SOLVING FUZZY LINEAR FRACTIONAL PROGRAMS WITH TRAPEZOIDAL FUZZY NUMBERS

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Abstract: In this paper, we have been pointed out the study of fuzzy linear fractional programming (FLFP) problems with trapezoidal fuzzy numbers. Where the objective functions are fuzzy numbers and the constraints are real numbers. A computational procedure has been presented to obtain an optimal solutions. To illustrate a numerical examples to check our efficiency of the proposed method.

Keywords: Fuzzy linear fractional programming; Ranking function; Symmetric Trapezoidal fuzzy numbers.

Introduction: Now a days the researchers are very much interested to doing research in the field of linear fractional programming (LFP) problem because of huge application in real life problem i.e.financial sector, hospitality, industrial sector etc. . It is a mathematical technique for optimal allocation to several activities on the basis of given decision of optimality. This type of problem is evidently an uncertain optimization problem due to its decision based system. So, it lead to propose a new concept in fuzzy optimization by Bellman and Zadeh [10]. Lotfi et al. [3] introduced a method to obtain the approximate the solution of fully fuzzy linear programming problems. Amit Kumar et al. [4] proposed a method for solving fully fuzzy linear programming problems using idea of crisp linear programming and ranking function.

Recently, Veeramani and Sumathi [5] established a new method for solving fuzzy linear fractional

Programming problem and they have transformed the problem into a multi objective linear programming problem. Ganesan and Veeramani [11] introduced the fuzzy linear programming problem with trapezoidal fuzzy numbers without converting them to crisp linear programming problem. A. Ebrahimnejad and M. Tavana [1] proposed a new concept which the coefficients of objective function and the values of the right hand side are represented by trapezoidal fuzzy numbers and other parts are represented by real numbers. They convert the fuzzy linear programming problem into an equivalent crisp linear programming problem and solved by simplex method.In this paper, a new method is proposed for finding the fuzzy optimal solution of FLFP problems with inequality constraints. The coefficients of the objective function are represented by trapezoidal fuzzy numbers and the constraints are represented by real numbers.

We introduce a new type of fuzzy arithmetic for symmetric trapezoidal fuzzy numbers and propose a method for solving fuzzy linear fractional programming problem with converting them to crisp linear fractional programming problems. After that, we used a new transformation technique used to solve these crisp linear fractional programming problem. To illustrate the proposed method, numerical examples are solved. This paper is organized as follows: In section 2 some basic definitions of fuzzy symmetric trapezoidal fuzzy number and some arithmetic results also. In section 3, formulation of FLFP problems and application of ranking function for solving FLFP problems are established. A new method is proposed for solving FLFP problems in section 4. In section 5, we give a numerical example involving symmetrical trapezoidal fuzzy numbers to illustrate the theory developed in this paper.

Preliminaries: The aim of this section is to present some notations and results of fuzzy set theory are discussed.

Definition 2.1 [1]. A convex fuzzy set $\tilde{A}$ on R is a fuzzy number if the following conditions hold:

(a) Its membership function is piecewise continuous.

(b) There exist three intervals [a, b], [b, c] and [c, d] such that $\mu_{\tilde{A}}$ is increasing on [a, b], equal to 1 on [b, c], decreasing on [c, d] and equal to 0 elsewhere.

Definition 2.2 [1]. The arithmetic operations on two symmetric trapezoidal fuzzy numbers

$\tilde{A} = (a^L, a^U, \alpha, \alpha)$ and $\tilde{B} = (b^L, b^U, \beta, \beta)$ are given by:

$$\tilde{A} + \tilde{B} = (a^L + b^L, a^U + b^U, \alpha + \beta, \alpha + \beta),$$

$$\tilde{A} - \tilde{B} = (a^L - b^L, a^U - b^U, \alpha - \beta, \alpha - \beta),$$

$$\tilde{A}\tilde{B} = ((\frac{a^L + a^U}{2})(\frac{b^L + b^u}{2}) - t, (\frac{a^L + a^U}{2})(\frac{b^L + b^U}{2}) + t, \left|a^U\beta + b^U\alpha\right|, \left|a^U\beta + b^U\alpha\right|),$$

Where

$$t = \frac{t_2 - t_1}{2}, \; t_1 = \min\{a^L b^L, a^U b^U, a^U b^L, a^U b^U\}, t_2 = \max\{a^L b^L, a^U b^U, a^U b^L, a^U b^U\}.$$

$$k\tilde{A}=\begin{cases}(ka^L,ka^U,k\alpha,k\alpha) & k\geq 0\\ (ka^U,ka^L,-k\alpha,-k\alpha) & k<0.\end{cases}$$

Definition 2.3 [1]. Let $\tilde{A}=(a^L,a^U,\alpha,\alpha)$ and $\tilde{B}=(b^L,b^U,\beta,\beta)$ be two symmetric trapezoidal fuzzy numbers. The relations $\tilde{\leq}$ and $\approx$ are defined as follows:

(i) $\dfrac{(a^L-\alpha)+(a^U-\alpha)}{2}<\dfrac{(b^L-\beta)+(b^U-\beta)}{2}$, that is $\dfrac{a^L+a^U}{2}<\dfrac{b^L+b^U}{2}$

(in this case, we may write $\tilde{A}\tilde{<}\tilde{B}$), or

(ii) $\dfrac{a^L+a^U}{2}=\dfrac{b^L+b^U}{2}, b^L<a^L, a^U<b^U$ (in this case we say $\tilde{A}\approx\tilde{B}$), or

(iii) $\dfrac{a^L+a^U}{2}=\dfrac{b^L+b^U}{2}, b^L=a^L, a^U=b^U, \alpha\leq\beta$ (in this case we say $\tilde{A}\approx\tilde{B}$).

Definition 2.4 [1] A fuzzy set $\tilde{A}$ on R is called a symmetric trapezoidal fuzzy number if its membership function is defined as follows:

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-(a^L-\alpha)}{\alpha} & a^L-\alpha\leq x\leq a^L\\ 1 & a^L\leq x\leq a^U\\ \dfrac{(a^u+\alpha)-x}{\alpha} & a^U\leq x\leq a^U+\alpha\\ 0 & else\end{cases}$$

3. Proposed method to find the fuzzy optimal solution of FLFP problems

In this section, a new method is proposed to find the fuzzy optimal solution of the following type of fuzzy linear fractional programming (FLFP) problems:

Maximize (or Minimize) $=\dfrac{\tilde{c}^t x+\tilde{\alpha}}{\tilde{d}^t x+\tilde{\beta}}$

Subject to $A\otimes x\leq b$, (1)

$x \geq 0$.

Where $\tilde{c}^t = [\tilde{c}_j]$ is 1 by n matrix; $\tilde{d}^t = [\tilde{d}_j]$ is 1 by n matrix; $x = [x_j]$ is n by 1 matrix; $A = [a_{ij}]$ is m by n matrix; $b = [b_{ij}]$ is a m by 1 matrix; $\tilde{\alpha} = [\tilde{\alpha}_j]$ and $\tilde{\beta} = [\tilde{\beta}_j]$ are the scalars.

Base on above definition 2.3, we define a rank for each symmetric trapezoidal fuzzy number for comparison purposes. Assuming that $\tilde{A} = (a^L, a^U, \alpha, \alpha)$ is a symmetric trapezoidal fuzzy number, then $R(\tilde{A}) = \dfrac{a^L + a^U}{2}$. This equation allows us to convert the fuzzy linear fractional programming (FLFP) problem in to a crisp linear fractional programming (CLFP) problem. We substitute the rank order of each fuzzy number for the corresponding fuzzy number in the fuzzy problem under consideration. This leads to an equivalent crisp linear fractional programming problem which can be solved by standard method. After we have got the crisp linear fractional programming we used a new transformation method for converting into an equivalent crisp linear programming problem and solved by simplex method.

4. Numerical Example:

In this section, we illustrate the proposed algorithm using a real life problem. Linear fractional programming problem is evidently an uncertain optimization problem due to its variations in the maximum daily requirements. So, the amount of each product of ingredient will be uncertain. Hence, we will model the problem as a Fuzzy linear fractional programming (FLFP) problem. We use trapezoidal fuzzy numbers for each uncertain value. Also, the mathematical programming problem will be solved by *Lingo*.

Example.4.1.

In Jamshedpur City, India, A Wooden company is the producer of two kinds of products A and B with profit around (4, 6, 3,3) and around (1, 5, 1, 1) dollar per unit, respectively. However the cost for each one unit of the above products is around (3, 7, 2, 2) and around (3, 1, 1, 1) dollars respectively. It is assume that a fixed cost of around (1, 1, 2, 2) dollar is added to the cost function due to expected duration through the process of production. Suppose the raw material needed for manufacturing product A and B is about 3 unitsper pound and about 5 units per pound respectively, the supply for this raw material is restricted to about 15 pounds. Man-hours per unit for the product A is about 5 hour and product B is about 2 hour per unit for manufacturing but total Man-hour available is about 10 hour daily. Determine how many products A and B should be manufactured in order to maximize the total profit.

This real life problem can be formulated to the following FLFP problem:

$$\text{Max } \frac{(4,6,3,3)x_1 + (1,5,1,1)x_2}{(3,7,2,2)x_1 + (3,1,1,1)x_2 + (1,1,2,2)}$$

s.t. $3x_1 + 5x_2 \le 15,$

(2)

$5x_1 + 2x_2 \le 10,$

$x_1, x_2 \ge 0.$

So, with respect to definition 2.3 we convert the problem (2) into an equivalent crisp linear programming problem as follows:

$$\text{Max } \frac{5x_1 + 3x_2}{5x_1 + 2x_2 + 1}$$

s.t. $3x_1 + 5x_2 \le 15,$

(3)

$5x_1 + 2x_2 \le 10,$

$x_1, x_2 \ge 0.$

Now, the crisp linear fractional programming problem (3) converting into an equivalent crisp linear programming problem by using transformation technique [9].

Max $5y_1 + 3y_2$

s. t. $78y_1 + 35y_2 \le 15,$

$55y_1 + 22y_2 \le 10,$

(4)

$y_1, y_2 \ge 0.$

The problem (4) is the crisp linear programming problem. Now solved by simplex method we get the result is:

Y_1=0, y_2=0.42, and the objective function value is Z=1.28.

As compare the existing method our method is very effective method as the iteration is very less then the other method.

Conclusion:

In this paper, we proposed a new method to find the fuzzy optimal solution of fuzzy linear fractional programming problems with inequality constraints occurring in daily real life problem. We showed that the method proposed in this paper is very reliability and applicability. To illustrate the proposed method numerical examples are solved.

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