STEADY STATE SOLUTION OF SERIAL QUEUING PROCESS WITH RENEGING AND FEEDBACK

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Abstract: The present paper provides steady state analysis of general queuing model in which a serial queuing network consisting of two sets of parallel service channels linked in series thorough a common service channel. The input process is Poisson and service time distribution is exponential. The concepts of reneging and feedback are also incorporated. Queue discipline follows FIFO (first in first out). Waiting space is infinite in the present queuing model. For this queuing model, formulas for mean queue length and steady state marginal probabilities have been derived. Numerical solutions have been discussed.

Keywords: Feedback, Numerical Solutions, Reneging, Steady State Solution.

Introduction: Queues or waiting lines are born of congestion which occurs due to some irregularities in arrival pattern or service mechanisms of a system of social and industrial activities. The pioneer work in the theory of queues was done by Erlang (1909) who studied the problem of congestion in telephone lines. Further contributions in the subject were made by Kendall(1951,1953), Cox and Smith(1961).O'Brien [7] studied the problem of two queues in series in steady state with Poisson inputs and exponential holding times. Jackson [3] solved the serial queuing problem having two or three phases for finite and infinite waiting spaces. Nelson[6] derived the probability of waiting longer than a prescribed time at all channels. Singh, Man [8] considered the Steady-state behavior of serial queuing processes with impatient customers. Vikram and Singh [9]obtained the steady state solution of serial and non serial queuing processes with reneging, balking and feedback phenomenon.

This paper studies a more general serial queuing network where two set of parallel service channels linked in series through a service channel. Here the two set of parallel channels $S_i'(i=1,2,\dots M)$ and S_2, S_3 are linked in series through a common channel S_1. Units demanding different types of service arrive at $S_i'(i=1,2,\dots M)$ and after being serviced by their respective servers either leave the system or enter the service channel S_1. Units serviced by S_1 may leave the system or join either of the service channels S_2 and S_3 which are in biseries. Units serviced by S_2 (or S_3) have also now the option to feedback to any of the servers S_i' of the first phase.

In this queuing model waiting space is considered to be infinite. The expressions for steady state marginal probabilities and mean queue length have also been derived whenever the queue discipline is first in first out.

Formulation Of Model: The problem dealt with may be stated as follows:

Units demanding different types of service arrive from outside the system in Poisson streams with parameters $\lambda_i'(i=1,2,\dots M); \lambda_1; \lambda_2; \lambda_3$ at the respective servers $S_i'(i=1,2,\dots M); S_1; S_2; S_3$ and form the queues $Q_i'(i=1,2,\dots M); Q_1; Q_2; Q_3$ respectively. After the completion of service at S_i', units either leave the system or join the queue Q_1 with respective probabilities p_i', q_i' $s.t.$ $p_i'+q_i'=1(i=1,2,\dots M)$. Now units serviced by S_1, leave the system with probability q_1 or join either of the queues Q_2 and Q_3 with respective probabilities q_2 and q_3 where $q_1+q_2+q_3=1$. Units which enter the service at S_2, after completion of service, either leave the system or join any of queues Q_3 ; $Q_i'(i=1,2,\dots M)$ with respective probabilities r_1, r_2, r_{i2} $where$ $r_1+r_2+\sum_{i=1}^{M} r_{i2}=1$.

Similarly, units which enter the service channel S_3, after completion of service, either leave the system or join any of the queues $Q_3, Q_i'(i=1,2,\dots M)$ with respective probabilities $p_1, p_2, p_{i3}(i=1,2\dots, M)$ such that $p_1+p_2+\sum_{i=1}^{M} p_{i3}=1$.

Thus we see that the concept of feedback is incorporated for units serviced by S_2 and S_3 may be directed to rejoin the queues $Q_i'(i=1,2,\dots M)$. The service time distributions for the servers $S_i'(i=1,2,\dots M); S_1; S_2; S_3$ are mutually independent negative exponential distributions with respective parameters $\mu_i'(i=1,2,\dots M); \mu_1; \mu_2; \mu_3$. Further it is assumed that units in $Q_i'(i=1,2,\dots M); Q_1; Q_2; Q_3$ may leave the system at random without getting service with mean reneging rates $\alpha_i'(i=1,2,\dots M); \alpha_1; \alpha_2; \alpha_3$ respectively.

Practical situations corresponding to the model studied in this paper are not uncommon. Consider a state health department which sets up dispensaries at village level to provide medical help. These village dispensaries correspond to the service facilities S_i' of our model. The patient visiting these dispensaries are either discharged after treatment or referred to the civil hospital where they visit the common counter (S_1) for registration etc. some of these patients are treated at the civil hospital while the others are referred back to their village dispensaries. It is also a common practice that patients may visit the civil hospital directly.

Formulation of Equations:

Define: $P(n_1, n_2, \dots n_M; m, n, r; t)$= the probability that at time t there are $n_1, n_2, \dots n_M$ units (which may renege, or after being serviced by $S_1', S_2', \dots S_M'$ respectively either leave the system or join S_1) waiting in the respective queues$Q_1' Q_2', \dots, Q_M'$; m units (which may renege, or after being serviced by S_1 leave the system or join either of S_2 and S_3) waiting in Q_1; n units (which may renege, or after being serviced by S_2 either leave the system or join any of S_3; $S_1', S_2', \dots S_M'$)in Q_2; r units (which may renege, or after being serviced by S_3 either leave the system or join any of S_2; $S_1', S_2', \dots S_M'$) waiting in Q_3 .

Probability reasoning leads to the following set of difference – differential equations:

$$\frac{d}{dt}P(n_1, n_2, \dots n_M; m, n, r; t) = -[\lambda_1 + \lambda_2 + \lambda_3 + \sum_{i=1}^{M} \lambda_i' + \sum_{i=1}^{M} \delta(n_i)(\mu_i' + \alpha_i') + \delta(m)(\mu_1 + \alpha_1) + \delta(n)(\mu_2 + \alpha_2) + \delta(r)(\mu_3 + \alpha_3)]\, P(n_1, n_2, \dots n_M; m, n, r; t) + \sum_{i=1}^{M} \lambda_i'\, P(n_1, n_2, \dots, n_i - 1, \dots n_M; m, n, r; t) + \lambda_1 P(n_1, n_2, \dots n_M; m-1, n, r; t) + \lambda_2 P(n_1, n_2, \dots n_M; m, n-1, r; t) + \lambda_3 P(n_1, n_2, \dots n_M; m, n, r-1; t) + \sum_{i=1}^{M}(\mu_i' p_i' + \alpha_i') P(n_1, n_2, \dots, n_i + 1, \dots n_M; m, n, r; t) + \sum_{i=1}^{M} \mu_i' q_i' P(n_1, n_2, \dots n_i{+}1, \dots n_M; m-1, n, r; t) + (\mu_1 q_1 + \alpha_1)\, P(n_1, n_2, \dots n_M; m+1, n, r; t) + \mu_1 q_2 P(n_1, n_2, \dots, n_M; m+1, n-1, r; t) + \mu_1 q_3 P(n_1, n_2, \dots n_M; m+1, n, r-1; t) + (r_1\mu_2 + \alpha_2) P(n_1, n_2, \dots n_M; m, n+1, r; t) + r_2\mu_2 P(n_1, n_2, \dots n_M; m, n+1, r-1; t) + \sum_{i=1}^{M} r_{i2}\mu_2\; P(n_1, n_2, \dots n_i - 1, \dots \dots . n_M; m, n+1, r; t) + (p_1\mu_3 + \alpha_3) P(n_1, n_2, \dots n_M; m, n, r+1; t) + p_2\mu_3 P(n_1, n_2, \dots n_M; m, n-1, r+1; t) + \sum_{i=1}^{M} p_{i3}\mu_3\; P(n_1, n_2, \dots n_i - 1, \dots, n_M; m, n, r+1; t) \quad \dots(1)$$

Where $n_i (i = 1,2, \dots, M); m, n, r \geq 0$ *and*

$$\delta(x) = \begin{cases} 1 & x \neq 0 \\ 0 & x = 0 \end{cases}$$

And $P(n_1, n_2, \dots n_M; m, n, r; t) = 0$ if any of arguments is negative.

Steady State Solution: The steady-state equations of the system are derived by equating the time-derivatives to zero in equation(1). The solution of the resulting equations can be verified to be

$$P(n_1, n_2, \dots n_M; m, n, r; t) = \left\{\prod_{i=1}^{M} (\rho_i')^{n_i}\right\} \rho_1{}^m \rho_2{}^n \rho_3{}^r P(0,0, \dots, 0)$$

$n_i = 1,2, \dots M; m, n, r \geq 0$ ...(2)

$$where\ \rho_i' = \frac{\lambda_i'}{\mu_i' + \alpha_i'} + \frac{N_i'}{D(\mu_i' + \alpha_i')} \quad ; i = 1,2, \dots, M$$

$$\rho_1 = \frac{\lambda_1}{(\mu_1 + \alpha_1)} + \frac{\sum_{i=1}^{M} \frac{\lambda_i' q_i' \mu_i'}{(\mu_i' + \alpha_i')}}{(\mu_1 + \alpha_1)} + \frac{N_1}{D(\mu_1 + \alpha_1)}$$

$\rho_2 = \frac{N_2}{D}; \rho_3 = \frac{N_3}{D}$; where

$$D = (\mu_2 + \alpha_2)(\mu_3 + \alpha_3) - p_2 r_2 \mu_2 \mu_3 - \left[\frac{\mu_1\mu_3 q_3(\mu_2 + \alpha_2) - \mu_1\mu_2\mu_3 r_2 q_2}{(\mu_1 + \alpha_1)}\right] \times \left[\sum_{i=1}^{M} \frac{\mu_i' q_i' r_{i2}}{(\mu_i' + \alpha_i')}\right]$$

$$N_i' = r_{i2}\mu_2[\lambda_3 p_2 \mu_3 + \lambda_2(\mu_3 + \alpha_3)] + p_{i3}\mu_3[r_2\mu_2\lambda_2 + \lambda_3(\mu_2 + \alpha_2)] + \left[\frac{r_{i2}\mu_2\mu_1 p_2 q_3 \mu_3 + \mu_1 q_2(\mu_3 + \alpha_3)}{(\mu_1 + \alpha_1)} + \frac{p_{i3}\mu_3\mu_1\mu_2 q_2 r_2 + \mu_1 q_3(\mu_2 + \alpha_2)}{(\mu_1 + \alpha_1)}\right] \times \left[\lambda_1 + \sum_{i=1}^{M} \frac{\mu_i' q_i' \lambda_i'}{(\mu_i' + \alpha_i')}\right] + (r_{i2}\mu_2) \times \left[\frac{\lambda_3\mu_1\mu_3 q_2 - \mu_1\mu_3 q_3 \lambda_2}{(\mu_1 + \alpha_1)}\right] \times \left[\sum_{i=1}^{M} \frac{\mu_i' q_i' p_{i3}}{(\mu_i' + \alpha_i')}\right] + (p_{i3}\mu_3) \times \left[\frac{\mu_1\mu_2 q_3 \lambda_2 - \mu_1\mu_2 q_3 \lambda_3}{(\mu_1 + \alpha_1)}\right] \times \left[\sum_{i=1}^{M} \frac{\mu_i' q_i' r_{i2}}{(\mu_i' + \alpha_i')}\right]$$

$$N_1 = [\lambda_3 p_2 \mu_3 + \lambda_2(\mu_3 + \alpha_3)] \times \left[\sum_{i=1}^{M} \frac{\mu_i' q_i' r_{i2}\mu_2}{(\mu_i' + \alpha_i')}\right] + [r_2\mu_2\lambda_2 + \lambda_3(\mu_2 + \alpha_2)] \times \left[\sum_{i=1}^{M} \frac{\mu_i' q_i' p_{i3}\mu_3}{(\mu_i' + \alpha_i')}\right] + \left\{\left[\frac{\mu_1 q_3 \mu_3 p_2 + \mu_1 q_2(\mu_3 + \alpha_3)}{(\mu_1 + \alpha_1)}\right] \times \left[\sum_{i=1}^{M} \frac{\mu_i' q_i' r_{i2}\mu_2}{(\mu_i' + \alpha_i')}\right] + \left[\frac{\mu_1 q_2 \mu_2 r_2 + \mu_1 q_3(\mu_2 + \alpha_2)}{(\mu_1 + \alpha_1)}\right] \times \left[\sum_{i=1}^{M} \frac{\mu_i' q_i' p_{i3}\mu_3}{(\mu_i' + \alpha_i')}\right]\right\} \times \left[\lambda_1 + \sum_{i=1}^{M} \frac{\mu_i' q_i' \lambda_i'}{(\mu_i' + \alpha_i')}\right]$$

$$N_2 = \lambda_3 p_2 \mu_3 + \lambda_2(\mu_3 + \alpha_3) + \left[\frac{\lambda_3 q_2 \mu_3 \mu_1 - \lambda_2 \mu_1 \mu_3 q_3}{(\mu_1 + \alpha_1)}\right] \times \left[\sum_{i=1}^{M} \frac{\mu_i' q_i' p_{i3}}{(\mu_i' + \alpha_i')}\right] + \left[\frac{\mu_1 p_2 q_3 \mu_2 + \mu_1 q_2(\mu_3 + \alpha_3)}{(\mu_1 + \alpha_1)}\right] \times \left[\lambda_1 + \sum_{i=1}^{M} \frac{\mu_i' q_i' \lambda_i'}{(\mu_i' + \alpha_i')}\right]$$

$$N_3 = \lambda_3(\mu_2+\alpha_2) + r_2\mu_2\lambda_2 + \left[\frac{\mu_1\mu_2 r_2 q_2 + \mu_1 q_3(\mu_2+\alpha_2)}{(\mu_1+\alpha_1)}\right] \times \left[\lambda_1 + \sum_{i=1}^{M}\frac{\mu_i' q_i' \lambda_i'}{(\mu_i'+\alpha_i')}\right] + \left[\frac{\lambda_2 q_3\mu_1\mu_2 - \lambda_3 q_2\mu_1\mu_2}{(\mu_1+\alpha_1)}\right] \times \left[\sum_{i=1}^{M}\frac{\mu_i' q_i' r_{i2}}{(\mu_i'+\alpha_i')}\right]$$

$P(0,0,0\ldots,0)$ is obtained by normalizing condition

$$\left[\sum_{n_i,m,n,r=0}^{\infty} P(n_1,n_2,\ldots,n_m;m,n,r)\right] = 1 \qquad \ldots(3)$$

Equations (2) and (3) gives

$P(0,0,\ldots,0) = (1-\rho_1)(1-\rho_2)(1-\rho_3)[\ \prod_{i=1}^{M}(1-\rho_i')]$

With the restriction that

$\rho_i'(i=1,2,..,M); \rho_1,\rho_2,\rho_3 < 1$. Thus

$$P(n_1,n_2,\ldots,n_m;m,n,r) = (\rho_1)^m(\rho_2)^n(\rho_3)^r\left[\prod_{i=1}^{M}(\rho_i')^{n_i}\right] \times \left[\prod_{i=1}^{M}(1-\rho_i')\right](1-\rho_1)\times(1-\rho_2) \times (1-\rho_3)$$

where n_i;m,n,r≥o

Steady State Marginal Probabilities: Let $P(n_i)$ be steady state marginal probabilities that there are n_i units before server S_i. This is determined as

$$P(n_i) = \sum P(n_1,n_2,\ldots,n_M;m,n,r)$$

Where summation is taken over $n_1,n_2,\ldots,n_M;m,n,r$ each varying from o to ∞ .

Thus $P(n_i) = (1-\rho_i')(\rho_i')^{n_i}$,

$n_i \geq 0; i = 1,2,\ldots.,M$ ... (4)

Similarly $P(m), P(n)$ *and* $P(r)$ are given by

$$P(m) = (1-\rho_1)(\rho_1)^m \quad ; m \geq 0$$
$$P(n) = (1-\rho_2)(\rho_2)^n \quad ; n \geq 0$$
$$P(r) = (1-\rho_3)(\rho_3)^r \quad ; r \geq 0$$

Mean Queue Length: The marginal mean queue length before server is found by using (4) in formula

$$L_i' = \left[\sum_{n_i=0}^{\infty} n_i P(n_i)\right] = \left[\frac{\rho_i'}{1-\rho_i'}\right] \quad ;(i=1,2,\ldots,M)$$

Similarly L_1, L_2 *and* L_3 are given by

$$L_1 = \left[\frac{\rho_1}{1-\rho_1}\right]$$
$$L_2 = \left[\frac{\rho_2}{1-\rho_2}\right]$$
$$L_3 = \left[\frac{\rho_3}{1-\rho_3}\right]$$

And mean queue length L of system is given by

$$L = \left[\sum_{i=1}^{M}\frac{\rho_i'}{1-\rho_i'}\right] + \left[\sum_{j=1}^{3}\frac{\rho_j}{1-\rho_j}\right]$$

Numerical Solution for Mean Queue Length:

Table: I

Index (i)	Arrival rate (λ_i') Per hour	Service Rate (μ_i') Per hour	Reneging Rate (α_i') Per hour	Arrival Rate (λ_i) Per hour	Service Rate (μ_i) Per hour	Reneging Rate (α_i) Per hour
1	10	13	5	9	12	6
2	12	16	7	11	15	5
3	14	17	8	14	16	7

Table: II

Index (i)	Probability of leaving the server S_i' (p_i')	Probability of joining the server S_1 (q_i')	Probability of joining the server S_i'after serviced by S_2 (r_{i2})	probability of joining the server S_i'after serviced by S_3 (p_{i3})
1	0.85	0.15	0.075	0.10
2	0.75	0.25	0.1	0.06
3	0.90	0.10	0.075	0.17

Values of remaining probabilities are as follows:

$q_1 = 0.5$, $q_2 = 0.3$, $q_3 = 0.2$;

$r_1 = 0.65$, $r_2 = 0.1$;

$p_1 = 0.5$, $p_2 = 0.17$;

such that

$p_i'+q_i' = 1, (i=1,2,3);\ q_1+q_2+q_3 = 1;$

$r_1 + r_2 + \sum_{i=1}^{3} r_{i2} = 1;\ p_1 + p_2 + \sum_{i=1}^{3} p_{13} = 1$

$$\rho_i' = \frac{\lambda_i'}{\mu_i'+\alpha_i'} + \frac{N_i'}{D(\mu_i'+\alpha_i')} \quad ;\ (i=1,2,3)$$

$$N_i' = r_{i2}\mu_2[\lambda_3 p_2\mu_3 + \lambda_2(\mu_3+\alpha_3)] + p_{i3}\mu_3[r_2\mu_2\lambda_2 + \lambda_3(\mu_2+\alpha_2)] + \left[\frac{r_{i2}\mu_2\mu_1 p_2 q_3\mu_3 + \mu_1 q_2(\mu_3+\alpha_3)}{(\mu_1+\alpha_1)} + \frac{p_{i3}\mu_3\mu_1\mu_2 q_2 r_2 + \mu_1 q_3(\mu_2+\alpha_2)}{(\mu_1+\alpha_1)}\right] \times \left[\lambda_1 + \sum_{i=1}^{3}\frac{\mu_i' q_i' \lambda_i'}{(\mu_i'+\alpha_i')}\right] + (r_{i2}\mu_2)\left[\frac{\lambda_3\mu_1\mu_3 q_2 - \mu_1\mu_3 q_3\lambda_2}{(\mu_1+\alpha_1)}\right] \times \left[\sum_{i=1}^{3}\frac{\mu_i' q_i' p_{i3}}{(\mu_i'+\alpha_i')}\right] + (p_{i3}\mu_3)\left[\frac{\mu_1\mu_2 q_3\lambda_2 - \mu_1\mu_2 q_3\lambda_3}{(\mu_1+\alpha_1)}\right] \times \left[\sum_{i=1}^{3}\frac{\mu_i' q_i' r_{i2}}{(\mu_i'+\alpha_i')}\right] \quad where\ i = (1,2,3)$$

$$\sum_{i=1}^{3}\frac{\mu_i' q_i' p_{i3}}{(\mu_i'+\alpha_i')}=0.03276$$
$$\sum_{i=1}^{3}\frac{\mu_i' q_i' r_{i2}}{(\mu_i'+\alpha_i')}=0.0306$$
$$\sum_{i=1}^{3}\frac{\mu_i' q_i' \lambda_i'}{(\mu_i'+\alpha_i')}=4.1223$$

$$N_1' = 327.465+474.4+101.134+0.683-0.9792 = 902.703$$
$$N_2' = 436.62+284.64+102.747+0.911-0.5875 = 824.3315$$
$$N_3' = 327.465+806.48+101.409+0.683-1.665 = 1234.372$$
$$D=(\mu_2+\alpha_2)(\mu_3+\alpha_3)-p_2r_2\mu_2\mu_3 - \left[\frac{\mu_1\mu_3q_3(\mu_2+\alpha_2)-\mu_1\mu_2\mu_3r_2q_2}{(\mu_1+\alpha_1)}\right]\times\left[\sum_{i=1}^{3}\frac{\mu_i' q_i' r_{i2}}{(\mu_i'+\alpha_i')}\right]$$
$$=460-4.08-1.159=454.761$$
$$N_1=[\lambda_3p_2\mu_3+\lambda_2(\mu_3+\alpha_3)]\times\left[\sum_{i=1}^{3}\frac{\mu_i' q_i' r_{i2}\mu_2}{(\mu_i'+\alpha_i')}\right] + [r_2\mu_2\lambda_2+\lambda_3(\mu_2+\alpha_2)]\times\left[\sum_{i=1}^{3}\frac{\mu_i' q_i' p_{i3}\mu_3}{(\mu_i'+\alpha_i')}\right] + \left\{\left[\frac{\mu_1q_3\mu_3p_2+\mu_1q_2(\mu_3+\alpha_3)}{(\mu_1+\alpha_1)}\right]\times\left[\sum_{i=1}^{3}\frac{\mu_i' q_i' r_{i2}\mu_2}{(\mu_i'+\alpha_i')}\right] + \left[\frac{\mu_1q_2\mu_2r_2+\mu_1q_3(\mu_2+\alpha_2)}{(\mu_1+\alpha_1)}\right]\times\left[\sum_{i=1}^{3}\frac{\mu_i' q_i' p_{i3}\mu_3}{(\mu_i'+\alpha_i')}\right]\right\}\times\left[\lambda_1+\sum_{i=1}^{3}\frac{\mu_i' q_i' \lambda_i'}{(\mu_i'+\alpha_i')}\right]$$
$$=133.61+155.413+[2.278+1.555]\times 13.1223 = 339.321$$
$$N_2=\lambda_3p_2\mu_3+\lambda_2(\mu_3+\alpha_3)+\left[\frac{\lambda_3q_2\mu_3\mu_1-\lambda_2\mu_1\mu_3q_3}{(\mu_1+\alpha_1)}\right]\times\left[\sum_{i=1}^{3}\frac{\mu_i' q_i' p_{i3}}{(\mu_i'+\alpha_i')}\right] + \left[\frac{\mu_1p_2q_3\mu_2+\mu_1q_2(\mu_3+\alpha_3)}{(\mu_1+\alpha_1)}\right]\times\left[\lambda_1+\sum_{i=1}^{3}\frac{\mu_i' q_i' \lambda_i'}{(\mu_i'+\alpha_i')}\right]$$
$$=38.08+253+0.699+65.122=356.901$$

$$N_3=\lambda_3(\mu_2+\alpha_2)+r_2\mu_2\lambda_2 + \left[\frac{\mu_1\mu_2r_2q_2+\mu_1q_3(\mu_2+\alpha_2)}{(\mu_1+\alpha_1)}\right]\times\left[\lambda_1+\sum_{i=1}^{3}\frac{\mu_i' q_i' \lambda_i'}{(\mu_i'+\alpha_i')}\right] + \left[\frac{\lambda_2q_3\mu_1\mu_2-\lambda_3q_2\mu_1\mu_2}{(\mu_1+\alpha_1)}\right]\times\left[\sum_{i=1}^{3}\frac{\mu_i' q_i' r_{i2}}{(\mu_i'+\alpha_i')}\right]$$
$$=280+16.5+38.929-0.612=334.817$$
$$\rho_i'=\frac{\lambda_i'}{\mu_i'+\alpha_i'}+\frac{N_i'}{D(\mu_i'+\alpha_i')} \qquad ; i=1,2,3$$
$$\rho_1'=\frac{\lambda_1'}{\mu_1'+\alpha_1'}+\frac{N_1'}{D(\mu_1'+\alpha_1')} = 0.56+0.11=0.67$$
$$\rho_2'=\frac{\lambda_2'}{\mu_2'+\alpha_2'}+\frac{N_2'}{D(\mu_2'+\alpha_2')} = 0.52+0.08=0.60$$
$$\rho_3'=\frac{\lambda_3'}{\mu_3'+\alpha_3'}+\frac{N_3'}{D(\mu_3'+\alpha_3')} = 0.56+0.11=0.67$$
$$\rho_1=\frac{\lambda_1}{(\mu_1+\alpha_1)}+\frac{\sum_{i=1}^{3}\frac{\lambda_i' q_i' \mu_i'}{(\mu_i'+\alpha_i')}}{(\mu_1+\alpha_1)}+\frac{N_1}{D(\mu_1+\alpha_1)} = 0.5+0.23+0.04=0.77$$

$$\rho_2=\frac{N_2}{D}=0.78;\ \rho_3=\frac{N_3}{D}=0.74$$
$$L_1'=\left[\frac{\rho_1'}{1-\rho_1'}\right]=2.03;$$
$$L_2'=\left[\frac{\rho_2'}{1-\rho_2'}\right]=1.5;$$
$$L_3'=\left[\frac{\rho_3'}{1-\rho_3'}\right]=2.03$$
$$L_1=\left[\frac{\rho_1}{1-\rho_1}\right]=3.35$$
$$L_2=\left[\frac{\rho_2}{1-\rho_2}\right]=3.55$$
$$L_3=\left[\frac{\rho_3}{1-\rho_3}\right]=2.85$$

Mean queue length L of system is given by
$$L=\left[\sum_{i=1}^{3}\frac{\rho_i'}{1-\rho_i'}\right]+\left[\sum_{j=1}^{3}\frac{\rho_j}{1-\rho_j}\right]=15.31\approx 15$$

Conclusion: The mean queue length of this system is found to be 15. Mean queue length can be controlled by giving higher service rates. During calculations it is observed that mean queue length depends upon probabilities q_i' of joining the server S_1 after service from S_i'. Queue length can be controlled if chances of joining server S_1 are less i.e. units serviced by S_i' leave the system.

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