BAIRE SPACE VIA WEAK FORM OF OPEN SETS IN BITOPOLOGY

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Abstract: This paper is to introduce the notions such as (1,2)*α_ψ–nowhere dense sets, (1,2)*α_ψ–dense sets etc. Also we introduce and discuss some of the properties of the (1,2)*α_ψ-Baire space in Bitopological spaces by using (1,2)*α_ψ-open sets.

Keywords: Bitopological spaces, (1,2)*α_ψ -open sets, (1,2)*α_ψ -nowhere dense sets, (1,2)*α_ψ -dense sets, (1,2)*α_ψ -first and second category.

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Introduction: In 1963, Kelly [4] initiated the study of the bitopological space which is to be a set X equipped with two topologies τ_1 and τ_2 on X. In this space, the Baire space concept was developed by Fukutake [1] in 1992. In bitopological spaces, Lellis Thivagar et al. [7] introduced (1,2)*α-open sets by defining a new class of open sets namely $\tau_{1,2}$-open sets and in it we can observe that the family of (1,2)*α-open sets is need not form a topology but an m-structure. This paper is to introduce the (1,2)*α_ψ-Baire space in Bitopological spaces by using (1,2)*α_ψ-open sets and for that we introduce (1,2)*α_ψ-nowhere dense sets, (1,2)*α_ψ-dense sets etc.

Preliminaries: In this section we recollect some properties of basic concepts which are useful in the sequel. In this paper, by (X, τ_1, τ_2) (or X) we always mean bitopological spaces on which no separation axioms are assumed, unless otherwise mentioned.

Definition 2.1. [4] A non-empty set X together with two arbitrary topologies τ_1 and τ_2 is called a bitopological space and is denoted by (X, τ_1, τ_2).

Definition 2.2. [5] A subset S of a bitopological space (X, τ_1, τ_2) is called $\tau_{1,2}$-open if and only if S=A∪B, where A is τ_1-open and B is τ_2-open.The complement of $\tau_{1,2}$-open sets are called $\tau_{1,2}$-closed sets. The family of all $\tau_{1,2}$-open sets is denoted by $\tau_{1,2}$O(X). Note that $\tau_{1,2}$ O(X) need not necessarily form a topology and τ_1O(X), τ_2O(X) ⊆ $\tau_{1,2}$O(X).

Remark 2.3. [5] Let A be a subset of a bitopological space (X, τ_1, τ_2). Then

1. $\tau_{1,2}$-int(A) =∪{G: G ⊆ A and G is $\tau_{1,2}$-open}
2. $\tau_{1,2}$-cl(A) =∩{F: A ⊆ F and F is $\tau_{1,2}$-closed}.

Definition 2.4. A subset A of a bitopological space (X, τ_1, τ_2) is called (1,2)*α-open [5] if A ⊆ $\tau_{1,2}$-int($\tau_{1,2}$-cl($\tau_{1,2}$-int(A))) and the complement of (1,2)*α-open sets are called (1,2)*α-closed sets. The family of all (1,2)*α-open sets need not form a topology and is denoted by (1,2)*αO(X).

Baire Space with Bitopological Open Sets: In this section we generate a topology by (1, 2)*α-open sets as its sub basis and in this topology we introduce and establish the properties of (1,2)*α_ψ-nowhere dense sets and (1,2)*α_ψ-Baire spaces.

Definition 3.1. A topology which is generated by the family (1,2)*αO(X) as its sub basis and the collection of elements of this topology is denoted by (1,2)*α_ψO(X). A subset A of X is called (1,2)*α_ψ-open if A∈(1,2)*α_ψO(X). From this it is very clear that every (1,2)*α-open set is (1,2)*α_ψ-open but not converse and the complement of a (1,2)*α_ψ-open set is (1,2)*α_ψ-closed set. The collection of all (1,2)*α_ψ-closed sets is denoted by (1,2)*α_ψC(X).

Remark 3.2. Let A be a subset of a bitopological space X. Then (1,2)*α_ψ-interior and (1,2)*α_ψ-closure of A are defined as follows:

1. (1,2)*α_ψ-int(A) =∪{G: G ⊆ A and G is (1,2)*α_ψ-open}
2. (1,2)*α_ψ-cl(A) =∩{F: A ⊆ F and F is (1,2)*α_ψ-closed}.

Example 3.3. Let X = {a, b, c}, τ_1 = {ϕ, X, {a, b}}, τ_2 = {ϕ, X, {b, c}}. Then $\tau_{1,2}$O(X) = {ϕ, X, {a, b}, {b, c}} = (1,2)*αO(X). Therefore, (1,2)*α_ψO(X) = { ϕ, X, {b}, {a, b}, {b, c}}.

Remark 3.4. We can easily prove that

1. a set A is (1,2)*α_ψ-closed set if (1,2)*α_ψ-cl(A) =A
2. a set A is (1,2)*α_ψ-open set if (1,2)*α_ψ-int(A) =A
3. (1,2)*α_ψ-int(A) ⊇ (1,2)*α-int(A)
4. (1,2)*α_ψ-cl(A) ⊆ (1,2)*α-cl(A).

Definition 3.5. A subset A of a bitopological space X is called

1. (1,2)*α_ψ-dense if (1,2)*α_ψ-cl(A) = X
2. (1,2)*α_ψ-nowhere dense if (1,2)*α_ψ- int((1,2)*α_ψ-cl(A)) = ϕ.

Example 3.6. Let X = {a, b, c}, τ_1 = {ϕ, X, {a, b}}, τ_2 = {ϕ, X, {b, c}}. Then $\tau_{1,2}$O(X) = {ϕ, X, {a, b}, {b, c}} = (1,2)*αO(X). Therefore, (1,2)*α_ψO(X) = {ϕ, X, {b}, {a, b}, {b, c}}, here {b} is (1,2)*α_ψ-dense and {a} is (1,2)*α_ψ-nowhere dense.

Theorem 3.7. A subset A of X is (1,2)*α_ψ-nowhere dense if and only if X–(1,2)*α_ψ-cl(A) is (1,2)*α_ψ-dense in X.

Proof: Suppose X–(1,2)*α_ψ-cl(A) is not (1,2)*α_ψ-dense in X, there exists p ∈ X and a (1,2)*α_ψ-open set G such that p ∈ G and G∩(X–(1,2)*α_ψ-cl(A)) = ϕ. Then p ∈ G

$\subset$ (1,2)*α_ψ-cl(A) and so p$\in$(1,2)*α_ψ-int((1,2)*α_ψ-cl(A)). This is not possible, because A is (1,2)*α_ψ-nowhere dense in X. Therefore, X – (1,2)*α_ψ-cl(A) is (1,2)*α_ψ-dense in X. Conversely, assume X – (1,2)*α_ψ-cl(A) is (1,2)*α_ψ-dense in X. Then (1,2)*α_ψ-cl(X–(1,2)*α_ψ-cl(A)) = X and so (1,2)*α_ψ-int((1,2)*α_ψ-cl(A)) = ϕ.

Theorem 3.8. The union of a finite number of (1,2)*α_ψ-nowhere dense sets is (1,2)*α_ψ-nowhere dense.

Proof: It is enough to show that the union of two (1,2)*α_ψ-nowhere dense sets A and B is (1,2)*α_ψ-nowhere dense. Without loss of generality, we may assume that A and B are (1,2)*α_ψ-closed. The Theorem is then equivalently to saying that the intersection of two (1,2)*α_ψ-dense open sets A^c and B^c is (1,2)*α_ψ-dense. Now if U is a non-empty (1,2)*α_ψ-open set, then U ∩ A^c is non-empty (1,2)*α_ψ-open. Hence (U ∩ A^c) ∩ B^c = U ∩ (A^c ∩ B^c) is non-empty (1,2)*α_ψ-open.

Theorem 3.9. If A ⊆ B ⊆ X and B is (1,2)*α_ψ-nowhere dense in X, A is (1,2)*α_ψ-nowhere dense in X.

Proof: Proof is trivially from the fact that if A ⊆ B ⊆ X, then (1,2)*α_ψ-cl(A) ⊆ (1,2)*α_ψ-cl(B) and (1,2)*α_ψ-int(A) ⊆ (1,2)*α_ψ-int(B).

Definition 3.10. A bitopological space X is said to be (1,2)*α_ψ-Baire space if for any countable collection {A_n} of (1,2)*α_ψ-closed subsets of X such that (1,2)*α_ψ-int(A_n) = ϕ ∀n, then (1,2)*α_ψ-int(∪$_n$ A_n)=ϕ.

Example 3.11. Let X = {a, b, c, d}, τ_1 = {ϕ,X,{a},{b},{a, b}}, τ_2 = {ϕ, X,{c},{d},{c, d}}. Then $\tau_{1,2}$O(X) = P(X) = (1,2)*αO(X) = (1,2)*α_ψO(X). Here X is (1,2)*α_ψ-Baire space.

Theorem 3.12. A bitopological space X is (1,2)*α_ψ-Baire space if and only if for any countable collection {A_n} of (1,2)*α_ψ-open subsets of X such that (1,2)*α_ψ-cl(A_n) = X ∀n, then (1,2)*α_ψ-cl(∩$_n$ An) =X.

Proof: Assume X is (1,2)*α_ψ-Baire space and {A_n} is a countable collection of (1,2)*α_ψ-open subsets of X such that (1,2)*α_ψ-cl(A_n) = X ∀n. Then (1,2)*α_ψ-int(X–A_n) = ϕ ∀n and implies (1,2)*α_ψ-int(∪$_n$(X–A_n)) = ϕ. Thus (1,2)*α_ψ-cl(∩$_n$ A_n) = X. Conversely, assume {A_n} is a countable collection of (1,2)*α_ψ-closed subsets of X such that (1,2)*α_ψ-int(A_n) = ϕ ∀n. Then (1,2)*α_ψ-cl(X–A_n) = X ∀n and by hypothesis, (1,2)*α_ψ-cl(∩$_n$(X–A_n)) =X and implies (1,2)*α_ψ-int(∪$_n$ A_n) = ϕ.

Definition 3.13. A subset A of a bitopological space X is said to be (1,2)*α_ψ-first category if A=∪$_n$ A_n, where each A_n is (1,2)*α_ψ-nowhere dense subset of X. If A is not (1,2)*α_ψ-first category, then A is said to be (1,2)*α_ψ-second category.

Example 3.14. Let X = {a, b, c}, τ_1 = {ϕ, X, {a, b}}, τ_2 = {ϕ, X, {b, c}}. Then $\tau_{1,2}$O(X) = {ϕ, X,{a, b},{b, c}}= (1,2)*αO(X). Therefore, (1,2)*α_ψO(X) = {ϕ, X,{b},{a, b},{b, c}}. Clearly, {a, c} = {a} ∪ {c} is (1,2)*α_ψ-first category and {b, c} is (1,2)*α_ψ-second category.

Theorem 3.15. If A⊆B⊆X and B is (1,2)*α_ψ-first category in X, A is (1,2)*α_ψ-first category in X.

Proof: Proof is trivial.

Theorem 3.16. Any (1,2)*α_ψ-closed set A such that (1,2)*α_ψ-int(A) = ϕ is (1,2)*α_ψ-first category.

Proof: If A is then (1,2)*α_ψ-closed, A=A ∪ ϕ ∪ ϕ ∪ ... is a union of a countable collection of (1,2)*α_ψ-nowhere dense sets.

Proposition 3.17. If A is a first category subset of a (1,2)*α_ψ-Baire space X, then (1,2)*α_ψ-int(A) = ϕ.

Proof: Since A is first category, A=∪$_n$ A_n, where each A_n is (1,2)*α_ψ-nowhere dense subset of X. Let G be a (1,2)*α_ψ-open set such that G ⊆ A. Then G ⊆∪$_n$ A_n⊆∪$_n$ (1,2)*α_ψ-cl(A_n) and so X–G ⊇ ∩$_n$ (X–(1,2)*α_ψ-cl(A_n)). Since each X – (1,2)*α_ψ-cl(A_n)) is (1,2)*α_ψ-open and (1,2)*α_ψ-dense in (1,2)*α_ψ-Baire Space X, ∩$_n$ (X–(1,2)*α_ψ-cl(A_n)) is (1,2)*α_ψ-dense in X and X–G is (1,2)*α_ψ-dense in X. So X–G = X and hence G = ϕ.

Proposition 3.18. Any (1,2)*α_ψ-open subspace Y of a (1,2)*α_ψ-Baire space X is itself a (1,2)*α_ψ-Baire Space.

Proof: Let $\{A_n\}_{n=1}^{\infty}$ be a countable collection of (1,2)*α_ψ-closed sets of Y such that each set (1,2)*α_ψ-int$_Y$(A_n) = ϕ (here interior in Y). We show that (1,2)*α_ψ-int$_Y$(∪$_n$ A_n) = ϕ. Let (1,2)*α_ψ-cl(A_n) be (1,2)*α_ψ-closure of A_n in X, then where each (1,2)*α_ψ-cl(A_n)∩Y=A_n and the set (1,2)*α_ψ-int((1,2)*α_ψ-cl(A_n)) = ϕ. For if U is a non-empty (1,2)*α_ψ-open set of X contained in (1,2)*α_ψ-cl(A_n), then U must intersect A_n. Then U∩Y is a non-empty (1,2)*α_ψ-open set of Y contained in A_n, contrary to hypothesis. If the union of the sets A_n contains the non-empty (1,2)*α_ψ-open set W of Y, then the union of the sets (1,2)*α_ψ-cl(A_n) also contains the set W, which is (1,2)*α_ψ-open in X because Y is (1,2)*α_ψ-open in X. But each set (1,2)*α_ψ-int((1,2)*α_ψ-cl(A_n)) = ϕ, contradicting the fact that X is (1,2)*α_ψ-Baire space.

Conclusion: In this paper, we have discussed some more properties of (1,2)*α_ψ-nowhere dense sets and (1,2)*α_ψ-locally compact sets. Finally we derived the properties of (1,2)*α_ψ-Baire space. In future, we can establish many research fields such as soft topology, fuzzy topology, digital topology via Baire Spaces.

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