DETERMINANT AND ADJOINT OF FUZZY SOFT SQUARE MATRIX

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Abstract: The present paper aims to introduce the concepts of fuzzy soft square matrix and their characteristics features. In this way, the present work contributes to find a new way to expand the determinant of fuzzy soft square matrices and lays foundation for defining the adjoint of fuzzy soft square in a quite different way. This investigation also attempted to find some of the new properties of determinant as well as adjoint of fuzzy soft square matrices which would be at most analogous to the properties of crisp cases.

Keywords: Fuzzy soft matrix, Determinant of fuzzy soft square matrices, Adjoint of fuzzy soft square matrices.

Introduction: Fuzzy set theory was proposed by Zadeh [12] in 1965 is a generalization of classical or crisp sets. It makes possible to describe vague notions and deals with uncertainty. In 1999, Molodtsov [6] introduced the theory of soft sets, which is a new mathematical approach to vagueness. In 2003, Maji et al.[5] studied the theory of soft sets initiated by Molodtsov[6] and developed several basic notions of soft set theory. At present, researchers are contributing a lot on the extension of soft set theory. In 2005, Pei and Miao[11] and Chen et al.[3] studied and improved the finding of Maji et al.[5]. In 2009, Ali et al.[1] gave some new notions such as the restricted intersection, the restricted difference and the extended intersection of two soft sets along with a new notion of relative complement of a soft set. Later in 2011, Neog and Sut[8] reintroduced the concept of complement of a soft set in such way that the fundamental properties related to complement are satisfied also by soft sets.

Matrices play an important role in the broad area of science and engineering. However, the classical matrix theory sometimes fails to solve the problems involving uncertainties, occurring in an imprecise environment. In Neog and Sut[10] proposed a matrix representation of a fuzzy soft set and successfully applied the proposed notion of fuzzy soft matrix in certain decision making problems.

In this paper, we state a definition for determinant and adjoint of a fuzzy soft square matrix. We establish some results including that $A(adjA) \geq |A| I$ and $(adjA)A \geq |A| I$, where $|A|$ denotes the determinant of a fuzzy soft square matrix and adjA denotes the adjoint of fuzzy soft square matrix A.

Preliminaries: This section recalls some basic definitions and results which will be needed in the sequel .

Definition 2.1.[7]: Suppose that U is an initial universe set and E is a set of parameters, let P(U) denotes the power set of U. A pair (F,E) is called a soft set over U where F is a mapping given by F : A $\rightarrow$ P(U).

Clearly a soft set is a mapping from parameters to P(U), and it is not a set but a parameterized family of subsets of the universe.

Definition 2.2[4]: Let U be an initial universe set and E be the set of parameters. Let A $\subseteq$ E. A pair (F,A) is called fuzzy soft set over U where F is a mapping given by $F : A \rightarrow F^u$,where F^u denotes the collection of all fuzzy subsets of U.

Definition 2.3[8]: Let U = $\{c_1, c_2, c_3, \ldots, c_n\}$ be the universal set and E be the set of parameters given by E= $\{e_1, e_2, e_3, \ldots, e_n\}$. Then the fuzzy soft square set (F,E) can be expressed in matrix form as

A = $[a_{ij}]_{n\times n}$ or simply by $[a_{ij}]$, i=1,2,3…,n; j=1,2,3…,n and $a_{ij} = (\mu_{j1}(c_i) - \mu_{j2}(c_i))$;where $\mu_{ji}(c_i)$ and $\mu_{j2}(c_i)$ represent the fuzzy membership function and fuzzy reference function respectively of c_i in the fuzzy set $F(e_j)$ so that $\delta_{ij} = (\mu_{j1}(c_i) - \mu_{j2}(c_i))$ gives the fuzzy membership value of c_i . We shall identify a fuzzy soft set with its fuzzy soft matrix and use these two concepts interchangeable. The set of all $n \times n$ fuzzy soft matrices over U will be denoted by $FSSM_{n\times n}$. For usual fuzzy sets with fuzzy reference function 0, it is obvious to see that $a_{ij} = (\mu_{j1}(c_i) - \mu_{j2}(c_i), 0) \forall i, j$

Definition 2.4[9]: Let U be a initial universe, E be the set of parameters and A $\subseteq$ E. Let (f_A, E) be fuzzy soft set (FS) over U. Then a subset of U $\times$ E is uniquely defined by $R_A = \{(u,e); e \in A, u \in f_A(e)\}$ which is called a relation form of (f_A, E).

Example 2.5: Assume that U = $\{u_1, u_2, u_3, \cdots, u_n\}$ is a universal set and E = $\{e_1, e_2, e_3, \cdots, e_n\}$ be the set of parameters and A = $\{e_1, e_2, e_3\} \subseteq E$

$$f_A(e_1) = \{u_1/(0.7,0), u_2/(0.1,0).u_3/(0.2,0)\}$$

$$f_A(e_2) = \{u_1/(0.8,0), u_2/(0.6,0).u_3/(0.1,0)\}$$

$f_A(e_3)=\{u_1/(0.1,0),u_2/(0.2,0).u_3/(0.7,0)\}$

Then the fuzzy soft set (F_A,E) is a parameterized family $\{f_A(e_1),f_A(e_2),f_A(e_3)\}$ of all fuzzy soft set over U. Then the relation form of (F_A,E) is written as;

Table 1: The relation form of (f_A,E)

R_A	e_1	e_2	e_3
u_1	(0.7,0)	(0.8,0)	(0.1,0)
u_2	(0.1,0)	(0.6,0)	(0.2,0)
u_3	(0.2,0)	(0.1,0)	(0.7,0)

Definition 2.6[10]:

R_A	e_1	e_2	e_3
u_1	$\chi_{R_A}(u_1,e_1)$	$\chi_{R_A}(u_1,e_2)$	$\chi_{R_A}(u_1,e_3)$
u_2	$\chi_{R_A}(u_2,e_1)$	$\chi_{R_A}(u_2,e_2)$	$\chi_{R_A}(u_2,e_3)$
u_3	$\chi_{R_A}(u_m,e_1)$	$\chi_{R_A}(u_m,e_2)$	$\chi_{R_A}(u_m,e_n)$

Let $U=\{c_1,c_2,c_3,...c_n\}$ be the universal set and E be the set of parameters given by $E=\{e_1,e_2,e_3...e_n\}$. Let the set of all $n\times n$ fuzzy soft square matrices over U be $FSSM_{n\times n}$. Let A,B$\in$ FSSM $_{n\times n}$, where $A=[a_{ij}]_{n\times n}$, $a_{ij}=(\mu_{j1}(c_i),\mu_{j2}(c_i))$ and $B=[b_{ij}]_{n\times n}, b_{ij}=(\chi_{j1}(c_i),\chi_{j2}(c_i))$. To avoid degenerate cases we assume that $min(\mu_{j1}(c_i,\chi_{j1}(c_i)\ge max(\mu_{j2}(c_i,\chi_{j2}(c_i)$ for all i and j. We define the operation 'addition $(+)$' between A and B as $A+B=C,$ where $C=[c_{ij}]_{n\times n}$,

$c_{ij}=\left(max(\mu_{j1}(c_i),\chi_{j1}(c_i)),min(\mu_{j2}(c_i),\chi_{j2}(c_i)\right)$

Definition 2.7[12]:

Let $A=[a_{ij}]_{m\times n}, a_{ij}=\left(\mu_{j1}(c_i),\mu_{j2}(c_i)\right)$; where $\mu_{j1}(c_i)$ and $\mu_{j2}(c_i)$ represent the fuzzy membership function and fuzzy reference of c_i, so that $\delta_{ij}(c_i)=\mu_{j1}(c_i)-\mu_{j2}(c_i)$ gives the fuzzy membership value of c_i. Also let $B=[b_{jk}]_{n\times p}, b_{jk}=\left(\chi_{k1}(c_j),\chi_{k2}(c_j)\right)$; where $\chi_{k1}(c_j)$ and $\chi_{k2}(c_j)$ represent the fuzzy membership function and fuzzy reference function respectively of c_j, so that $\delta_{jk}(c_j)=\chi_{k1}(c_j)-\chi_{k2}(c_j)$ gives the fuzzy membership value of c_j. We now define A.B, the product of A and B as,

$A.B=[d_{ik}]_{m\times p}$

$=\left[maxmin(\mu_{j1}(c_i),\chi_{k2}(c_i)),minmax(\mu_{j2}(c_i),\chi_{k2}(c_i))\right]_{m\times p}$

where $1\le i\le m, 1\le k\le p$ for $j=1,2,\ldots,n$.

Definition 2.8[2]: Let (f_A,E) be a soft set defined over the universe U. Then a subset of $U\times E$ is uniquely defined by $R_A=\{(u,e):e\in A,u\in f_A(e)\}$ which is called a relation form of (f_A,E). The characteristic function of R_A is written by

$$\chi_{R_A}:U\times E\to\{0,1\},\chi_{R_A}(u,e)=\begin{cases}1,(u,e)\in R_A\\0,(u,e)\notin R_A\end{cases}$$

If $U=\{u_1,u_2,...,u_m\},E=\{e_1,e_2,...,e_n\}$ and $A\subseteq E$, then the R_A can be presented by a table as in the following form

If $a_{ij}=\chi_{R_A}(u_i,e_j)$, we can define a matrix

$$[a_{ij}]_{m\times n}=\begin{bmatrix}a_{11}&a_{12}&\cdots&a_{1n}\\a_{21}&a_{22}&\cdots&a_{2n}\\\vdots&\vdots&\ddots&\vdots\\a_{m1}&a_{m2}&\cdots&a_{mn}\end{bmatrix}$$

which is called an $m\times n$ soft matrix of the soft set (f_A,E) over U. According to this definition, a soft set (f_A,E) is uniquely characterized by the matrix $[a_{ij}]_{m\times n}$. It means that a soft set (f_A,E) is formally equal to its soft matrix $[a_{ij}]_{m\times n}$. Therefore, we shall identify any soft set with its soft matrix and use these two concepts as interchangeable. The set of all $m\times n$ soft matrices over U will be denoted by $FSM_{m\times n}$

Definition 2.9

Let (f_A,E) be a fuzzy soft set over U. Then a subset of $U\times E$ is uniquely defined by $R_A=\{(u,e):e\in A,u\in f_A(e)\}$ which is called a relation form of (f_A,E). The characteristic function of R_A is written by $\mu_{R_A}:U\times E\to[0,1]$, where $\mu_{R_A}(u,e)\in[0,1]$ is the membership value of $u\in U$

for each $e \in E$. If, $\mu_{ij} = \mu_{R_A}(u_i, e_j)$ we can define a matrix

$$\left[\mu_{ij}\right]_{m\times n} = \begin{bmatrix} \mu_{11} & \mu_{12} & \cdots & \mu_{1n} \\ \mu_{21} & \mu_{22} & \cdots & \mu_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{m1} & \mu_{m2} & \cdots & \mu_{mn} \end{bmatrix}$$

which is called an $m\times n$ fuzzy soft matrix of fuzzy soft set (f_A, E) over U.

Therefore, we can say that a fuzzy soft set (f_A, E) is uniquely characterized by the matrix $[\mu_{ij}]_{m\times n}$ and both concept are interchangeable. If $m = n$ [(i.e) the number of row is equal to the number of columns] and whose elements belong to the unit interval [0,1] is called a Fuzzy Soft Square Matrix and its denoted by $FSSM_n$.

Example 2.10: Assume that $U = \{u_1, u_2, u_3, u_4\}$ is a universal set and $E = \{e_1, e_2, e_3, e_4, e_5\}$ be the set of parameters. If $A = \{e_1, e_2, e_3, e_4\} \subseteq E$ and

$$f_A(e_1) = \{(u_1,(0.8,0)),(u_2,(0.6,0)),(u_3,(0.2,0)),(u_4,(0.7,0))\}$$
$$f_A(e_2) = \{(u_1,(0.2,0)),(u_2,(0.5,0)),(u_3,(0.6,0)),(u_4,(0.1,0))\}$$
$$f_A(e_3) = \{(u_1,(1,0)),(u_2,(0.7,0)),(u_3,(0.4,0)),(u_4,(0.3,0))\}$$
$$f_A(e_4) = \{(u_1,(0.7,0)),(u_2,(0.2,0)),(u_3,(0.8,0)),(u_4,(0,0))\}$$

Then the relative form of (f_A, E) is written by,

R_A	e_1	e_2	e_3	e_4
u_1	(0.8,0)	(0.2,0)	(1,0)	(0.7,0)
u_2	(0.6,0)	(0.5,0)	(0.4,0)	(0.2,0)
u_3	(0.2,0)	(0.6,0)	(0.4,0)	(0.8,0)
u_4	(0.7,0)	(0.1,0)	(0.3,0)	(0,0)

Hence, the fuzzy soft square matrix $[\mu_{ij}]$ is written as

$$\left[\mu_{ij}\right] = \begin{bmatrix} (0.8,0) & (0.2,0) & (1,0) & (0.7,0) \\ (0.6,0) & (0.5,0) & (0.7,0) & (0.2,0) \\ (0.2,0) & (0.6,0) & (0.4,0) & (0.8,0) \\ (0.7,0) & (0.1,0) & (0.3,0) & (0,0) \end{bmatrix}$$

Determinant of The Fuzzy Soft Square Matrix:

Definition 3.1: The determinant $|A|$ of an $n\times n$ fuzzy matrix A is defined as

$$|A| = \sum_{\sigma\in S_n} a_{1\sigma(1)} a_{2\sigma(2)} \cdots a_{n\sigma(n)} = \sum_{\sigma\in S_n} \prod_{k=1}^{n} a_{i\sigma(i)}$$

where $a_{i\sigma(i)} = (\mu_{j1\sigma(i)}(c_i), \mu_{j2\sigma(i)}(c_i)$ and S_n denote the symmetric group of all permutations of the indices $(1, 2, 3, \ldots, n)$

Definition 3.2: Let FSSM $_n$ be the set of all $(n\times n)$ fuzzy soft square matrices over [0,1], for every A in FSSM $_n$. Define determinant of A denoted by |A| $as\ |A| =$ det[A], where $A = [a_{ij}]$

Determinant of 2×2 fuzzy soft square matrix:

Let $A = \begin{bmatrix} (a_1,0) & (b_1,0) \\ (a_2,0) & (b_2,0) \end{bmatrix}$

The determinant of the fuzzy soft square matrix A would be denoted by,

$$|A| = \begin{vmatrix} (a_1,0) & (b_1,0) \\ (a_2,0) & (b_2,0) \end{vmatrix}$$

$$= [\max\{\min(a_1,b_2), \min(b_1,a_2)\}, \min\{\max(0,0), \max(0,0)\}],$$

Determinant of {3×3} fuzzy soft square matrix

Let $A = \begin{bmatrix} (a_1,0) & (b_1,0) & (c_1,0) \\ (a_2,0) & (b_2,0) & (c_2,0) \\ (a_3,0) & (b_3,0) & (c_3,0) \end{bmatrix}$

The determinant of the fuzzy soft square matrix A would be denoted by,

$$|A_1| = \begin{vmatrix} (a_1,0) & (b_1,0) & (c_1,0) \\ (a_2,0) & (b_2,0) & (c_2,0) \\ (a_3,0) & (b_3,0) & (c_3,0) \end{vmatrix}$$

$$= (a_1,0)[\max\{\min(b_2,c_3), \min(b_3,c_2)\}, 0] + (b_1,0)[\max\{\min(a_2,c_3), \min(a_3,c_2)\}, 0] + (c_1,0)[max\{min(a_2,b_3), min(a_3,b_2)\}, 0]$$

Example 3.3: Let $U = \{u_1, u_2, u_3\}$ is a universal set and $E = \{e_1, e_2, e_3, e_4, e_5\}$ be the set of parameters and $A = \{e_1, e_2, e_3\} \subseteq E$ and

$$(f, A) = \begin{Bmatrix} f_A(e_1) = \{u_1/(0.8,0), u_2/(0.1,0) u_3/(0.7,0)\} \\ f_A(e_2) = \{u_1/(0.4,0), u_2/(0.7,0) u_3/(0.5,0)\} \\ f_A(e_3) = \{u_1/(0.8,0), u_2/(0.4,0) u_3/(0.7,0)\} \end{Bmatrix}$$

The fuzzy soft square matrix represents the fuzzy soft set as

$$A = \begin{bmatrix} (0.8,0) & (0.4,0) & (0.8,0) \\ (0.1,0) & (0.7,0) & (0.4,0) \\ (0.7,0) & (0.5,0) & (0.7,0) \end{bmatrix}$$

Then the determinant of fuzzy soft square matrix A would be,

$$|A| = \begin{vmatrix} (0.8,0) & (0.4,0) & (0.8,0) \\ (0.1,0) & (0.7,0) & (0.4,0) \\ (0.7,0) & (0.5,0) & (0.7,0) \end{vmatrix}$$

$$= (0.8,0)[\max\{\min(0.7,0.7),\min(0.4,0.5)\},0]$$
$$+ (0.4,0)[\max\{\min(0.1,0.7),\min(0.4,0.7)\},0]$$
$$+ (0.8,0)[\max\{\min(0.1,0.5),\min(0.7,0.7)\},0]$$
$$= (0.8,0)[\max(0.7,0.4),0] + (0.4,0)[\max(0.1,0.4),0]$$
$$+ (0.8,0)[\max(0.1,0.7),0]$$
$$= (0.8,0)[(0.7,0)] + (0.4,0)[(0.4,0)] + (0.8,0)[(0.7,0)]$$
$$|A| = (0.7,0)$$

Theorem 3.4: Let FSSM $_n$ be the set of all $(n \times n)$ fuzzy soft square matrices over $F = [0,1]$ then for all matrices A and B in FSSM $_n$ and scalar α in [0,1], we have

1. $|A| = \det[A] \geq 0$ and $|A| = 0$ if and only if A=0
2. $|\alpha A| = \alpha \det[A]$ for any α in [0,1]
3. $|A+B| = \det[A] + \det[B]$ for A,B in FSSM $_n$

Proof: Let $A = [a_{ij}]_{n\times n}$, $a_{ij} = (\mu_{j1}(c_i), \mu_{j2}(c_i))$

$B = [b_{ij}]_{n\times n}$, $b_{ij} = (\chi_{j1}(c_i), \chi_{j2}(c_i))$

To avoid degenerate case we assume that $min(\mu_{j1}(c_i), \chi_{j1}(c_i)) \geq max(\mu_{j2}(c_i)), \chi_{j2}(c_i)) \; \forall i, j.$

(i)

1. If $|A|$ is a fuzzy soft matrix in FSSM $_n$, since all $a_{ij} \in [0,1]$,

$$det[A] = |A| \geq 0 \forall A \in FSSM_n.$$

(ii)

2. If $|A| = 0$, then $det[A] = 0; a_{ij} = 0 \forall ij, A = 0$

Conversely, if A=0, then $det[A] = 0, |A| = 0$ (iii)

Therefore $|A| = 0 \Leftrightarrow A = 0$,

If $\alpha \in [0,1]$ then, $\alpha A = \alpha[a_{ij}]_{n\times n}$

$$|\alpha A| = det[\alpha A] = det[A] = \alpha |A|$$

$$(3) Let |A| = det[A] and |B| = det[B]$$

Now, $|A+B| = det[C]$,

where $c_{ij} = [\max(\mu_{j1}(c_i), \chi_{j1}(c_i)),$
$\min(\mu_{j2}(c_i), \chi_{j2}(c_i))]$

$$|A+B| = det[[A]+[B]] = \det[A] + \det[B] = |A| + |B|$$

Definition 3.5: The adjoint matrix of an $n \times n$ fuzzy soft square matrix A is denoted by, adjA and its defined as, $b_{ij} = |A_{ij}|$, where $|A_{ij}|$ is the determinant of the $(n-1)\times(n-1)$ fuzzy soft matrix formed by deleting row j and column i from A. Then $B = adjA$. We can rewrite the element b_{ij} of $B = adjA = [b_{ij}]$ as follows

$adjA = b_{ij} = \sum_{\Pi \in S_{n_j n_i}} \prod_{t \in n_j} \mu_{t\sigma(t)}$ where $n_j = \{1,2,...n\}/\{j\}$, $n_i = \{1,2,...n\}/\{i\}$ and $S_{n_j n_i}$ is the set of all permutations of set n_j over the set n_i.

Proposition 3.6: Comparison of the adjoint of two fuzzy soft matrices for any two FSSMn, A and B we have the following, $A \leq B \Rightarrow adjA \leq adjB$

$adjA + adjB \leq adj(A+B)$

Proof:

(i) Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be the two FSSMn, where $a_{ij} = [(\mu_{j1}(c_i), \mu_{j2}(c_i))]$ and $b_{ij} = [(\chi_{j1}(c_i), \chi_{j2}(c_i))]$.

Let $C = adjA$ and $D = adjB$ (i.e)

$$c_{ij} = \sum_{\sigma \in s_{njni}} \prod_{t\in n_j} (\mu_{j1\sigma(t)}(c_i), (\mu_{j2\sigma(t)}(c_i))$$

and $d_{ij} = \sum_{\sigma \in s_{njni}} \prod_{t\in n_j} (\chi_{j1\sigma(t)}(c_i), (\chi_{j2\sigma(t)}(c_i))$.

Now it is clear that, $c_{ij} \leq d_{ij}$ because $\mu_{j1\sigma(t)}(c_i) \leq \chi_{j1\sigma(t)}(c_i)$ and $\mu_{j2\sigma(t)}(c_i) \leq \chi_{j2\sigma(t)}(c_i)$ for every $t \in n_j, t \neq j, \sigma(t) \neq \sigma(j)$. Therefore $C \leq D$

(i.e) $adjA \leq adjB$

Because $A \leq A+B$ and $B \leq A+B$. It is clear that $adjB \leq adj(A+B)$ and so that $adjA + adjB \leq adj(A+B)$.

Proposition 3.7: For a fuzzy soft square matrix $A, adjA' = (adjA)'$.

Proof: Let $B = adjA$ and $C = adjA'$.

Then $b_{ij} = \sum_{\sigma \in s_{njni}} \prod_{t\in n_j} (\mu_{j1\sigma(t)}(c_i), (\mu_{j2\sigma(t)}(c_i))$

and $c_{ij} = \sum_{\sigma \in s_{njni}} \prod_{\sigma(t)\in n_j} (\mu_{j1\sigma(t)}(c_i), (\mu_{j2\sigma(t)}(c_i))$,

$= (b_{ji})$

Hence $adjA' = (adjA')$.

Proposition 3.8: Let A be a $n \times n$ fuzzy soft matrix. Then $A(adjA) \geq |A| I_n$.

(i) $(adjA)A \geq |A| I_n$ .Where I_n is a unit matrix of order n.

Proof: Let $C = A(adjA)$.

The i^{th} row of A is given by, $(\mu_{i1}(c_1), \mu_{i2}(c_1))(\mu_{i1}(c_2), \mu_{i2}(c_2)) \ldots(\mu_{i1}(c_n), \mu_{i2}(c_n))$

Then by definition of $adjA$, then j^{th} column of $adjA$ is given by,

$(\chi_{j1}(c_1), \chi_{j2}(c_1))(\chi_{j1}(c_2), \chi_{j2}(c_2)) \ldots(\chi_{j1}(c_n), \chi_{j2}(c_n))$

Where each element is the cofactor of the element a_{ij} in A.

Therefore $c_{ij} = \sum_{k=1}^{n} (\mu_{i1}(c_k), \mu_{i2}(c_k))(\chi_{j1}(c_k), \chi_{j2}(c_k)) \geq 0$

And where

$$c_{ii} = \sum_{k=1}^{n} (\mu_{i1}(c_k), \mu_{i2}(c_k))(\chi_{i1}(c_k), \chi_{i2}(c_k)) = |A|.$$

Thus $C = A(adjA) \geq |A| I_n$

$$\text{where } |A| = \begin{bmatrix} |A| & 0 & 0 & \cdots 0 \\ 0 & |A| & 0 & \ldots 0 \\ \vdots & \vdots & \ddots & \vdots \vdots \\ 0 & 0 & 0 & \ldots |A| \end{bmatrix}$$

(i) Let $A = [a_{ij}]_{n \times n}$. Then j^{th} column of A is $(\mu_{1j}(c_1), \mu_{2j}(c_1))(\mu_{1j}(c_2), \mu_{2j}(c_2)) \ldots(\mu_{1j}(c_n), \mu_{2j}(c_n))$.

Let $B = [\chi_{ij}]_{n \times n}; b_{ij} = \chi_{ij}(e_j) = adjA$. Then i^{th} row of B is $(\chi_{i1}(e_1), \chi_{i2}(e_2), \ldots, \chi_{in}(e_n)) = (|A_{1i}|, |A_{2i}|, \ldots, |A_{ni}|)$

(ii) Let $C = [\eta_{ij}]_{n \times n} = (adjA)A$.

Then $(i, j)^{th}$ elements of $C = [\eta_{ij}]_{n \times n} = (adjA)A$ is

$\eta_{ij} = \sum_{k=1}^{n} |A_{ki} \mu_{kj}| \geq 0$ and where $\eta = \sum_{k=1}^{n} |A_{ki} \mu_{ki}| = |A|$.

Thus $C = (adjA)A \geq |A| I_n$ where I_n is a unit matrix of order n.

Proposition 3.9: Let A be a Fuzzy soft square Matrix, then the following properties hold

1. If A contains a zero row then $(adjA)A = [0,0]$ (the zero matrix)
2. If A contains a zero column then $A(adjA) = [0,0]$ (the zero matrix)

Proof: Suppose $A = [a_{ij}]$ be a FSSMn, where $a_{ij} = (\mu_{j1}(c_i), \mu_{j2}(c_i))$

Let $B = adjA$, then $b_{ij} = |A_{ji}|$ and $C = (adjA).A = B.A$

where $c_{ij} = \sum_{k=1}^{n} b_{ki}.a_{kj} = \sum_{k=1}^{n} |A_{ki}|.a_{kj}$

If the i^{th} row of A is zero, A_{ki} contains zero row where $k \neq i$ and therefore $|A_{ki}| = [0,0]$ $(i.e) a_{ij} = (\mu_{j1}(c_i), \mu_{j_2}(c_i))$,

then $\sum_{k=1}^{n} |A_{ki}|.a_{kj} = [0,0]$, $c_{ij} = \sum_{k=1}^{n} |A_{ki}|.a_{kj} = [0,0]$

Hence $C = (adjA).A = [0,0]$.

(i) Let $C = A(adjA) = A.B$, where

$$c_{ij} = \sum_{k=1}^{n} a_{ik}.b_{jk} = \sum_{k=1}^{n} a_{ik} |A_{jk}|.$$

If the j^{th} column of A is zero, then A_{jk} is zero, therefore $|A_{jk}| = [0,0]$ .Then

$$\sum_{k=1}^{n} a_{ik} |A_{jk}| = [0,0] \forall i, j$$

$$c_{ij} = \sum_{k=1}^{n} a_{ik}.b_{jk} = \sum_{k=1}^{n} a_{ik} |A_{jk}|$$

Hence $C = (adjA)A = [0,0]$

(ii) Let $C = A(adjA) = A.B$,

where $c_{ij} = \sum_{k=1}^{n} a_{ik}.b_{jk} = \sum_{k=1}^{n} a_{ik} |A_{jk}|$.

If the j^{th} column of A is zero, then A_{jk} contains zero column where $k \neq j$ and therefore $|A_{jk}| = [0,0]$, $\sum_{k=1}^{n} a_{ik} |A_{jk}| = [0,0] \forall i, j$

$$c_{ij} = \sum_{k=1}^{n} a_{ik} |A_{jk}| = [0,0].$$

Hence $C = (adjA)A = [0,0]$.

Theorem 3.10: For a fuzzy soft square matrix A, we have |A|=|adjA|

Proof: $|adjA| = \sum_{\sigma \in S_n} \prod_{t=n_k}^{n} |A_{k\sigma(k)}|$

$$= \sum_{\sigma \in s_n} \left[\prod_{i=1}^{n} \Big(\sum_{\theta \in S_{n_j n_{\sigma(k)}}} \prod_{t=n_k}^{n} (\mu_{j1\theta(t)}(c_i), \mu_{j2\theta(t)}(c_i)) \Big) \right]$$

$$= \sum_{\sigma \in s_n} \left[\sum_{\theta \in S_{n_1 n_{\sigma(1)}}} \left(\prod_{t=n_1}^{n} (\mu_{j1\theta(t)}(c_i), \mu_{j2\theta(t)}(c_i)) \right) \right]$$

$$\left(\sum_{\theta \in S_{n_2 n_{\sigma(2)}}} \left(\prod_{t=n_2}^{n} (\mu_{j1\theta(t)}(c_i), \mu_{j2\theta(t)}(c_i)) \right) \right)$$

$$\left(\sum_{\theta \in S_{n_n n_{\sigma(n)}}} \left(\prod_{t=n_n}^{n} (\mu_{j1\theta(t)}(c_i), \mu_{j2\theta(t)}(c_i)) \right) \right)$$

$$= \sum_{\sigma \in s_n} \left[\left(\prod_{t \in n_1} (\mu_{j1\theta_1(t)}(c_i), \mu_{j2\theta_1(t)}(c_i)) \right) \right.$$

$$\left(\prod_{t \in n_2} (\mu_{j1\theta_2(t)}(c_i), \mu_{j2\theta_2(t)}(c_i)) \right)$$

$$\cdots \left(\prod_{t \in n_n} (\mu_{j1\theta n(t)}(c_i), \mu_{j2\theta n(t)}(c_i)) \right)$$

for some $\theta_1 \in S_{n_1 n_{\sigma(1)}}, \theta_2 \in S_{n_2 n_{\sigma(2)}} \ldots \theta_n \in S_{n_n n_{\sigma(n)}}$

$$= \sum_{\sigma \in S_n} \Big\{ \Big[\big(\mu_{j1\theta_1(2)}(c_i), \mu_{j2\theta_1(2)}(c_i)\big)$$

$$\big(\mu_{j1\theta_1(3)}(c_i), \mu_{j2\theta_1(3)}(c_i)\big)$$

$$\ldots \big(\mu_{j1\theta_1(n)}(c_i), \mu_{j2\theta_1(n)}(c_i)\big) \Big]$$

$$\Big[\big(\mu_{j1\theta_2(1)}(c_i), \mu_{j2\theta_2(1)}(c_i)\big) \big(\mu_{j1\theta_2(3)}(c_i), \mu_{j2\theta_2(3)}(c_i)\big)$$

$$\ldots \big(\mu_{j1\theta_2(n)}(c_i), \mu_{j2\theta_2(n)}(c_i)\big) \Big]$$

$$\cdots \Big[\big(\mu_{j1\theta_n(1)}(c_i), \mu_{j2\theta_n(1)}(c_i)\big) \big(\mu_{j1\theta_n(2)}(c_i), \mu_{j2\theta_n(2)}(c_i)\big)$$

$$\ldots \big(\mu_{j1\theta_n(n-1)}(c_i), \mu_{j2\theta_n(n-1)}(c_i)\big) \Big] \Big\}$$

$$= \sum_{\sigma \in S_n} \Big[\big(\mu_{j1\theta_1(f_1)}(c_i), \mu_{j2\theta_1(f_1)}(c_i)\big)$$

$$\big(\mu_{j1\theta_2(f_2)}(c_i), \mu_{j2\theta_2(f_2)}(c_i)\big)$$

$$\ldots \big(\mu_{j1\theta_n(f_n)}(c_i), \mu_{j2\theta_n(f_n)}(c_i)\big) \Big]$$

where $f_n \in \{1, 2, \ldots n\}, n = 1, 2, \ldots, n$ .Then,

$\big(\mu_{j1\theta_n(f_n)}(c_i), \mu_{j2\theta_n(f_n)}(c_i)\big)$

$= \big(\mu_{j1\sigma(i)}(c_i), \mu_{j2\sigma(i)}(c_i)\big)$

therefore $|adjA| = \sum_{\sigma \in s_n} \prod_{i=1}^{n} \big(\mu_{j1\sigma(i)}(c_i), \mu_{j2\sigma(i)}(c_i)\big) = |A|$

$|adjA| = |A|$.

Proposition 3.11: Let A be an $n \times n$ constant fuzzy soft square matrix then

1. $(adjA)'$ is constant.
2. $C = A(adjA)$ is constant and $c_{ij} = |A|$, which is the least element in A.

Proof: Let A be an $n \times n$ constant FSSM, where its all rows are equal to each other. (i.e) $a_{ik} = a_{jk} \; \forall i, j$

(i) Let $B = (adjA)$,

then $b_{ij} = \sum_{\sigma \in S_{n_j n_i}} \prod_{t \in n_j} (\mu_{j1\sigma(t)}(c_i), \mu_{j2\sigma(t)}(c_i))$ and

$b_{ik} = \sum_{\sigma \in S_{n_k n_i}} \prod_{t \in n_k} (\mu_{j1\sigma(t)}(c_i), \mu_{j2\sigma(t)}(c_i))$

since the numbers $\sigma(t)$ of columns cannot be changed in the two expansions of b_{ij} and b_{ik} as A is constant and so $b_{ij} = b_{ik} \; \forall i, j, k$ .In order that $b_{jk} = b_{ki} \; \forall i, j, k$ we must have $(adjA)'$ is constant.

(ii) Since A is constant (i.e) $a_{ik} = a_{jk} \; \forall i, j, k$

then $A_{ik} = A_{jk} \; \forall i, j, k$ and so $|A_{ik}| = |A_{jk}| \; \forall i, j, k$

Let $C = [c_{ij}] = A(adjA)$,

then $c_{ij} = \sum_{k=1}^{n} a_{ik} |A_{jk}| = \sum_{k=1}^{n} a_{ik} |A_{ik}| = |A| \forall i, j$.

Thus $= A(adjA)$ is constant.

Now $|A| = \sum_{\sigma \in S_n} \prod_{i=1}^{n} (\mu_{j1\sigma(i)}(c_i), \mu_{j2\sigma(i)}(c_i))$

$$= \sum_{\sigma \in S_n} \Big[(\mu_{j1\sigma(i)}(c_1), \mu_{j2\sigma(1)}(c_1)),$$

$$(\mu_{j1\sigma(2)}(c_2), \mu_{j_2\sigma(2)}(c_2))$$

$$\ldots.(\mu_{j1\sigma(n)}(c_n), \mu_{j2\sigma(n)}(c_n)) \Big]$$

$$= (\mu_{j1\sigma(1)}(c_1), \mu_{j2\sigma(1)}(c_1)),$$

$$(\mu_{j1\sigma(2)}(c_2), \mu_{j2\sigma(2)}(c_2))$$

$\ldots(\mu_{j1\sigma(n)}(c_n), \mu_{j2\sigma(n)}(c_n))$ for any $\sigma \in S_n$.

Since A is constant $a_{ik} = a_{jk} \; \forall i, j, k$. Taking σ , the identity permutation. (i.e) $\sigma(i) = i, \forall i$ we get $|A| = a_{11}, a_{12} \ldots a_{nn}$; which is the least element in a constant fuzzy soft matrix A.

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