GENERALIZED DEFINITION OF IMAGE OF AN FS-SUBSET UNDER AN FS-FUNCTION-RESULTANT PROPERTIES OF IMAGES

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Abstract: Vaddiparthi Yogeswara, G.Srinivas and Biswajit Rath introduced the concept of Fs-set ,Fs-subset, complement an of Fs-subset and proved important results like De Morgan laws for Fs-sets which are called Fs- De Morgan laws. In the paper[5] Vaddiparthi Yogeswara, Biswajit Rath and S.V.G.Reddy introduced the concept of Fs-Function between two Fs-subsets of a given Fs-set and defined images of Fs-subset under various Fs-functions and studied the properties of images. In another paper[17], they studied the properties of images of Fs-subsets after changing the definition of images. In this paper we give the generalized definition of an image of an Fs-subset under a given Fs-function and again study the properties of images of Fs-subsets with this new definition.

Keywords: Fs-set, Fs-subset, Fs-empty set, Fs-union, Fs-intersection, Fs-complement, Fs-De Morgan laws and Fs-Function and images of Fs-subsets

Introduction: Murthy[1] introduced F-set in order to prove Axiom of choice for fuzzy sets which is not true for L-fuzzy sets introduced by Goguen[2]. In the paper[3], Tridiv discussed fuzzy complement of an extended fuzzy subset and proved De Morgan laws etc. The extended Fuzzy set Tridiv considered contains the membership value $\mu_1(x) - \mu_2(x)$.Even thought this definition is meaningful $-\mu_2(x)$, a term in this expression will not be in the interval [0,1].To answer this incomprehensiveness, Vaddiparthi Yogeswara , G.Srinivas and Biswajit Rath introduced the concept of Fs-set and developed the theory of Fs-sets in order to prove the collection of all Fs-subsets of given Fs-set is a complete Boolean algebra under Fs-unions, Fs-intersections and Fs-complements [4]. The Fs-sets they introduced contain Boolean valued membership functions .All most they are successful in their efforts in proving that result with some conditions. In the paper[5] Vaddiparthi Yogeswara, Biswajit Rath and S.V.G.Reddy introduced the concept of Fs-Function between two Fs-subsets of a given Fs-set and defined images of Fs-subset under various Fs-functions and studied the properties of images In another paper[17],Vaddiparthi Yogeswara, Biswajit Rath studied the properties of images of Fs-subsets after changing the definition of images. In this paper we will give the generalized definition of an image of an Fs-subset under a given Fs-function and again study the properties of images of Fs-subsets with this new definition. For convenience of readers before beginning the paper, we mention various definitions and results in paper[4]. We denote the largest element of a complete Boolean algebra L_A[1.1] byM_A. We denote Fs-union and crisp set union by same symbol ∪ and similary Fs-intersection and crisp set intersection by the same symbol ∩. [X] denote the complete ideal generated by X and (X) denote the complete subalgebra generated by X in a complete Boolean algebra. For all lattice theoretic properties and Boolean algebraic properties we refer Szasz [7], Garret Birkhoff[8],Steven Givant • Paul Halmos[8] and Thomas Jech[9]

Theory Of Fs-Sets

1.1 Fs-set: Let U be a universal set, $A_1 \subseteq U$ and let A⊆U be non-empty. A four tuple $\mathcal{A} = (A_1, A, \bar{A}\,(\mu_{1A_1}, \mu_{2A}), L_A)$ is said be an Fs-set if, and only if

(1) $A \subseteq A_1$

(2) L_A is a complete Boolean Algebra

(3) $\mu_{1A_1}: A_1 \longrightarrow L_A$, $\mu_{2A}: A \longrightarrow L_A$,are functions such that $\mu_{1A_1}|A \geq \mu_{2A}$

(4) $\bar{A}$:A⟶ L_A is defined by

$\bar{A}x = \mu_{1A_1}x \wedge (\mu_{2A}x)^c$,for each $x \in A$

1.2 Fs-subset

Let $\mathcal{A}$=$(A_1, A, \bar{A}(\mu_{1A_1}, \mu_{2A}), L_A)$ and $\mathcal{B}$=$(B_1, B, \bar{B}(\mu_{1B_1}, \mu_{2B}), L_B)$ be a pair of Fs-sets. $\mathcal{B}$ is said to be an Fs-subset of $\mathcal{A}$, denoted by $\mathcal{B}$⊆$\mathcal{A}$, if, and only if

(1) $B_1 \subseteq A_1$, $A \subseteq B$

(2) L_B is a complete subalgebra of L_A or $L_B \leq L_A$

(3) $\mu_{1B_1} \leq \mu_{1A_1}|B_1$, and $\mu_{2B}|A \geq \mu_{2A}$

1.3 Proposition: Let $\mathcal{B}$ and $\mathcal{A}$ be a pair of Fs-sets such that $\mathcal{B} \subseteq \mathcal{A}$. Then $\overline{B}x \leq \overline{A}x$ is true for each x ∈A

1.4 Definition: For some L_X, such that $L_X \leq L_A$ a four tuple $\mathcal{X} = (X_1, X, \bar{X}(\mu_{1X_1}, \mu_{2X}), L_X)$ is not an Fs-set if, and only if

(a)$X \nsubseteq X_1$ or

(b) $\mu_{1X_1}x \ngeq \mu_{2X}x$, for some $x \in X \cap X_1$

Here onwards, any object of this type is called an Fs-empty set of first kind and we accept that it is an Fs-subset of $\mathcal{B}$ for any $\mathcal{B} \subseteq \mathcal{A}$.

Definition: An Fs-subset $\mathcal{Y}$=$(Y_1, Y, \bar{Y}(\mu_{1Y_1}, \mu_{2Y}), L_Y)$ of $\mathcal{A}$, is said to be an Fs-empty set of second kind if, and only if

(a') $Y_1 = Y = A$

(b') $L_Y \leq L_A$

(c') $\bar{Y} = 0$

1.4.1 Remark: we denote Fs-empty set of first kind or Fs-empty set of second kind by $\Phi_{\mathcal{A}}$and we prove later (1.15), $\Phi_{\mathcal{A}}$ is the least Fs-subset among all Fs-subsets of $\mathcal{A}$.

1.5 Definition: Let $\mathcal{B}_1 = \left(B_{11}, B_1, \overline{B}_1\left(\mu_{1B_{11}}, \mu_{2B_1}\right), L_{B_1}\right)$ and $\mathcal{B}_2 = \left(B_{12}, B_2, \overline{B}_2\left(\mu_{1B_{12}}, \mu_{2B_2}\right), L_{B_2}\right)$ be a pair of Fs-subsets.

(1) We say that $\mathcal{B}_1$and $\mathcal{B}_2$ are (1,5)-equal, if $B_{11} = B_{12}$ and $L_{B_1} = L_{B_2}$
(2) We say that $\mathcal{B}_1$and $\mathcal{B}_2$ are (2,5)-equal, if $B_1 = B_2$ and $L_{B_1} = L_{B_2}$
(3) We say that $\mathcal{B}_1$ and $\mathcal{B}_2$ are 3-equal, if $\mathcal{B}_1$ and $\mathcal{B}_2$ are (1,5)-equal and $\mu_{1B_{11}} = \mu_{1B_{12}}$
(4) We say that $\mathcal{B}_1$ and $\mathcal{B}_2$ are 4-equal, if $\mathcal{B}_1$ and $\mathcal{B}_2$ are (2,5)-equal and $\mu_{2B_1} = \mu_{2B_2}$
(5) We say that $\mathcal{B}_1$and $\mathcal{B}_2$ are Total equal denoted $\mathcal{B}_1 = \mathcal{B}_2(\mathrm{T})$, if $\mathcal{B}_1$and $\mathcal{B}_2$ are (2,5)-equal and $\overline{B}_1 = \overline{B}_2$
(6) We say that$\mathcal{B}_1$, $\mathcal{B}_2$ are Full-equal, denoted $\mathcal{B}_1 = \mathcal{B}_2$, if $\mathcal{B}_1$and $\mathcal{B}_2$ are 3-equal and 4-equal.

1.6 Proposition:
$\mathcal{B}_1 = \left(B_{11}, B_1, \overline{B}_1\left(\mu_{1B_{11}}, \mu_{B_1}\right), L_{B_1}\right)$ and $\mathcal{B}_2 = \left(B_{12}, B_2, \overline{B}_2\left(\mu_{1B_{12}}, \mu_{B_2}\right), L_{B_2}\right)$ are equal if, only if $\mathcal{B}_1 \subseteq \mathcal{B}_2$ and $\mathcal{B}_2 \subseteq \mathcal{B}_1$

1.7 Definition of Fs-union for a given pair of Fs-subsets of $\mathcal{A}$:
Let $\mathcal{B}$=$\left(B_1, B, \bar{B}\left(\mu_{1B_1,}\mu_{2B}\right), L_B\right)$and
$\mathcal{C}$= $\left(C_1, C, \overline{C}\left(\mu_{1C_1}, \mu_{2C}\right), L_C\right)$, be a pair of Fs-subsets of$\mathcal{A}$. Then,
the Fs-union of $\mathcal{B}$ and $\mathcal{C}$,denoted by $\mathcal{B}\cup\mathcal{C}$ is defined as
$\mathcal{B}\cup\mathcal{C}$=$\mathcal{D}$=$\left(D_1, D, \overline{D}\left(\mu_{1D_1}, \mu_{2D}\right), L_D\right)$,where

(1) $D_1 = B_1 \cup C_1$, $D = B \cap C$
(2) $L_D = L_B \vee L_C$=complete subalgebra generated by $L_B \cup L_C$
(3) $\mu_{1D_1}: D_1 \longrightarrow L_D$ is defined by
$\mu_{1D_1}x = (\mu_{1B_1} \vee \mu_{1C_1})x$
$\mu_{2D} : D \longrightarrow L_D$ is defined by
$\mu_{2D}x = \mu_{2B}x \wedge \mu_{2C}x$
$\overline{D}: D \longrightarrow L_D$ is defined by
$\overline{D}x = \mu_{1D_1}x \wedge (\mu_{2D}x)^c$

1.8 Proposition: *$\mathcal{B}\cup\mathcal{C}$ is an Fs-subset of $\mathcal{A}$.*

1.9 Definition of Fs-intersection for a given pair of Fs-subsets of $\mathcal{A}$:
Let $\mathcal{B}$=$\left(B_1, B, \bar{B}\left(\mu_{1B_1,}\mu_{2B}\right), L_B\right)$ and $\mathcal{C}$= $\left(C_1, C, \overline{C}\left(\mu_{1C_1}, \mu_{2C}\right), L_C\right)$ be a pair of Fs-subsets of$\mathcal{A}$ satisfying the following conditions:

(i) $B_1 \cap C_1 \supseteq B \cup C$
(ii) $\mu_{1B_1}x \wedge \mu_{1C_1}x \geq (\mu_{2B} \vee \mu_{2C})x$,for each $x \in A$

Then, the Fs-intersection of $\mathcal{B}$ and $\mathcal{C}$, denoted by $\mathcal{B}\cap\mathcal{C}$ is defined as

$\mathcal{B}\cap\mathcal{C}$ =$\mathcal{E}$=$(E_1, E, \bar{E}(\mu_{1E_1}, \mu_{2E}), L_E)$,where

(a) $E_1 = B_1 \cap C_1$, $E = B \cup C$

(b) $L_E = L_B \wedge L_C$=$L_B \cap L_C$

(c) $\mu_{1E_1}: E_1 \longrightarrow L_E$ is defined by $\mu_{1E_1}x = \mu_{1B_1}x \wedge \mu_{1C_1}x$

$\mu_{2E} : E \longrightarrow L_E$ is defined by

$\mu_{2E}x = (\mu_{2B} \vee \mu_{2C})x$

$\bar{E}: E \longrightarrow L_E$ is defined by

$\bar{E}x = \mu_{1E_1}x\wedge(\mu_{2E}x)^c$.

1.9.1 Remark: If (i) or (ii) fails we define $\mathcal{B}\cap\mathcal{C}$ as $\mathcal{B}\cap\mathcal{C}$=$\Phi_{\mathcal{A}}$, which is the Fs-empty set of first kind.

1.10 Proposition: *For any pair of Fs-subsets* $\mathcal{B}$=$(B_1, B, \bar{B}(\mu_{1B_1}, \mu_{2B}), L_B)$*and* $\mathcal{C}$=$(C_1, C, \bar{C}(\mu_{1C_1}, \mu_{2C}), L_C)$ *of* $\mathcal{A}$*, the following results are true*

(1) $\mathcal{B}\subseteq\mathcal{B}\cup\mathcal{C}$ and $\mathcal{C}\subseteq\mathcal{B}\cup\mathcal{C}$

(2) $\mathcal{B}\cap\mathcal{C}\subseteq\mathcal{B}$ and $\mathcal{B}\cap\mathcal{C}\subseteq\mathcal{C}$ provided $\mathcal{B}\cap\mathcal{C}$ exists

(3) $\mathcal{B}\subseteq\mathcal{C}$ implies $\mathcal{B}\cup\mathcal{C}$=$\mathcal{C}$

(4) $\mathcal{B}\cap\mathcal{C}$=$\mathcal{B}$ when $\mathcal{B}\neq \Phi_{\mathcal{A}}$ and $\mathcal{B}\subseteq\mathcal{C}$ and $\Phi_{\mathcal{A}} \cap\mathcal{C}$=$\Phi_{\mathcal{A}}$

(5) $\mathcal{B}\cup\mathcal{C}$= $\mathcal{C} \cup \mathcal{B}$ (commutative law of Fs-union)

(6) $\mathcal{B}\cap\mathcal{C}$= $\mathcal{C} \cap \mathcal{B}$ provided $\mathcal{B}\cap\mathcal{C}$ exists. (commutative law of Fs-intersection)

(7) $\mathcal{B}\cup\mathcal{B}$=$\mathcal{B}$

(8) $\mathcal{B}\cap\mathcal{B}$=$\mathcal{B}$ ((7)and (8) are Idempotent laws of Fs-union and Fs-intersection respectively)

1.11 Proposition: For any Fs-subsets $\mathcal{B}$, $\mathcal{C}$and $\mathcal{D}$ of $\mathcal{A} = (A_1, A, \bar{A}(\mu_{1A_1}, \mu_{2A}), L_A)$, the following associative laws are true:

(I) $\mathcal{B} \cup (\mathcal{C} \cup \mathcal{D}) = (\mathcal{B} \cup \mathcal{C}) \cup \mathcal{D}$

(II) $\mathcal{B} \cap (\mathcal{C} \cap \mathcal{D}) = (\mathcal{B} \cap \mathcal{C}) \cap \mathcal{D}$, whenever Fs-intersections exist.

1.12 Arbitrary Fs-unions and arbitrary Fs-intersections:

Given a family $(\mathcal{B}_i)_{i\in I}$ of Fs-subsets of $\mathcal{A}$=$(A_1, A, \bar{A}(\mu_{1A_1}, \mu_{2A}), L_A)$, where $\mathcal{B}_i = (B_{1i}, B_i, \bar{B}_i(\mu_{1B_{1i}}, \mu_{2B_i}), L_{B_i})$,for any $i \in I$

1.13 Definition of Fs-union is as follows

Case (1): For I=Φ,define Fs-union of $(\mathcal{B}_i)_{i\in I}$,denoted by $\bigcup_{i\in I}\mathcal{B}_i$ as$\bigcup_{i\in I}\mathcal{B}_i = \Phi_{\mathcal{A}}$, which is the Fs-empty set

Case (2): Define for I≠Φ, Fs-union of $(\mathcal{B}_i)_{i\in I}$ denoted by $\bigcup_{i\in I}\mathcal{B}_i$ as follow

$$\bigcup_{i\in I}\mathcal{B}_i = \mathcal{B} = (B_1, B, \bar{B}(\mu_{1B_1}, \mu_{2B}), L_B),$$

where

(a) $B_1 = \bigcup_{i\in I} B_{1i}$,$B = \bigcap_{i\in I} B_i$

(b) $L_B = \bigvee_{i\in I} L_{B_i}$ = complete subalgebra generated by $\bigcup L_i (L_i = L_{B_i})$

(c) $\mu_{1B_1}: B_1 \rightarrow L_B$ is defined by

$\mu_{1B_1}x = (\vee_{i\in I}\mu_{1B_{1i}})x = \vee_{i\in I_x}\mu_{1B_{1i}}x$, where
$I_x = \{i \in I \mid x \in B_i\}$
$\mu_{2B}: B \longrightarrow L_B$ is defined by $\mu_{2B}x = (\wedge_{i\in I}\mu_{2B_i})x$
$=\wedge_{i\in I}\mu_{2B_i}x$
$\bar{B}: B \longrightarrow L_B$ is defined by $\bar{B}x = \mu_{1B_1}x \wedge (\mu_{2B}\,x)^c$

1.13.1Remark: We can easily show that (d) $B_1 \supseteq B$ and $\mu_{1B_1}|B \geq \mu_{2B}$.

1.14 Definition of Fs-intersection:

Case (1): For I=Φ, we define Fs-intersection of $(\mathcal{B}_i)_{i\in I}$, denoted by $\cap_{i\in I}\mathcal{B}_i$ as $\cap_{i\in I}\mathcal{B}_i = \mathcal{A}$

Case (2): Suppose $\cap_{i\in I}B_{1i} \supseteq \cup_{i\in I}B_i$ and $\wedge_{i\in I}\mu_{1B_{1i}}|(\cup_{i\in I}B_i) \geq \vee_{i\in I}\mu_{2B_i}$

Then, we define Fs-intersection of $(\mathcal{B}_i)_{i\in I}$, denoted by $\cap_{i\in I}\mathcal{B}_i$ as follows

$$\bigcap_{i\in I}\mathcal{B}_i = \mathcal{C} = (C_1, C, \bar{C}(\mu_{1C_1}, \mu_{2C}), L_C)$$

(a') $C_1 = \cap_{i\in I}B_{1i}$, $C = \cup_{i\in I}B_i$

(b') $L_C = \wedge_{i\in I}L_{B_i}$

(c') $\mu_{1C_1}: C_1 \longrightarrow L_C$ is defined by $\mu_{1C_1}x = (\wedge_{i\in I}\mu_{1B_{1i}})x = \wedge_{i\in I}\mu_{1B_{1i}}x$

$\mu_{2C}: C \longrightarrow L_C$ is defined by $\mu_{2C}x = (\vee_{i\in I}\mu_{2B_i})x = \vee_{i\in I_x}\mu_{2B_i}x$,

where,$I_x = \{i \in I \mid x \in B_i\}$

$\bar{C}: C \longrightarrow L_C$ is defined by $\bar{C}x = \mu_{1C_1}x \wedge (\mu_{2C}\,x)^c$

Case (3): $\cap_{i\in I}B_{1i} \not\supseteq \cup_{i\in I}B_i$ or $\wedge_{i\in I}\mu_{1B_{1i}}|(\cup_{i\in I}B_i) \not\geq \vee_{i\in I}\mu_{2B_i}$

We define

$$\bigcap_{i\in I}\mathcal{B}_i = \Phi_{\mathcal{A}}$$

1.14.1Lemma: For any Fs-subset $\mathcal{B}= (B_1, B, \bar{B}(\mu_{1B_1}, \mu_{2B}), L_A)$ and $\mathcal{B} \subseteq \mathcal{B}_i = (B_{1i}, B_i, \bar{B}_i(\mu_{1B_{1i}}, \mu_{2B_i}), L_{B_i})$

1.15 Proposition: $(\mathcal{L}(\mathcal{A}), \cap)$ is ∧-complete lattics.

1.15.1 Corollary: For any Fs-subset $\mathcal{B}$ of $\mathcal{A}$, the following results are true

(i) $\Phi_{\mathcal{A}} \cup \mathcal{B} = \mathcal{B}$

(ii) $\Phi_{\mathcal{A}} \cap \mathcal{B} = \Phi_{\mathcal{A}}$.

1.16 Proposition:$(\mathcal{L}(\mathcal{A}), \cup)$ is ∨-complete lattics.

1.16.1 Corollary: $(\mathcal{L}(\mathcal{A}), \cup, \cap)$ is a complete lattice with∨and∧

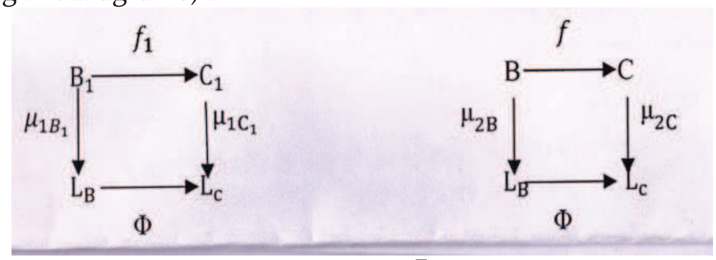
1.17 Proposition: Let $\mathcal{B}=(B_1, B, \bar{B}(\mu_{1B_1}, \mu_{2B}), L_B)$, $\mathcal{C}=(C_1, C, \bar{C}(\mu_{1C_1}, \mu_{2C}), L_C)$and $\mathcal{D}=(D_1, D, \bar{D}(\mu_{1D_1}, \mu_{2D}), L_D)$.Then $\mathcal{B} \cup (\mathcal{C} \cap \mathcal{D})=(\mathcal{B} \cup \mathcal{C}) \cap (\mathcal{B} \cup \mathcal{D})$ provided $\mathcal{C} \cap \mathcal{D}$ exists.

1.18 Proposition: Let $\mathcal{B}=(B_1,B,\bar{B}(\mu_{1B_1},\mu_{2B}),L_B)$, $\mathcal{C}=(C_1,C,\bar{C}(\mu_{1C_1},\mu_{2C}),L_C)$and $\mathcal{D}=(D_1,D,\bar{D}(\mu_{1D_1},\mu_{2D}),L_D)$.Then $\mathcal{B}\cap(\mathcal{C}\cup\mathcal{D})=(\mathcal{B}\cap\mathcal{C})\cup(\mathcal{B}\cap\mathcal{D})$ provided in R.H.S
$(\mathcal{B}\cap\mathcal{C})$ and $(\mathcal{B}\cap\mathcal{D})$ exist.

THEORY OF FS-FUNCTIONS

2.1 Fs-Function

A Triplet (f_1, f, Φ) is said to be is an Fs-Function between two given Fs-subsets $\mathcal{B}=\left(B_1,B,\bar{B}\left(\mu_{1B_1},\mu_{2B}\right),L_B\right)$ and $\mathcal{C}=\left(C_1,C,\bar{C}\left(\mu_{1C_1},\mu_{2C}\right),L_C\right)$ of $\mathcal{A}$, denoted by $(f_1,f,\Phi):\mathcal{B}=\left(B_1,B,\bar{B}\left(\mu_{1B_1},\mu_{2B}\right),L_B\right)\longrightarrow\mathcal{C}=\left(C_1,C,\bar{C}\left(\mu_{1C_1},\mu_{2C}\right),L_C\right)$ if, and only if (using the diagrams) .



(Fig-1:Fs-function $\bar{f}:\mathcal{B}\to\mathcal{C}$)

(1a) $f_1|_B = f$ is onto
(1b) $\Phi: L_B \longrightarrow L_c$ is complete homomorphism
In general (f_1, f, Φ) is denoted by $\bar{f}$

2.2 Proposition: (i) $\mu_{1C_1}|_C \circ f_1|_B \geq \mu_{2C}\circ f$
(ii) $\Phi\circ\mu_{1B_1}|_B \geq \Phi\circ\mu_{2B}$

2.2.1 Remark: Φ is a complete homomorphism between complete Boolean algebras implies $\Phi(0)=0$ and $\Phi(1)=1$ and$[\Phi(a)]^c=\Phi(a^c)$
Therefore $\Phi(a)\wedge\Phi(a^c)=\Phi(a\wedge a^c)=\Phi(0)=0$
$\Phi(a)\vee\Phi(a^c)=\Phi(a\vee a^c)=\Phi(1)=1$
2.3 Def: Increasing Fs-function:$\bar{f}$ is said to be an increasing Fs- function, and denoted by $\bar{f}_i$ if ,and only if(using fig-1)
(2a) $\mu_{1C_1}|_C\circ f_1|_B \geq \Phi\circ\mu_{1B_1}$
(2b) $\mu_{2C}\circ f \leq \Phi\circ\mu_{2B}$
2.4 Proposition: $\Phi\circ(\mu_{2B}x)^c=[(\Phi\circ\mu_{2B})x]^c$
Proof: LHS: $\Phi\circ(\mu_{2B}x)^c=\Phi\left[(\mu_{2B}x)^c\right]=[\Phi(\mu_{2B}x)]^c=[(\Phi\circ\mu_{2B})x]^c$
2.5 Proposition: $\Phi\circ\bar{B}\leq\bar{C}\circ f$,provided $\bar{f}$ is an increasing Fs-function
2.6 Def: Decreasing Fs-function:$\bar{f}$ is said to be decreasing Fs-function denoted as $\bar{f}_d$ and if and only if

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(3a) $\mu_{1C_1}|_C \circ f_1|_B \leq \Phi \circ \mu_{1B_1}$

(3b) $\mu_{2C} \circ f \geq \Phi \circ \mu_{2B}$

2.7 Proposition: $\Phi \circ \overline{B} \geq \overline{C} \circ f$,provided $\overline{f}$ is a decreasing Fs-function

2.8 Def: Preserving Fs- function: $\overline{f}$ is said to be preserving Fs-function and denoted as $\overline{f}_p$ if ,and only if

(4a) $\mu_{1C_1}|_C \circ f_1|_B = \Phi \circ \mu_{1B_1}$

(4b) $\mu_{2C} \circ f = \Phi \circ \mu_{2B}$

2.9 Proposition: $\Phi \circ \overline{B} = \overline{C} \circ f$,provided $\overline{f}$ is Fs- preserving function

2.10 Def: Composition of two Fs-function: Given two Fs-functions $\overline{f}: \mathcal{B} \rightarrow \mathcal{C}$ and $\overline{g}: \mathcal{C} \rightarrow \mathcal{D}$.We denote composition of $\overline{g}$ and $\overline{f}$ as $\overline{g} \circ \overline{f}$ and define as $(\overline{g} \circ \overline{f}) = (g_1, g, \Psi) \circ (f_1, f, \Phi) = [g_1 \circ f_1, g \circ f, \Psi \circ \Phi]$

2.11 Proposition: Composition of two increasing Fs-function are increasing.

2.12 Proposition: Composition of two decreasing Fs-function are decreasing.

2.13 Proposition: Composition of two preserving Fs-function are preserving.

2.13.1 Remark: (f_1, f, Φ) is preserving if, and only if (f_1, f, Φ) simultaneously both increasing and decreasing

2.14 Proposition: The class of all Fs-sets as objects together with morphism sets Fs-functions under the partial operation denoted by $\circ$ is called composition between Fs-functions whenever it exists is a category denoted by $\mathbb{F}$s-SET

Where $(g_1, g, \Psi) \circ (f_1, f, \Phi) = (g_1 \circ f_1, g \circ f, \Psi \circ \Phi)$

2.15 Proposition: The class of all Fs-sets as objects together with morphism sets increasing Fs-functions under the partial operation denoted by $\circ$ is called composition between increasing Fs-functions whenever it exists is a category denoted by $\mathbb{F}$s-SET_i

2.16 Proposition: The class of all Fs-sets as objects together with morphism sets decreasing Fs-functions under the partial operation denoted by $\circ$ is called composition between decreasing Fs-functions whenever it exists is a category denoted by $\mathbb{F}$s-SET_d

2.17 Proposition: The class of all Fs-sets as objects together with morphism sets preserving Fs-functions under the partial operation denoted by $\circ$ is called composition between preserving Fs-functions whenever it exists is a category denoted by $\mathbb{F}$s-SET_p

IMAGES OF FS-SUBSETS UNDER A GIVEN FS-FUNCTION

3.1 Definition Let $\mathcal{D} \subseteq \mathcal{B}$ and $\overline{f}: \mathcal{B} \longrightarrow \mathcal{C}$ be an Fs-function, where $\mathcal{B} = \left(B_1, B, \overline{B}\left(\mu_{1B_1}, \mu_{2B}\right), L_B\right), \mathcal{C} = \left(C_1, C, \overline{C}\left(\mu_{1C_1}, \mu_{2C}\right), L_C\right), \mathcal{D} = \left(D_1, D, \overline{D}\left(\mu_{1D_1}, \mu_{2D}\right), L_D\right)$.

Define $\overline{f}(\mathcal{D})$ as follows

$\overline{f}(\mathcal{D}) = \mathcal{E} = \left(E_1, E, \overline{E}\left(\mu_{1E_1}, \mu_{2E}\right) L_E\right)$,

where

(a) $E_1 = f_1(D_1)$

(b) $E = f_1(D)$

(c) $\mu_{1E_1}: E_1 \longrightarrow L_C$ is defined by

$$\mu_{1E_1}y = \begin{cases} \mu_{2C}y \vee \left[\mu_{1C_1}y \wedge \left(\bigvee_{\substack{y=f_1x \\ x\in D_1}} \Phi\mu_{1D_1}x\right)\right], \text{if } y \in C \\ \mu_{1C_1}y \wedge \left(\bigvee_{\substack{y=f_1x \\ x\in D_1}} \Phi\mu_{1D_1}x\right), \text{if } y \notin C \end{cases}$$

(d) $\mu_{2E}: E \longrightarrow L_C$ is defined by

$$\mu_{2E}y = \begin{cases} \mu_{2C}y \vee \left[\mu_{1C_1}y \wedge \left(\bigvee_{\substack{y=f_1x \\ x\in D}} \Phi\mu_{2D}x\right)\right], \text{if } y \in C \\ \mu_{1C_1}y \wedge \left(\bigvee_{\substack{y=f_1x \\ x\in D}} \Phi\mu_{2D}x\right), \text{if } y \notin C \end{cases}$$

(e) $L_E = \left(\left[\mu_{1E_1}(E_1)\right]\right)$ =The complete subalgebra generated by $\left[\mu_{1E_1}(E_1)\right]$, where $\left[\mu_{1E_1}(E_1)\right]$ = The complete ideal generated by $\mu_{1E_1}(E_1)$

3.2 Remark Observe that $\mu_{1E_1}y, \mu_{2E}y$ and $\overline{E}y \in L_E$ for any $y \in C$ or $y \in E_1 - C$ where $L_E = \left(\left[\mu_{1E_1}(E_1)\right]\right)$

3.3 Proposition

(1) $E_1 \subseteq C_1$

(2) $E \supseteq C$

(3) $L_E \leq L_C$.

(4) $\mu_{1E_1}y \in L_E$, for each $y \in E_1$

(5) $\mu_{1E_1}y \leq \mu_{1C_1}y$, for each $y \in E_1$

(6) $\mu_{1E_1}y \geq \mu_{2E}y$, for each $y \in E$

(7) $\mu_{2E}y \geq \mu_{2C}y$, for each $y \in C$

3.4 Propositions $\overline{f}(\mathcal{D})$ is an Fs-subset of $\mathcal{C} = \left(C_1, C, \overline{C}\left(\mu_{1C_1}, \mu_{2C}\right), L_C\right)$

Proof follows from proposition 3.3

3.5 Proposition Let $\mathcal{B}$ and $\mathcal{C}$ be any pair of Fs-subsets and $\overline{f}: \mathcal{B} \longrightarrow \mathcal{C}$ be an Fs-function. Let $\mathcal{H}_1$ and $\mathcal{H}_2$ be Fs-subsets $\mathcal{B}$ such that $\mathcal{H}_1 \subseteq \mathcal{H}_2$, then $\overline{f}(\mathcal{H}_1) \subseteq \overline{f}(\mathcal{H}_2)$

Proof Let $\mathcal{B} = \left(B_1, B, \overline{B}\left(\mu_{1B_1}, \mu_{2B}\right), L_B\right), \mathcal{C} = \left(C_1, C, \overline{C}\left(\mu_{1C_1}, \mu_{2C}\right), L_C\right)$,

$\mathcal{H}_1 = \left(H_{11}, H_1, \overline{H}_1\left(\mu_{1H_{11}}, \mu_{2H_1}\right), L_{H_1}\right)$ and $\mathcal{H}_2 = \left(H_{12}, H_2, \overline{H}_2\left(\mu_{1H_{12}}, \mu_{2H_2}\right), L_{H_2}\right)$

Suppose $\bar{f}(\mathcal{H}_1) = \mathcal{G}_1 = \left(G_{11}, G_1, \overline{G}_1\left(\mu_{1G_{11}}, \mu_{2G_1}\right) L_{G_1}\right)$ and $\bar{f}(\mathcal{H}_2) = \mathcal{G}_2 = \left(G_{12}, G_2, \overline{G}_2\left(\mu_{1G_{12}}, \mu_{2G_2}\right), L_{G_2}\right)$,
where
(a) $G_{11} = f_1(H_{11})$
(b) $G_1 = f_1(H_1)$
(c) $L_{G_1} = \left(\left[\mu_{1G_{11}}(G_{11})\right]\right)$
(d) $\mu_{1G_{11}}: G_{11} \longrightarrow L_{G_1}$ is defined by

$$\mu_{1G_{11}}y = \begin{cases} \mu_{2C}y \vee \left[\mu_{1C_1}y \wedge \left(\bigvee_{\substack{y=f_1x \\ x\in H_{11}}} \Phi\mu_{1H_{11}}x\right)\right], \text{if } y \in C \\ \mu_{1C_1}y \wedge \left(\bigvee_{\substack{y=f_1x \\ x\in H_{11}}} \Phi\mu_{1H_{11}}x\right), \text{if } y \notin C \end{cases}$$

(e) $\mu_{2G_1}: G_1 \longrightarrow L_{G_1}$ is defined

$$\text{by } \mu_{2G_1}y = \begin{cases} \mu_{2C}y \vee \left[\mu_{1C_1}y \wedge \left(\bigvee_{\substack{y=f_1x \\ x\in H_1}} \Phi\mu_{2H_1}x\right)\right], \text{if } y \in C \\ \mu_{1C_1}y \wedge \left(\bigvee_{\substack{y=f_1x \\ x\in H_1}} \Phi\mu_{2H_1}x\right), \text{if } y \notin C \end{cases}$$

(f) $G_{12} = f_1(H_{12})$
(g) $G_2 = f_1(H_2)$
(h) $L_{G_2} = \left(\left[\mu_{1G_{12}}(G_{12})\right]\right)$
(i) $\mu_{1G_{12}}: G_{12} \longrightarrow L_{G_2}$ is defined by

$$\mu_{1G_{12}}y = \begin{cases} \mu_{2C}y \vee \left[\mu_{1C_1}y \wedge \left(\bigvee_{\substack{y=f_1x \\ x\in H_{12}}} \Phi\mu_{1H_{12}}x\right)\right], \text{if } y \in C \\ \mu_{1C_1}y \wedge \left(\bigvee_{\substack{y=f_1x \\ x\in H_{12}}} \Phi\mu_{1H_{12}}x\right), \text{if } y \notin C \end{cases}$$

(j) $\mu_{2G_2}: G_2 \longrightarrow L_{G_2}$ is defined by

$$\mu_{2G_2}y = \begin{cases} \mu_{2C}y \vee \left[\mu_{1C_1}y \wedge \left(\bigvee_{\substack{y=f_1x \\ x\in H_2}} \Phi\mu_{2H_2}x\right)\right], \text{if } y \in C \\ \mu_{1C_1}y \wedge \left(\bigvee_{\substack{y=f_1x \\ x\in H_2}} \Phi\mu_{2H_2}x\right), \text{if } y \notin C \end{cases}$$

$\mathcal{H}_1 \subseteq \mathcal{H}_2$ imply
(k) $H_{11} \subseteq H_{12}, H_1 \supseteq H_2$
(l) $L_{H_1} \leq L_{H_2}$
(m) $\mu_{1H_{11}}x \leq \mu_{1H_{12}}x, \forall x \in H_{11}$ and $\mu_{2H_1}x \geq \mu_{2H_2}x, \forall x \in H_2$
Need to show $\mathcal{G}_1 \subseteq \mathcal{G}_2$ i.e. to show the following:

(n) $G_{11} \subseteq G_{12}$, $G_2 \subseteq G_1$

(o) $\mu_{1G_{11}} \leq \mu_{1G_{12}}|G_{11}$, and $\mu_{2G_1}|G_2 \geq \mu_{2G_1}$

(p) $L_{G_1} \leq L_{G_2}$

Proof of (n) (k) implies

$$\left.\begin{array}{l} f_1(H_{11}) \subseteq f_1(H_{12}) \text{ or } G_{11} \subseteq G_{12} \text{ and} \\ f_1(H_1) \supseteq f_1(H_2) \text{ or } G_1 \supseteq G_2 \end{array}\right\} \quad \ldots (I)$$

Proof of (o) From (m)

$\bigvee_{\substack{y=f_1x \\ x\in H_{11}}} \Phi\mu_{1H_{11}}x \leq \bigvee_{\substack{y=f_1x \\ x\in H_{12}}} \Phi\mu_{1H_{12}}x$ also

Case:1: For $y \in C$

$$\mu_{1C_1}y \wedge \left(\bigvee_{\substack{y=f_1x \\ x\in H_{11}}} \Phi\mu_{1H_{11}}x\right) \leq \mu_{1C_1}y \wedge \left(\bigvee_{\substack{y=f_1x \\ x\in H_{12}}} \Phi\mu_{1H_{12}}x\right)$$

$$\Rightarrow \mu_{2C}y \vee \left[\mu_{1C_1}y \wedge \left(\bigvee_{\substack{y=f_1x \\ x\in H_{11}}} \Phi\mu_{1H_{11}}x\right)\right] \leq \mu_{2C}y \vee \left[\mu_{1C_1}y \wedge \left(\bigvee_{\substack{y=f_1x \\ x\in H_{12}}} \Phi\mu_{1H_{12}}x\right)\right]$$

i.e. $\mu_{1G_{11}}y \leq \mu_{1G_{12}}y$

Case:2: For $y \notin C$,

$$\mu_{1C_1}y \wedge \left(\bigvee_{\substack{y=f_1x \\ x\in H_{11}}} \Phi\mu_{1H_{11}}x\right) \leq \mu_{1C_1}y \wedge \left(\bigvee_{\substack{y=f_1x \\ x\in H_{12}}} \Phi\mu_{1H_{12}}x\right)$$

i.e. $\mu_{1G_{11}}y \leq \mu_{1G_{12}}y$

From Case 1 and Case 2 for each $y \in G_{11}$ we can conclude that,

$\mu_{1G_{11}}y \leq \mu_{1G_{12}}y$ for $y \in C$ or $y \notin C$ ... (II)

Again From (m) we have

$\bigvee_{\substack{y=f_1x \\ x\in H_1}} \Phi\mu_{2H_1}x \geq \bigvee_{\substack{y=f_1x \\ x\in H_2}} \Phi\mu_{2H_2}x$ also

Case:3: For $y \in C$

$$\mu_{1C_1}y \wedge \left(\bigvee_{\substack{y=f_1x \\ x\in H_1}} \Phi\mu_{2H_1}x\right) \geq \mu_{1C_1}y \wedge \left(\bigvee_{\substack{y=f_1x \\ x\in H_2}} \Phi\mu_{2H_2}x\right)$$

$$\Rightarrow \mu_{2C}y \vee \left[\mu_{1C_1}y \wedge \left(\bigvee_{\substack{y=f_1x \\ x\in H_1}} \Phi\mu_{2H_1}x\right)\right] \geq \mu_{2C}y \vee \left[\mu_{1C_1}y \wedge \left(\bigvee_{\substack{y=f_1x \\ x\in H_2}} \Phi\mu_{2H_2}x\right)\right]$$

i.e. $\mu_{2G_1}y \geq \mu_{2G_2}y$

Case:4: For $y \notin C$,

$$\mu_{1C_1}y \wedge \left(\bigvee_{\substack{y=f_1x \\ x\in H_1}} \Phi\mu_{2H_1}x\right) \geq \mu_{1C_1}y \wedge \left(\bigvee_{\substack{y=f_1x \\ x\in H_2}} \Phi\mu_{2H_2}x\right)$$

i.e. $\mu_{2G_1}y \geq \mu_{2G_2}y$

From Case 3 and Case 4 for each $y \in G_2$ we can conclude that,

$\mu_{2G_1}y \geq \mu_{2G_2}y$ for $y \in C$ or $y \notin C$ ... (III)
Proof of (p) for $y \in G_{11}$ from (2) we have
$\mu_{1G_{11}}y \leq \mu_{1G_{12}}y$, so that
$\mu_{1G_{11}}y \in \left[\mu_{1G_{12}}(G_{12})\right]$ i.e

$$\mu_{1G_{11}}(G_{11}) \subseteq \left[\mu_{1G_{12}}(G_{12})\right]$$

$$\Rightarrow \left[\mu_{1G_{11}}(G_{11})\right] \subseteq \left[\mu_{1G_{12}}(G_{12})\right]$$

$$\Rightarrow \left(\left[\mu_{1G_{11}}(G_{11})\right]\right) \leq \left(\left[\mu_{1G_{12}}(G_{12})\right]\right) \text{ i.e. } L_{G_1} \leq L_{G_2} \quad \text{... (IV)}$$

From (I),(II),(III) and (IV) we can conclude that,
$\mathcal{G}_1 \subseteq \mathcal{G}_2$ i.e. $\bar{f}(\mathcal{H}_1) \subseteq \bar{f}(\mathcal{H}_2)$.

Image of Fs-empty set of first kind under an Fs-function.

3.6 Definition

Let $\Phi_{\mathcal{A}} = \mathcal{X} = \left(X_1, X, \bar{X}\left(\mu_{1X_1}, \mu_{2X}\right), L_X\right)$, where

(1) $A \subseteq X_1 \cap X$ and $X_1 \not\supseteq X$ or

(2) $\mu_{1D_1}x \ngeq \mu_{2D}x$,for $x \in X_1 \cap X$

We define $\bar{f}(\Phi_{\mathcal{A}}) = \Phi_{\mathcal{A}}$.

3.7 Result $\bar{f}(\Phi_{\mathcal{A}}) = \Phi_{\mathcal{A}}$, where $\Phi_{\mathcal{A}} = \mathcal{D} = \left(D, D, \bar{D}\left(\mu_{1D_1}, \mu_{2D}\right), L_D\right)$,where $D_1 = D$ and $\Phi_{\mathcal{A}}$ is Fs-empty set of second

3.8 Proposition For any Fs-function $\bar{f}: \mathcal{B} \to \mathcal{C}$, $\bar{f}(\Phi_{\mathcal{A}}) = \Phi_{\mathcal{A}}$ where $\Phi_{\mathcal{A}}$ is Fs-empty set of first or Fs-empty set of second kind.

3.9 Proposition For any Fs-function $\bar{f}: \mathcal{B} \to \mathcal{C}$ and any two Fs-subsets $\mathcal{H}_1$ and $\mathcal{H}_2$ of $\mathcal{B}$, the following are true.

(1) $\bar{f}(\mathcal{H}_1 \cup \mathcal{H}_2) \supseteq \bar{f}(\mathcal{H}_1) \cup \bar{f}(\mathcal{H}_2)$

(2) $\bar{f}(\mathcal{H}_1 \cap \mathcal{H}_2) \subseteq \bar{f}(\mathcal{H}_1) \cap \bar{f}(\mathcal{H}_2)$

3.10 Proposition: For any Fs-function $\bar{f}: \mathcal{B} \to \mathcal{C}$ and any family of Fs-subsets $\mathcal{H}_i$, $i \in I$ of $\mathcal{B}$ the following are true.

3.11 Proposition For any Fs-function $\bar{f}: \mathcal{B} \to \mathcal{C}$ and any two Fs-subsets $\mathcal{H}_1$ and $\mathcal{H}_2$ of $\mathcal{B}$,
$\bar{f}(\mathcal{H}_1 \cup \mathcal{H}_2)$ and $\bar{f}(\mathcal{H}_1) \cup \bar{f}(\mathcal{H}_2)$ are 3-equal

3.12 Proposition For any Fs-function $\bar{f}: \mathcal{B} \to \mathcal{C}$ such that f_1 is one-one and for any two Fs-subsets $\mathcal{H}_1$ and $\mathcal{H}_2$ of $\mathcal{B}$, $\bar{f}(\mathcal{H}_1 \cup \mathcal{H}_2)$ and $\bar{f}(\mathcal{H}_1) \cup \bar{f}(\mathcal{H}_2)$ are full-equal

3.13 Proposition For any Fs-function $\bar{f}: \mathcal{B} \to \mathcal{C}$ and any family of Fs-subsets $\mathcal{H}_i$, $i \in I$ of $\mathcal{B}$,$\bar{f}(\bigcup_{i \in I} \mathcal{H}_i)$ and $\bigcup_{i \in I} \bar{f}(\mathcal{H}_i)$ are 3-equal

3.14 Proposition For any Fs-function $\bar{f}: \mathcal{B} \to \mathcal{C}$ such that f_1 is one-one and any family of Fs-subsets $\mathcal{H}_i$, $i \in I$ of $\mathcal{B}$, $\bar{f}(\bigcup_{i \in I} \mathcal{H}_i)$ and $\bigcup_{i \in I} \bar{f}(\mathcal{H}_i)$ are full-equal

3.15 Result If $\overline{f}$ is increasing Fs-function, $\mathcal{D}\subseteq\mathcal{B}$ and $\overline{f}_i(\mathcal{D}) = \mathcal{E} = \left(E_1, E, \overline{E}\left(\mu_{1E_1}, \mu_{2E}\right), L_E\right)$ then $\mu_{1E_1}y = \bigvee_{\substack{y=f_1x\\x\in D_1}} \Phi\mu_{1D_1}x$ and $\mu_{2E}y = \bigvee_{\substack{y=f_1x\\x\in D}} \Phi\mu_{2D}x$.

3.15 Result If $\overline{f}$ is decreasing Fs-function, $\mathcal{D}\subseteq\mathcal{B}$ and $\overline{f}_i(\mathcal{D}) = \mathcal{E} = \left(E_1, E, \overline{E}\left(\mu_{1E_1}, \mu_{2E}\right), L_E\right)$ then

$$\mu_{1E_1}y = \begin{cases} \mu_{2C}y\vee\left[\mu_{1C_1}y\wedge\left(\bigvee_{\substack{y=f_1x\\x\in D_1}}\Phi\mu_{1D_1}x\right)\right], \text{if } y\in C \\ \mu_{1C_1}y\wedge\left(\bigvee_{\substack{y=f_1x\\x\in D_1}}\Phi\mu_{1D_1}x\right), \text{if } y\notin C \end{cases} \quad \text{and}$$

$$\mu_{2E}y = \begin{cases} \mu_{2C}y\vee\left[\mu_{1C_1}y\wedge\left(\bigvee_{\substack{y=f_1x\\x\in D}}\Phi\mu_{2D}x\right)\right], \text{if } y\in C \\ \mu_{1C_1}y\wedge\left(\bigvee_{\substack{y=f_1x\\x\in D}}\Phi\mu_{2D}x\right), \text{if } y\notin C \end{cases}$$

3.16 Result If $\overline{f}$ is preserving Fs-function, $\mathcal{D}\subseteq\mathcal{B}$ and $\overline{f}_p(\mathcal{D}) = \mathcal{E} = \left(E_1, E, \overline{E}\left(\mu_{1E_1}, \mu_{2E}\right) L_E\right)$ then $\mu_{1E_1}y = \bigvee_{\substack{y=f_1x\\x\in D_1}} \Phi\mu_{1D_1}x$ and $\mu_{2E}y = \bigvee_{\substack{y=f_1x\\x\in D}} \Phi\mu_{2D}x$.

.17 Proposition: For any pair of Fs-functions $\overline{f}_*: \mathcal{B} \to \mathcal{C}$ and $\overline{g}_*: \mathcal{C} \to \mathcal{D}$ and any Fs-subset $\mathcal{H}$ of $\mathcal{B}$, $\left(\overline{g}\circ\overline{f}\right)_*(\mathcal{H})$ and $\overline{g}_*\left(\overline{f}_*(\mathcal{H})\right)$ are full-equal whenever $* = i$ or p

Proof For $* = i$,

LHS: $\left(\overline{g}\circ\overline{f}\right)_i(\mathcal{H}) = [g_1\circ f_1, g\circ f, \Psi\circ\Phi]_i(\mathcal{H}) = \mathcal{G} = \left(G_1, G, \overline{G}\left(\mu_{1G_1,}\mu_{2G}\right), L_G\right)$ say

(1) $G_1 = (g_1\circ f_1)(H_1)$

(2) $G = (g_1\circ f_1)(H)$

(3) $L_G = \left(\left[\mu_{1G_1}(G_1)\right]\right) = \left(\left[\mu_{1G_1}((g_1\circ f_1)(H_1))\right]\right)$

(4) $\mu_{1G_1}: G_1 \longrightarrow L_G$ is defined by $\mu_{1G_1}z = \bigvee_{\substack{z=(g_1\circ f_1)x\\x\in H_1}} \Psi\Phi\mu_{1H_1}x$

(5) $\mu_{2G}: G \longrightarrow L_G$ is defined by $\mu_{2G}z = \bigvee_{\substack{z=(g_1\circ f_1)x\\x\in H}} \Psi\Phi\mu_{2H}x$

Let $\overline{f}_i(\mathcal{H}) = \mathcal{K} = \left(K_1, K, \overline{K}\left(\mu_{1K_1,}\mu_{2K}\right), L_K\right)$, where

(6) $K_1 = f_1(H_1)$

(7) $K = f_1(H)$

(8) $L_K = \left(\left[\mu_{1K_1}(K_1)\right]\right)$

(9) $\mu_{1K_1}: K_1 \longrightarrow L_K$ is defined by $\mu_{1K_1}y = \bigvee_{\substack{y=f_1x\\x\in H_1}} \Phi\mu_{1H_1}x$

(10) $\mu_{2K}: K \longrightarrow L_K$ is defined by $\mu_{2K} y = \bigvee_{\substack{y=f_1x \\ x\in H}} \Phi\mu_{2H} x$

RHS: $\bar{g}_i\left(\bar{f}_i(\mathcal{H})\right) = \bar{g}_i(\mathcal{K}) = \mathcal{M} = \left(M_1, M, \bar{M}\left(\mu_{1M_1}, \mu_{2M}\right), L_M\right)$ say

(11) $M_1 = g_1(K_1) = g_1\left(f_1(H_1)\right) = (g_1 \circ f_1)(H_1)$

(12) $M = g_1(K) = g_1\left(f_1(H)\right) = (g_1 \circ f_1)(H)$

(13) $L_M = \left(\left[\mu_{1M_1}(M_1)\right]\right) = \left(\left[\mu_{1M_1}\left((g_1 \circ f_1)(H_1)\right)\right]\right)$

(14) $\mu_{1M_1}: M_1 \longrightarrow L_M$ is defined by $\mu_{1M_1} z = \bigvee_{\substack{z=g_1y \\ y\in K_1}} \Psi\mu_{1K_1} y$ =

$\bigvee_{\substack{z=g_1y \\ y\in K_1}} \Psi\left(\bigvee_{\substack{y=f_1x \\ x\in H_1}} \Phi\mu_{1H_1} x\right)$

$= \bigvee_{\substack{z=g_1f_1x \\ x\in H_1}} \Psi\Phi\mu_{1H_1} x$

$= \bigvee_{\substack{z=(g_1\circ f_1)x \\ x\in H_1}} \Psi\Phi\mu_{1H_1} x$

(15) $\mu_{2M}: M \longrightarrow L_M$ is defined by $\mu_{2M} z = \bigvee_{\substack{z=g_1y \\ y\in K}} \Psi\mu_{2K} y$

$= \bigvee_{\substack{z=g_1y \\ y\in K}} \Psi\left(\bigvee_{\substack{y=f_1x \\ x\in H}} \Phi\mu_{2H} x\right)$

$= \bigvee_{\substack{z=g_1f_1x \\ x\in H}} \Psi\Phi\mu_{2H} x = \bigvee_{\substack{z=(g_1\circ f_1)x \\ x\in H}} \Psi\Phi\mu_{2H} x$

Need to prove $\mathcal{G} = \left(\bar{g} \circ \bar{f}\right)_i(\mathcal{H})$ and $\mathcal{M} = \bar{g}_i\left(\bar{f}_i(\mathcal{H})\right)$ are full-equal i.e.

(16) $G_1 = M_1$

(17) $G = M$

(18) $\mu_{1G_1} = \mu_{1M_1}$

(19) $\mu_{2G} = \mu_{2M}$

(20) $L_G = L_M$

Proof of (16) follows from (1) and(11)

Proof of (17) follows from (2) and(12)

Proof of (18) follows from (4) and(14)

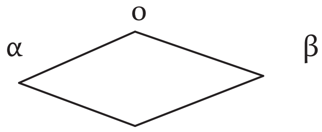
Proof of (19) follows from (5) and(115)

Proof of (20) follows from (3) ,(13) and $\mu_{1G_1} = \mu_{1M_1}$

For $*= p$ the proof is obvious.

In General $(\bar{g} \circ \bar{f})(\mathcal{H})$ and $\bar{g}\left(\bar{f}(\mathcal{H})\right)$ Are Different

3.17.1 Example Let $\bar{f}: \mathcal{B} \to \mathcal{C}$ and $\bar{g}: \mathcal{C} \to \mathcal{D}$ be a pair Fs-function, where



1 (fig-4)

$\mathcal{B}=\left(B_1, B, \bar{B}\left(\mu_{1B_1}, \mu_{2B}\right), L_B\right)$

$B_1 = \{a, b\}, B = \{b\}$, $L_B = \{0, \| \beta, 1\}$

$\mu_{1B_1}: B_1 \longrightarrow L_B$ is given by $\mu_{1B_1} = 1$

$\mu_{2B}: B \longrightarrow L_B$ is given by $\mu_{2B} = 0$

$\mathcal{C}=\left(C_1, C, \bar{C}\left(\mu_{1C_1}, \mu_{2C}\right), L_C\right)$ where $C_1 = \{a, b\}, C = \{a\}, L_C = L_B$

$\mu_{1C_1}: C_1 \longrightarrow L_C$ is given by $\mu_{1C_1} = 0$

$\mu_{2C}: C \longrightarrow L_C$ is given by $\mu_{2C} = 0$

$f_1: B_1 \longrightarrow C_1$ is given by $f_1 a = b, f_1 b = a$

$f: B \longrightarrow C$ is given by $fb = a$

$\Phi: L_B \longrightarrow L_C$ is given by $\Phi(0) = 0, \Phi(\alpha) = \beta, \Phi(\beta) = \alpha, \Phi(1) = 1$

$\mathcal{D} = \left(D_1, D, \bar{D}\left(\mu_{1D_1}, \mu_{2D}\right), L_D\right)$ where

$D_1 = \{a, b\}, D = \{b\}$,

$L_D = L_B$

$\mu_{1D_1}: D_1 \longrightarrow L_D$ is given by $\mu_{1D_1} = \alpha$

$\mu_{2D}: D \longrightarrow L_D$ is given by $\mu_{2D} = 0$

$g_1: C_1 \longrightarrow D_1$ is given by $g_1 a = b, g_1 b = a$

$g: C \longrightarrow D$ is given by $gb = a$

$\Psi: L_C \longrightarrow L_D$ is given by $\psi(0) = 0, \Psi(\alpha) = \beta, \Psi(\beta) = \alpha, \Psi(1) = 1$

$\mathcal{D} = \left(H_1, H, \bar{H}\left(\mu_{1H_1}, \mu_{2H}\right), L_H\right)$ where $H_1 = \{b\}, H = \{b\}, L_H = L_B$

$\mu_{1H_1}: H_1 \longrightarrow L_H$ is given by $\mu_{1H_1} = \alpha$

$\mu_{2H}: H \longrightarrow L_H$ is given by $\mu_{2H} = 0$

$(\bar{g} \circ \bar{f})(\mathcal{H}) = [g_1 \circ f_1, g \circ f, \Psi \circ \Phi](\mathcal{H}) = \mathcal{G} = \left(G_1, G, \bar{G}\left(\mu_{1G_1}, \mu_{2G}\right), L_G\right)$ say

(a) $G_1 = (g_1 \circ f_1)(H_1) = (g_1 \circ f_1)(\{b\}) = g_1\big(f_1(\{b\})\big) = g_1(\{f_1 b\}) = g_1(\{a\}) = \{g_1(a)\} = \{b\}$

(b) $G = (g_1 \circ f_1)(H) = (g_1 \circ f_1)(\{b\}) = g_1\big(f_1(\{b\})\big) = g_1(\{f_1 b\}) = g_1(\{a\}) = \{g_1(a)\} = \{b\}$

(c) $L_G = \left(\left[\mu_{1G_1}(G_1)\right]\right) = L_B$

(d) $\mu_{1G_1}: G_1 \longrightarrow L_G$ is defined by

$$\mu_{1G_1} b = \mu_{2D} b \vee \left[\mu_{1D_1} b \wedge \left(\vee_{\substack{b=(g_1 \circ f_1)x \\ x \in H_1}} \Psi\Phi\mu_{1H_1} x\right)\right] = 0 \vee [\alpha \wedge \alpha] = \alpha$$

(e) $\mu_{2G}: G \longrightarrow L_G$ is defined by $\mu_{2G}z = \mu_{2D}b \vee \left[\mu_{1D_1}b \wedge \left(\vee_{\substack{b=(g_1 \circ f_1)x \\ x \in H_1}} \Psi\Phi\mu_{2H}x\right)\right] =$
$0 \vee [\alpha \wedge 0] = 0$

Let $\bar{f}(\mathcal{H}) = \mathcal{K} = \left(K_1, K, \bar{K}\left(\mu_{1K_1,}\mu_{2K}\right), L_K\right)$, where

(a) $K_1 = f_1(H_1) = f_1(\{b\}) = \{f_1 b\} = \{a\}$

(b) $K = f_1(H) = f_1(\{b\}) = \{f_1 b\} = \{a\}$

(c) $L_K = \left(\left[\mu_{1K_1}(K_1)\right]\right) = ([0]) = \{0,1\}$

(d) $\mu_{1K_1}: K_1 \longrightarrow L_K$ is defined by $\mu_{1K_1}a = \mu_{2C}a \vee \left[\mu_{1C_1}a \wedge \left(\vee_{\substack{a=f_1x \\ x \in H_1}} \Phi\mu_{1H_1}x\right)\right] =$
$0 \vee [0 \wedge (\alpha)] = 0$

(e) $\mu_{2K}: K \longrightarrow L_K$ is defined by $\mu_{2K}y == \mu_{2C}a \vee \left[\mu_{1C_1}a \wedge \left(\vee_{\substack{a=f_1x \\ x \in H_1}} \Phi\mu_{2H}x\right)\right] =$
$0 \vee [0 \wedge (0)] = 0$

Let $\bar{g}\left(\bar{f}(\mathcal{H})\right) = \bar{g}(\mathcal{K}) = \mathcal{M} = \left(M_1, M, \bar{M}\left(\mu_{1M_1,}\mu_{2M}\right), L_M\right)$ say

(f) $M_1 = g_1(K_1) = g_1\left(f_1(H_1)\right) = g_1\left(f_1(\{b\})\right) = g_1(\{f_1 b\}) = g_1(\{a\}) = \{g_1(a)\} =$
$\{b\}$

(g) $M = g_1(K) = g_1\left(f_1(H)\right) = g_1\left(f_1(\{b\})\right) = g_1(\{f_1 b\}) = g_1(\{a\}) = \{g_1(a)\} = \{b\}$

(f) $L_M = \left(\left[\mu_{1M_1}(M_1)\right]\right) = ([0]) = \{0,1\}$

(g) $\mu_{1M_1}: M_1 \longrightarrow L_M$ is defined by $\mu_{1M_1}b = \mu_{2D}b \vee \left[\mu_{1D_1}b \wedge \left(\vee_{\substack{z=g_1y \\ y \in K_1}} \Psi\mu_{1K_1}y\right)\right] =$
$0 \vee [\alpha \wedge 0] = 0$

(h) $\mu_{2M}: M \longrightarrow L_M$ is defined by $\mu_{2M}b = \mu_{2D}b \vee \left[\mu_{1D_1}b \wedge \left(\vee_{\substack{b=g_1y \\ y \in K}} \Psi\mu_{2K}y\right)\right] =$
$0 \vee [\alpha \wedge 0] = 0$

$\therefore \mu_{1G_1}b = \alpha \neq \mu_{1M_1}b = 0$

Hence $\left(\bar{g} \circ \bar{f}\right)(\mathcal{H}) \neq \bar{g}\left(\bar{f}(\mathcal{H})\right)$.

Conclusion: We can observe that similarities between results in theory of Fs-functions and some results in the theory of crisp functions .

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