabla.\mathbf{V}=0, \qquad ...(5)$$

$$\rho_0\left(\frac{\partial\mathbf{V}}{\partial t}+\left(\mathbf{V}.\nabla\right)\mathbf{V}\right)=-\nabla p+\nabla.\boldsymbol{\tau}+\mathbf{r}-\vec{k}g\rho, \qquad ...(6)$$

$$\varepsilon\frac{\partial T}{\partial t}+\mathbf{V}.\nabla T=\kappa\nabla^2 T. \qquad ...(7)$$

where $\vec{\mathbf{k}}$ is a unit vector along the z-direction which is vertically upward, **g** the acceleration due to gravity, **r** the flow resistance, i.e., the force offered by the solid matrix of the anisotropic porous medium, $\varepsilon=\left\{\phi\left(\rho c\right)_f+\left(1-\phi\right)\left(\rho c\right)_s\right\}/\left(\rho c\right)_f$ and κ the thermal diffusivity. The assumptions for equations (5) – (7) are that the fluid and the anisotropic porous medium are in local thermodynamic equilibrium, the radiative effects are also negligible, the fluid temperature is below the boiling point and the fluid properties are homogeneous and anisotropic. Since we are only interested in investigating the onset of thermal convection, the velocity is assumed sufficiently small and then the quadratic drag is also negligible in momentum equation [**22**].

Due to the volume averaging process, some information was lost, thus requiring supplementary empirical relation for the flow resistance [**28**]. Since the pressure gradient in equation (1) can be interpreted as a measure of the resistance to Maxwell fluid flow in the bulk of anisotropic porous medium and **r** is also a measure of the flow resistance offered by the solid matrix, thus it can be inferred from equation (1) to satisfy the following equation [**28,30**].

$$\left(1+t_m\frac{\partial}{\partial t}\right)\mathbf{r}=-\frac{\mu}{\left(k_x,k_y,k_z\right)}\mathbf{V} \qquad ...(8)$$

Substituting equation (8) into equation (6), we can find that equation (6) can be simplified in the equation (1) if the inertia and viscous terms are ignored.

In this paper we assume that at quiescent state the temperature varies linearly across the layer thickness. The basic state of the system is quiescent and is described by

$$\mathbf{V}_b=\left(0,0,0\right), \qquad \rho=\rho_b\left(z\right)$$

$$p=p_b\left(z\right), \qquad T=T_b\left(z\right) \qquad ...(9)$$

Substituting equation (9) into equations (3) – (7) yields

$$\frac{dp_b}{dz}+\rho_b g=0, \quad ...(10)$$

$$\rho_b=\rho_0\left\{1-\alpha\left(T_b-T_0\right)\right\}, \quad ...(11)$$

$$T_b=T_1-\frac{\Delta T}{d}z, \quad ...(12)$$

$$\tau_b=0. \quad ...(13)$$

We now superimpose small perturbations on the basic state in the form

$$V=V_b+V', \quad T=T_b+T', \quad \rho=\rho_b+\rho', \quad p=p_b+p', \quad \tau=\tau_b+\tau', \quad ...(14)$$

where primes denote the perturbed quantities relative to those of the basic state indicated by the subscript 'b'. Substituting equation (14) into equations (3) to (7) and eliminating τ' in equation (6) with the help of equation (3), and when all higher order terms of the small quantities are neglected, one has

$$\nabla.\mathbf{V}'=0, \quad ...(15)$$

$$\rho_0\left(1+t_m\frac{\partial}{\partial t}\right)\frac{\partial V'}{\partial t}=-\left(1+t_m\frac{\partial}{\partial t}\right)\left(\nabla p'-\vec{k}g\rho_0\alpha T'\right)+\mu\nabla^2V'-\frac{\mu}{\left(k_x,k_y,k_z\right)}V', \quad ...(16)$$

$$\varepsilon\frac{\partial T'}{\partial t}-\frac{\Delta T}{d}w'=\kappa\nabla^2T' \quad ...(17)$$

Performing the divergence operation on both sides of equation (16), we get

$$\nabla^2p'-g\rho_0\alpha\frac{\partial T'}{\partial z}=0. \quad ...(18)$$

From equation (16), the vertical motion of the fluid is governed by

$$\rho_0\left(1+t_m\frac{\partial}{\partial t}\right)\frac{\partial u'}{\partial t}=-\mu\nabla^2u'+\frac{\mu}{k_x}u'=-\left(1+t_m\frac{\partial}{\partial t}\right)\left(\frac{\partial p'}{\partial x}\right) \quad ...(19)$$

$$\rho_0\left(1+t_m\frac{\partial}{\partial t}\right)\frac{\partial v'}{\partial t}-\mu\nabla^2v'+\frac{\mu}{k_y}v'=-\left(1+t_m\frac{\partial}{\partial t}\right)\left(\frac{\partial p'}{\partial y}\right) \quad ...(20)$$

$$\rho_0\left(1+t_m\frac{\partial}{\partial t}\right)\frac{\partial w'}{\partial t}-\mu\nabla^2w'+\frac{\mu}{k_z}w'=-\left(1+t_m\frac{\partial}{\partial t}\right)\left(\frac{\partial p'}{\partial z}-g\rho\alpha T'\right) \quad ...(21)$$

Eliminating p' in equation (19), (20) and (21)by using equation (18) yields $u'=v'$, $k_x=k_y=k$, k_z as

$$\rho_0\left(1+t_m\frac{\partial}{\partial t}\right)\frac{\partial w'}{\partial t}\nabla^2=\mu\left[\left(\nabla^2-\frac{1}{k}\right)\frac{\partial w'}{\partial z^2}+\left(\nabla^2-\frac{1}{k_z}\right)\nabla_1^2w'\right]+g\rho\alpha\left(1+t_m\frac{\partial}{\partial t}\right)\nabla_1^2T' \quad ...(22)$$

Where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$, $\nabla_1^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$.

$$\frac{\partial T}{\partial t}-w=\nabla^2T \quad ...(24)$$

The problem can be non-dimensional by scaling length with d, time with $\frac{d^2}{\kappa}$ temperature with ΔT, velocities with $\frac{\kappa}{d}$. Keeping the same notation for all the variables (the primes are omitted hereafter for brevity) and assuming $\varepsilon=1$, the dimensionless equations are

$$\frac{1}{Pr}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial w}{\partial t}\nabla^2-\left\{\left(\nabla^2-\frac{d^2}{k}\right)\frac{\partial^2w}{\partial z^2}+\left(\nabla^2-\frac{d^2}{k_z}\right)\nabla_1^2w\right\}$$

$$=R\left(1+\lambda\frac{\partial}{\partial t}\right)\nabla_1^2T$$

It can be written as

$$\frac{1}{Pr}\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial w}{\partial t}\nabla^2-\left\{\left(\nabla^2-\eta\right)\frac{\partial^2w}{\partial z^2}+\left(\nabla^2-\eta_z\right)\nabla_1^2w\right\}$$

$$=R\left(1+\lambda\frac{\partial}{\partial t}\right)\nabla_1^2T \quad ...(23)$$

Where v is the kinematic viscosity and

$Pr=\frac{v}{\kappa}$ (Prandtl number),

$\eta=\frac{d^2}{k},\eta_z=\frac{d^2}{k_z}$ (Anisotropic porous parameter),

$R=\frac{g\alpha d^3\Delta T}{v\kappa}$ (Rayleigh number),

$\lambda=\frac{t_m\kappa}{d^2}$ (Dimensionless relaxation time).

The anisotropic porous parameter η,η_z are related to the Darcy number $Da\left(=k/d^2\right)$, $Da_z\left(=k_z/d^2\right)$ by $Da=1/\eta$, $Da_z=1/\eta_z$ respectively, that is, the Darcy number is the reciprocal of the anisotropic porous parameter η and η_z. λ is the dimensionless stress relaxation time of the Maxwell fluid, which represents the elastic property of a viscoelastic fluid [**29**].

Stability Analysis:

According to the normal mode analysis, convective motion is assumed to exhibit horizontal periodicity [**29**]. Then the perturbed quantities can be expressed as follows

$$\begin{bmatrix} w \\ T \end{bmatrix} = \begin{bmatrix} \psi(z) \\ \theta(z) \end{bmatrix} e^{ilx} e^{imy} e^{\sigma t}, \quad \dots (25)$$

Where $i = \sqrt{-1}$, l and m are the dimensionless wave numbers in the x - y horizontal plane, respectively and σ is the temporal growth rate, which can be rewritten as $\sigma = \sigma_r + i\sigma_i$. For $\rho_r < 0$, the system is always stable. On the other hand, for $\sigma_r > 0$ the system becomes unstable. When $\sigma = 0$, the system is marginally stable under the principle of exchange of stabilities. In particular, when $\sigma_r = 0$ and $\sigma_i \neq 0$, the overstability of periodic motion is possible and oscillatory motion occurs [**18,32**]. The minimum value of the Rayleigh numbers at the marginal condition of $\sigma_r = 0$ is regarded as the critical value to mark the onset of convection.

Substituting equation (25) into equations (24) yields

$$\left[\frac{\sigma}{\Pr}(1+\lambda\sigma)\right](D^2-\beta^2)\psi - \left[\left\{(D^2-\beta^2)-\eta\right\}D^2\psi - \left\{(D^2-\beta^2)-\eta_z\right\}\beta^2\psi\right]$$

$$= -\beta^2 R(1+\lambda\sigma)T \quad \dots(26)$$

$$\sigma T - \psi = (D^2-\beta^2)T \quad \dots(27)$$

$$\psi = -(D^2-\beta^2-\sigma)T$$

Where $D \equiv d/dz$, the differential operator in z-direction $\beta = \sqrt{l^2+m^2}$ is the horizontal wave number. We now solve equations (26) and (27) for eigenvalues under the boundary conditions:

$$\psi = D^2\psi = T = 0 \text{ at } z = 0, 1 \quad \dots(28)$$

Equations (26) and (27) can be combined together by operating $(D^2-\beta^2-\sigma)$ on both sides of equation (26) and then substituting equation (27) into it. The resulting equation is

$$\left[\frac{\sigma}{\Pr}(1+\lambda\sigma)\right](D^2-\beta^2)(D^2-\beta^2-\sigma)$$

$$-\left[\left\{(D^2-\beta^2)-\eta\right\}D^2\psi - \left\{(D^2-\beta^2)-\eta_z\right\}\beta^2\psi\right](D^2-\beta^2-\sigma)$$

$$= \beta^2 R(1+\lambda\sigma)\psi \quad \dots(29)$$

The boundary conditions in equation (28) indicate that the required solution is of the form

$$\psi(z) = A\sin(n\pi z), \quad n = 1, 2, 3, \dots\dots\dots \quad \dots(30)$$

Where A is constant. The critical mode number is n = 1. Substitution of equation (30) into equation (29) gives the characteristic equation for R

$$(A_1 + A_2 - A_3)u_{p,pm} + (A_2 + A_3)u_{m,pp}$$

$$+\frac{\Pr}{\lambda T}\left\{T^3 + (\eta\pi^2 + \eta_z\beta^2)(\eta^2+\beta^2) - \beta^2 R\right\} = 0 \quad \dots(31)$$

Where $\Gamma = \pi^2 + \beta^2$. If $\sigma = 0$, it is easily observed that the exchange of stabilities will occur in the Maxwell fluid in anisotropic porous layer, then

$$\frac{\Pr}{\lambda\Gamma}\left\{\Gamma^3 + (\eta\pi^2 + \eta_z\beta^2)(\pi^2+\beta^2) - \beta^2 R_s\right\} = 0$$

$$R_s = \frac{\Gamma^3}{\beta^2} + \frac{(\eta\pi^2+\eta_z\beta^2)(\pi^2+\beta^2)}{\beta^2} \quad \dots(32)$$

where R_s is the Rayleigh number for the exchange of the stabilities.

The critical wave number β_c can be obtained by minimizing R_s with respect to β. Setting $\partial R_s/\partial\beta$ = 0, β_c can be given by the following equation:

$$6(\pi^2+\beta_c^2) - \frac{2(\pi^2+\beta_c^2)^2}{\beta_c^2} - \frac{2}{\beta^2}(\eta\pi^2+\eta_z\beta_c^2) + 2\left[\eta_z + \frac{(\eta\pi^2+\eta_z\beta_c^2)}{(\pi^2+\beta_c^2)}\right] = 0$$

. ...(33)

The corresponding critical Rayleigh number is

$$R_{sc} = \frac{(\pi^2+\beta_c^2)^3}{\beta_c^2} + \frac{(\eta\pi^2+\eta_z\beta_c^2)(\pi^2+\beta_c^2)}{\beta_c^2} \quad \dots(34)$$

It is not worthy that only the anisotropic porous parameter is relevant to the exchange of stabilities. Equations (34) and (35) are independent of the relaxation time and the Prandtl number. Vest and Rosenbolt also reported that the elasticity of viscoelastic fluid had no effect on the exchange of stabilities in nonporous media [**31,35**]. For the detailed analysis one can refer to their papers. From equation (3) we find that it is also true for viscoelastic fluids in porous media.

In the limiting case when $\eta \to 0, \eta_z \to 0$, i.e. $Da \to \infty, Da_z \to \infty$, the system can be simplified to that of a pure viscoelastic fluid layer in a non-anisotropic porous medium. Equations (33) and (34) are then reduced to

$$\beta_c = \frac{\pi}{\sqrt{2}}, \qquad R_{sc} = \frac{27\pi^4}{4}. \qquad ...(35)$$

These are the exact result previously published for viscoelastic fluids in non-anisotropic porous media.

However, in addition to the exchange of stability, it is of most interest here to investigate overstability, i.e. the condition for a stable state to transit into an unstable state (or vice versa) is not $\sigma = 0$, but $\sigma = i\omega$ with ω a real number. In this case substituting $\sigma = i\omega$ into equation (31) and setting both the real part and the imaginary part equal to zero yields

$$\left(\frac{1}{\lambda}+\Gamma\right)\omega^2 - \frac{\Pr}{\lambda\Gamma}\left[\Gamma^3 + \left(\eta\pi^2 + \eta_z\beta^2\right)\left(\pi^2+\beta^2\right) - \beta^2 R_p\right] = 0, \qquad ...(36)$$

$$\omega^2 - \left[\frac{1}{\lambda}\left\{\left(1+\Pr+\frac{\eta\Pr}{\Gamma}\right)\pi^2 + \left(1+\Pr+\frac{\eta_z\Pr}{\Gamma}\right)\beta^2\right\}\right] + \frac{\Pr R_p}{\Gamma}\beta^2 = 0, \qquad ...(37)$$

Where ω frequency for the overstability, Rp is is the Rayleigh number for the overstability. Clearly, we could now find Rp from equations (36) and (37)

$$Rp = \frac{\Gamma^2}{\lambda\beta^2 \Pr} + \frac{(1+\Pr)\Gamma}{\lambda^2\beta^2\Pr} + \frac{\left(\eta\pi^2+\eta_z\beta^2\right)}{\lambda\Gamma\beta^2}. \qquad ...(38)$$

Substituting equation (38) into (37) yields

$$\omega^2 = \frac{1}{\lambda^2\Gamma}\left[\Gamma\lambda\left\{\left(\Pr+\eta\frac{\Pr}{\Gamma}\right)\pi^2 + \left(\Pr+\eta_z\frac{\Pr}{\Gamma}\right)\beta^2\right\} - \left\{\left(1+\Pr+\eta\frac{\Pr}{\Gamma}\right)\pi^2 + \left(1+\Pr+\eta_z\frac{\Pr}{\Gamma}\right)\beta^2\right\}\right] \qquad ...(39)$$

From equation (39) it follows that overstability can occur for a particular wave number only if

$$\lambda > \frac{\left(\Gamma+\Gamma\Pr+\eta\Pr\right)\pi^2 + \left(\Gamma+\Gamma\Pr+\eta_z\Pr\right)\beta^2}{\Gamma\left\{\left(\Gamma\Pr+\eta\Pr\right)\pi^2 + \left(\Gamma\Pr+\eta_z\Pr\right)\beta^2\right\}} \qquad ...(40)$$

i.e., the elasticity is sufficiently large. It implies that the overstability is due entirely to the elasticity of the fluid.

The critical Rayleigh number for the overstability is obtained by minimizing equation (38) with respect to β. Although the minimization can be done by letting $\partial Rp/\partial\beta = 0$, for nonzero η and η_z, it is not analytically tractable. From equation (38), we can find that unlike the exchange of stabilities, the overstability are dependent of the relaxation time, the Prandtl number and the anisotropic porous parameter.

In the limiting case when $\eta \to 0$, $\eta_z \to 0$, i.e. $Da \to \infty$, $Da_z \to \infty$, the system can be simplified to that of a Maxwell fluid layer in a non-anisotropic porous medium. Equations (36) and (39) are then reduced to

$$R_p = \frac{\Gamma^3}{\beta^2} - \frac{\omega^2\Gamma}{\beta^2\Pr}\left(1+\lambda\Gamma\right) \qquad ...(41)$$

$$\omega^2 = \frac{\Gamma\Pr}{\lambda}\left[\Gamma - \left(1+\frac{1}{\Pr}\right)\frac{1}{\lambda}\right] \qquad ...(42)$$

Equations (41) and (42) are the same results as obtained by Kolkka and Ierley in nonporous media.

Fig. 1 Rs as a function of wave number for $\eta = 20, \eta_z = 20, \lambda = 0.5, P_r = 10$. The solid curve corresponds to Rs in anisotropic medium. The broken curve corresponds to Rs in a nonporous medium

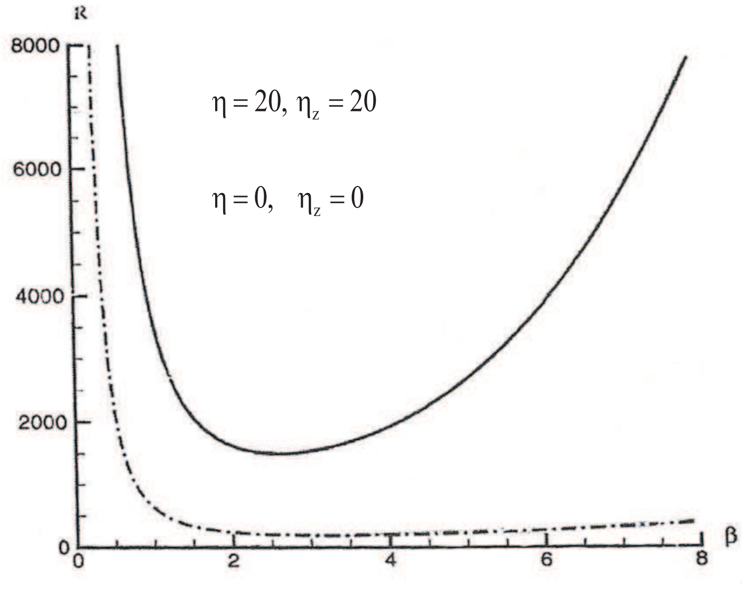

Fig. 2 Rayleigh number for the onset of instabilities as function of wave number for $\eta = 20, \eta_z = 20, \lambda = 0.5, P_r = 10$. The solid curve corresponds to Rs, the broken curve corresponds to Rp.

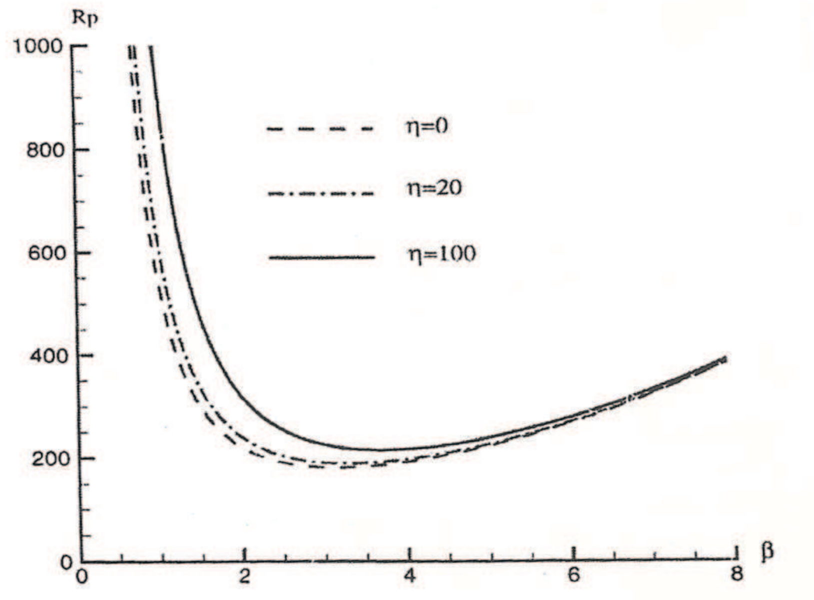

Fig. 3 Rp as a function of wave number for different values of η, η_z and $\lambda = 0.5, P_r = 10$.

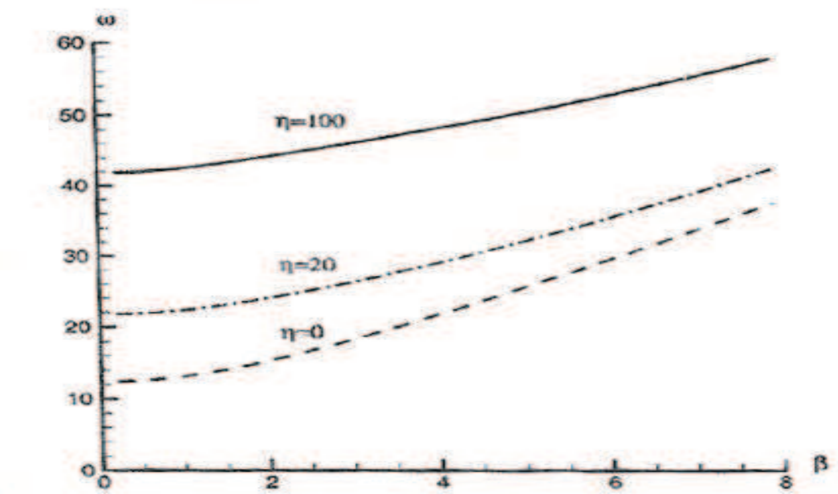

Fig. 4. Frequency overstability as a function of two wave numbers for different values of η, η_z and $\lambda = 0.5$, $P_r = 10$.

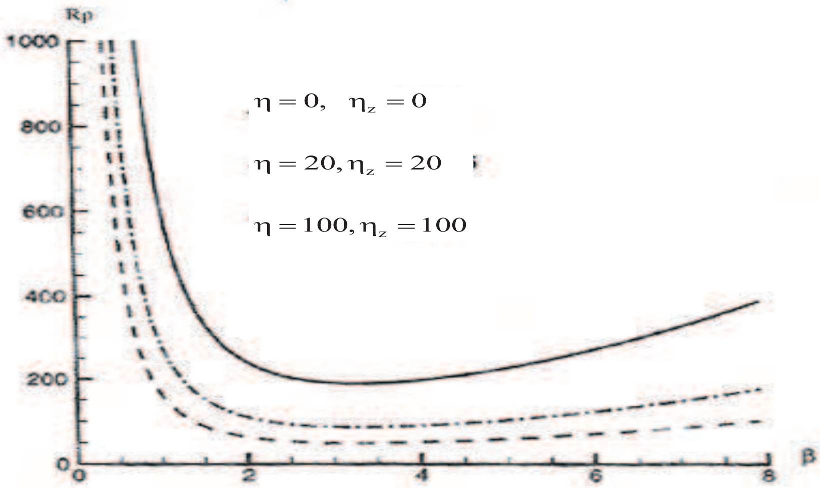

Fig. 5. Rp as a function of wave number for different value of $\lambda, \eta = 20, \eta_z = 20$ and $P_r = 10$.

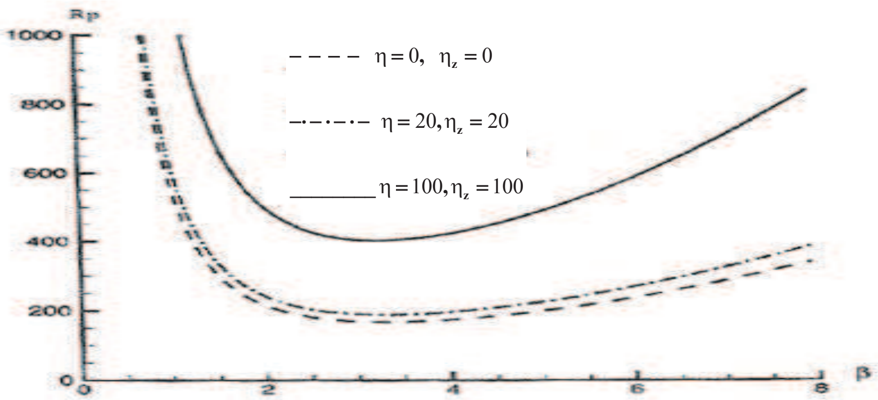

Fig.6. Rp as a function of wave number for different values of P_r and $\eta = 20, \eta_z = 20, \lambda = 0.5$.

Results and Discussion: In order to illustrate the effect of the anisotropic porous media on the exchange of stabilities, we plot the typical curves of the Rayleigh number as a function of β in Fig. 2. The solid curve represents Rs for a viscoelastic fluid in anisotropic porous medium with $\eta = 20$, $\lambda = 0.5$ and Pr = 10. The broken curve depicts Rs for the same viscoelastic fluid in nonporous case. It can be seen that the critical Rayleigh number for the exchange of stabilities in the porous case is larger than that in the nonporous case. The critical wave number β_c in the porous medium is also larger than that in the nonporous case. It indicates that the porous media have a stabilizing influence on a liquid layer heated from below. Because the critical number for the exchange of stabilities is only relevant to the porous property and is independent of the relaxation time and the Prandtl number, this result is true for both Newtonian fluids and non-Newtonian fluids.

For comparison, an overstability curve and an exchange stability curve in the porous medium are shown in Fig. 3. The solid and broken curves represents Rs and Rp as functions of β for the viscoelastic fluid in the porous medium with $\eta = 20, \eta_z = 20$, $\lambda = 0.5$ and Pr = 10, respectively. Examining, Fig. 3, it is clear that the overstability curve lies far below that of the exchange of stability, i.e., Rp < Rs for the same wave number. This implies that oscillatory instabilities can set in before stationary modes. We also observe that the overstability curve has a very flat bottom, indicating instability can occur within a broad wave number band.

This effect of the anisotropic porous parameter η, η_z on overstability is plotted in Fig. 4. It can be seen that the critical Rayleigh number increases with the increase of the value of the anisotropic porous parameter η, η_z, indicating that the effect of increasing η, η_z is to stabilize the system. The effect of the anisotropic porous parameter on the frequency of overstability is shown in Fig. 5. The frequency also increases as η, η_z increases. It implies that for the anisotropic porous medium with a high value of η, η_z the frequency mode for overstability is also high.

The effect of the relaxation time of the fluid on the overstability is shown in Fig. 6. Examining Fig. 6, we observe that the Maxwell fluid with a higher value of the relaxation time will exhibits overstability at a lower Rayleigh number. The critical Rayleigh number for overstability decreases as the relaxation time increases. This result indicates that the elasticity of a Maxwell fluid has a destabilizing influence on a fluid layer in anisotropic porous medium heated from below. The effect of the Prandtl number is also important, because many practical viscoelastic fluids have large Prandtl numbers. From Fig. 7, it is interesting that when the value of the Prandtl number is larger than 10, increase of the Prandtl number has almost no effect on overstability.

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Ruchi Goel/Department of Mathematics/ D.N. (P.G.) College/ Meerut/ India.
Sudhir Kumar Yadav/Department of Applied Sciences/
Accurate Institute of Management & Technology/ Gr. Noida/ India.