PROPERTIES OF Δ^*-OPEN SETS IN TOPOLOGICAL SPACES

K. MEENA, K. SIVAKAMASUNDARI

Abstract: In this paper a new class of generalised open sets called Δ^*-open sets which are the complement of Δ^*-closed sets are introduced and some of its properties are proved. The characteristics of Δ^*-open sets using δg-kernel are also analysed in this paper.

Keywords: δg-closed sets, Δ^*-closed sets, δg-kernel and Δ^*-closure operator.

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Introduction: The concept of generalised closed (briefly, g-closed) sets and δ-open sets were introduced and investigated by Norman Levine [3] and Velicko [2] in 1970 and 1968 respectively. By combining the concepts of δ-closed sets and g-closed sets, Julian Dontchev [4] proposed a class of generalised closed sets called δg-closed sets in 1996. R.Sudha and K.Sivakamasundari [5] developed the concept of δg-closed sets and defined δg^*-closed sets which are stronger than δg-closed sets in 2012. K.Meena and K.Sivakamasundari [6] introduced and analysed a new class of generalised closed sets called Δ^*-closed sets which are weaker than δg^*-closed sets and independent of δg-closed sets in 2014.

Throughout this paper (X, τ) represents a non empty topological space on which no separation axioms are mentioned unless otherwise specified. For a subset A of (X, τ), the closure of A and interior of A denoted by cl(A) and int(A) respectively.

In the paper [6] of K.Meena and K. Sivakamasundari, Δ^*-closed sets were introduced and initially denoted by $\delta(\delta g)^*$-closed sets.

Preliminaries:

Definition 2.1 A subset A of (X, τ) is called a

i) Regular open set if A = int(cl(A)).[1]

ii)δ-open set if it is the union of regular open sets[2]

Definition 2.2 A subset A of a topological space (X, τ) is called a

i) Δ^*-closed set if $\delta cl(A)\subseteq U$ whenever $A\subseteq U$, U is δg-open in (X, τ). The class of all Δ^*-closed sets of (X, τ) is denoted by $\Delta^*C(X, \tau)$. [6]

ii) δ-generalized closed (briefly δg-closed) if $\delta cl(A)\subseteq U$ whenever $A\subseteq U$, U is open in (X, τ). [4]

Definition 2.3 The Δ^*-closure operator of a topological space (X, τ) is denoted by Δ^***cl(A)** and defined as follows.

$\Delta^*cl(A)= \cap\{F\subseteq X : A\subseteq F \text{ and } F \text{ is } \Delta^*\text{-closed in } (X, \tau)\}$. [7]

Definition 2.4 Let U be any subset of (X,τ). Using Δ^*-closure operator, a new class of sets denoted by $\Delta^*\tau^\#$ is defined as follows. $\Delta^*\tau^\#=\{ U:\Delta^*cl(X-U)=X-U \}$. [7]

Remark 2.5 Every δ-closed set is Δ^*-closed but not conversely. (Proposition 3.2 of [6])

Remark 2.6 If A is a Δ^*-closed set in the space X and $A \subseteq B \subseteq \delta cl(A)$ then B is also Δ^*-closed set. (Theorem 3.7 of [7])

Remark 2.7 For any topology τ, we have $\tau_\delta\subseteq\Delta^*\tau^\#$. (Proposition 4.6 of [7])

Remark 2.8 Let A be a Δ^*-closed set of X. Then δcl(A)–A does not contain a non empty δg-closed set. (Theorem 3.4 of [7])

Remark 2.9 The finite union of Δ^*-closed sets is closed. (Theorem 3.1 of [7])

Remark 2.10 For a subset A of (X,τ),

$A \subseteq\Delta^*cl(A) \subseteq \delta g^*cl(A) \subseteq\delta cl(A)$. (Remark 4.3 of[7])

III Δ^*-open Sets

Definition 3.1 A subset A of a topological space(X,τ) is called Δ^*-open if its complement A^c is Δ^*-closed in (X,τ). The collection of all Δ^*-open sets in (X,τ) is denoted by $\Delta^*O(X,\tau)$.

Theorem 3.2 If a subset A of a topological space (X,τ) is δ-open, then it is Δ^*-open in (X, τ).

Proof: Let A be a δ-open set in a topological space (X, τ). Then A^c is δ-closed in (X,τ). By Remark 2.5, A^c is Δ^*-closed in (X,τ). Hence A is Δ^*open in (X,τ).

Remark 3.3 The converse of the above theorem need not be true as seen in the following example.

Example 3.4 Let X={a, b, c}, τ={ϕ, X, {a},{a ,b}}. Then the subset {a} is Δ^*-open but not δ-open in (X, τ).

Theorem 3.5 A subset A of a topological space (X,τ) is Δ^*-open if and only if $G\subseteq\delta Int(A)$ whenever $A\supseteq G$ and G is δg-closed.

Proof: Assume that A is Δ^*-open . Then A^c is Δ^*-closed. Let G be a δg-closed set in (X,τ) contained in A. Then G^c is a δg-open set in (X,τ) containing A^c. Since A^c is Δ^*-closed, $\delta cl(A^c)\subseteq G^c$, equivalently $G\subseteq\delta Int(A)$.

Conversely assume that $G\subseteq\delta Int(A)$, whenever $G\subseteq A$ and G is δg-closed in(X, τ). Let A^c be contained in F where F is δg-open. Then $F^c\subseteq A$. By criteria, $F^c\subseteq\delta Int(A)$.

This implies $\delta cl(A^c)\subseteq F$. Thus A^c is Δ^*-closed and hence A is Δ^*-open.

Proposition 3.6 If $\delta Int(A)\subseteq B\subseteq A$ and A is Δ^*-open in (X, τ), then B is Δ^*-open in

(X, τ).

Proof: The proof follows by the fact that for a subset of A of (X, τ), $\delta cl(X-A) = X-\delta Int(A)$ and by the Remark 2.6

Theorem 3.7 If A and B are Δ^*-open sets in (X, τ) then $A\cap B$ is Δ^*-open in (X, τ).

Proof: Let A and B be Δ^*-open sets in X. Then X–A and X–B are Δ^*-closed sets and $(X-A)\cup(X-B)=X-(A\cap B)$ is Δ^*-closed. Hence $A\cap B$ is Δ^*-open.

Theorem 3.8 A subset A of (X, τ) is Δ^*-open if and only if G=X whenever G is δg-open and $\delta Int(A)\cup A^c\subseteq G$.

Proof:(Necessary): Let A be a Δ^*-open set and G be a δg-open and $\delta Int(A)\cup A^c\subseteq G$. This gives $G^c\subseteq [\delta Int(A)\cup A^c]^c = [\delta Int(A)]^c\cap A = [\delta Int(A)]^c-A^c$. Since A^c is Δ^*-closed and G^c is δg-closed (Remark 2.8), it follows that $G^c=\phi$. Therefore G = X.

(Sufficiency): Suppose that F is δg-closed and $F\subseteq A$. Then $\delta Int(A)\cup A^c\subseteq\delta Int(A)\cup F^c$. As δ-open implies δg-open, we get δInt(A)is δg-open. Also F^c is δg-open. Hence $\delta Int(A)\cup F^c$ is δg-open. It follows by hypothesis that $\delta Int(A)\cup A^c = X$ and hence $F\subseteq\delta Int(A)$. Therefore by proposition 3.5, A is Δ^*-open in (X, τ).

Proposition 3.9: For each $a\in(X, \tau)$ either {a} is δg-closed or {a} is Δ^*-open in (X, τ). That is for any space (X,τ), $X = \delta GC(X,\tau) \cup \Delta^*O(X, \tau)$.

Proof: Suppose that {a} is not δg-closed in (X,τ). Then $\{a\}^c$ is not δg-open and the only δg-open set containing $\{a\}^c$ is the space (X, τ) itself. That is $\{a\}^c\subseteq X$.

Therefore $\delta cl(\{a\}^c)\subseteq X$ and so $\{a\}^c$ is Δ^*-closed and hence {a} is Δ^*-open (X, τ).

Definition 3.10: The intersection of all δg-open subsets of (X, τ) containing A is called the δg- kernel of A and is denoted by **δg-ker(A).** i.e., δg-ker(A)= $\cap\{U$ / U is δg-open in (X, τ) and $A\subseteq U\}$.

Theorem 3.11 For a subset A of (X, τ), the following properties are equivalent.

a) A is Δ^*-closed.

b) $\delta cl(A)\subseteq$ δg-ker(A) holds.

c) i) $\delta cl(A)\cap\delta GC(X,\tau)\subseteq A$

ii) $\delta cl(A)\cap\Delta^*O(X, \tau)\subseteq$ δg-ker(A).

Proof: (a)⟹(b) : Let $x\notin$ δg-ker(A). Then there exists a set $U\in\delta GO(X, \tau)$ such that $x\notin U$ and $A\subseteq U$. Since A is Δ^*-closed, $\delta cl(A)\subseteq U$ and therefore $x\notin\delta cl(A)$.

(b)⟹(c) : i) First we claim that δg-ker(A)$\cap\delta GC(X,\tau)\subseteq A$.

Let $x\in$ δg-ker(A)$\cap\delta GC(X,\tau)$ and assume that $x\notin A$. Since the set $X-\{x\}\in\delta GO(X, \tau)$ and $A\subseteq X-\{x\}$, δg-ker(A)$\subseteq X-\{x\}$. Then we have $x\in X-\{x\}$ which is a contradiction. Thus we show that

δg-ker(A) $\cap\delta GC(X,\tau)\subseteq A$.

By using (b), $\delta cl(A)\cap\delta GC(X,\tau)\subseteq$ δg-ker(A) $\cap\delta GC(X,\tau)\subseteq A$.

ii) Follows from (i) and (b).

(c)⟹(b) : By Remark 2.9 and (C),

$\delta cl(A)=\delta cl(A)\cap X=\delta cl(A)\cap[\delta GC(X,\tau)\cup \Delta^*O(X,\tau)]=[\delta cl(A)\cap \delta GC(X,\tau)] \cup [\delta cl(A) \cap \Delta^*O(X,\tau)] \subseteq A \cup$ δg-ker(A)=δg-ker(A).

That is $\delta cl(A)\subseteq$δg-kernel(A)holds.

(b)⟹(a) : Let $U\in\delta GO(X,\tau)$ such that $A\subseteq U$. Then we have that δg-ker(A)$\subseteq U$ and so by (b) $\delta cl(A)\subseteq U$. Therefore A is Δ^*-closed.

Theorem 3.12 For each $x\in X$, $x\in\Delta^*cl(A)$ if and only if $U\cap A\neq\phi$ for every Δ^*-open set U in (X, τ) containing x.

Proof: (Necessary): Let $x\in\Delta^*cl(A)$.

Suppose that there exists a Δ^*-open set U in (X,τ) containing x such that $U\cap A=\phi$. Hence X–U is Δ^*-closed in (X,τ) containing A which implies that $\Delta^*cl(A)\subseteq X-U$. Hence $x\notin\Delta^*cl(A)$ which is a contradiction. Hence $U\cap A \neq \phi$.

(Sufficiency): Let us assume that $U\cap A \neq \phi$ for every Δ^*-open set U in (X, τ) containing x. Suppose that $x\notin\Delta^*cl(A)$. By the definition of $\Delta^*cl(A)$, there exists a Δ^*-closed set U in (X, τ) containing A such that $x\notin U$. Hence X–U is Δ^*-open set in (X,τ) containing x. Since $A\subseteq U$, we have $(X-U) \cap A=\phi$ which is a contradiction. Hence $x\in\Delta^*cl(A)$.

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K. Meena/Senior Grade Assistant Professor/Department of Mathematics/Kumaraguru College of Technology/Coimabtore-641049/TamilNadu/India.

K. Sivakamasundari/Professor/Department of Mathematics/Avinashilingam Institute for Home Science and Higher Education for Women University/Coimabtore-641043/TamilNadu/ India.