S130AFsB-TOPOLOGY AND THEORY OF FsB- PRODUCT TOPOLOGICAL SPACE

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Abstract: In this paper we introduce the concept of FsB-topology on a given Fs-set and FsB-subspace topology on a given Fs-subset. Also we introduce the concept of FsB-product topology on a Fs-Cartesian product of a given non-empty family of Fs-sets and we discuss some relevant results.

Keywords: Fs-set, Fs-subset, complement of an Fs-subset, FsB-topology, FsB-subspace topology and FsB-product topology.

Introduction: Ever since Zadeh [18] introduced the notion of fuzzy sets in his pioneering work, several mathematicians studied numerous aspects of fuzzy sets.

Murthy[8] introduced f-set in order to prove Axiom of choice for fuzzy sets which is not true for L-fuzzy sets introduced by Goguen[11]. Murthy [10] introduced the definition of f-complement of an f-subset in [10]. We can easily see that the collection all f-subsets of a given f-set with this definition of f-complement could not form a Boolean algebra. Recently many researchers put their efforts in order to prove collection of all fuzzy subsets of given fuzzy set is Boolean algebra under suitable operations and it seems among them the efforts of Tridiv [4],[5],[20] are most successful. The definition of fuzzy set given by Tridiv is based on the definition of fuzzy set given by H.K. Baruah [6]. Particularly in the definition of membership function of Tridiv [4] namely, $\mu_1(x) - \mu_2(x)$, $-\mu_2(x)$ will not be in the real interval [0,1].To eliminate this lacuna Vaddiparthi Yogeswara, G.Srinivas and Biswajit Rath introduced the concept of Fs-set and developed the theory of Fs-sets in order to prove collection of all Fs-subsets of given Fs-set is a complete Boolean algebra under Fs-unions, Fs-intersections and Fs-complements. The Fs-sets they introduced contain Boolean valued membership functions .They are successful in their efforts in proving that result with some conditions. In this paper we introduce the concept of FsB-topology on a given Fs set for any given Fs-subset of an Fs-set, also we introduce FsB-subspace topology. That after we developed the concept of FsB-product topology on an Fs-Cartesian product of a family of given Fs sets and we proved some results. We can observe that in the development of FsB-product topology we need the axiom of choice for Fs-sets [21].Here the operations on collection of Fs-subsets of $\mathcal{A}$ are Fs-union, Fs-intersection and Fs-complement. For smooth reading of the paper, the theory of Fs-sets in brief is dealt with in first two sections. We denote the largest element of a complete Boolean algebra L_A[1.1] by M_A or 1_A. We denote Fs-union and crisp set union by the same symbol $\cup$ and similarly Fs-intersection and crisp set intersection by the same symbol $\cap$. For all lattice theoretic properties and Boolean algebraic properties one can refer Szasz [13], Garret Birkhoff [14],Steven Givant • Paul Halmos[12] and Thomas Jech[15]. For results in topology one can refer[22]

Theory of Fs-Sets:

Fs-set: Let U be a universal set, $A_1 \subseteq U$ and let $A \subseteq U$ be non-empty. A four tuple $\mathcal{A} = (A_1, A, \overline{A}(\mu_{1A_1}, \mu_{2A}), L_A)$ is said be an Fs-set if, and only if

1. $A \subseteq A_1$
2. L_A is a complete Boolean Algebra
3. $\mu_{1A_1}: A_1 \longrightarrow L_A$, $\mu_{2A}: A \longrightarrow L_A$,are functions such that $\mu_{1A_1}|A \geq \mu_{2A}$
4. $\overline{A}$:$A \longrightarrow L_A$ is defined by $\overline{A}x = \mu_{1A_1}x \wedge (\mu_{2A}x)^c$,for each $x \in A$

Fs-subset: Let $\mathcal{A}=(A_1, A, \overline{A}(\mu_{1A_1}, \mu_{2A}), L_A)$ and $\mathcal{B}=(B_1, B, \overline{B}(\mu_{1B_1}, \mu_{2B}), L_B)$ be a pair of Fs-sets. $\mathcal{B}$ is said to be an Fs-subset of $\mathcal{A}$, denoted by $\mathcal{B} \subseteq \mathcal{A}$, if, and only if

1. $B_1 \subseteq A_1$, $A \subseteq B$
2. L_B is a complete subalgebra of L_A or $L_B \leq L_A$
3. $\mu_{1B_1} \leq \mu_{1A_1}|B_1$, and $\mu_{2B}|A \geq \mu_{2A}$

Proposition: Let $\mathcal{B}$ and $\mathcal{A}$ be a pair of Fs-sets such that $\mathcal{B} \subseteq \mathcal{A}$. Then $\overline{B}x \leq \overline{A}x$ is true for each $x \in A$

Definition: For some L_X, such that $L_X \leq L_A$ a four tuple $\mathcal{X} = (X_1, X, \overline{X}(\mu_{1X_1}, \mu_{2X}), L_X)$ is not an Fs-set if, and only if

1. (a)$X \nsubseteq X_1$ or
2. (b) $\mu_{1X_1}x \ngeq \mu_{2X}x$, for some $x \in X \cap X_1$

Here onwards, any object of this type is called an Fs-empty set of first kind and we accept that it is an Fs-subset of $\mathcal{B}$ for any $\mathcal{B} \subseteq \mathcal{A}$.

Definition: An Fs-subset $\mathcal{Y}=(Y_1, Y, \overline{Y}(\mu_{1Y_1}, \mu_{2Y}), L_Y)$ of $\mathcal{A}$, is said to be an Fs-empty set of second kind if, and only if

1. $Y_1 = Y = A$
2. $L_Y \leq L_A$
3. $\overline{Y} = 0$

Remark: we denote Fs-empty set of first kind or Fs-empty set of second kind by $\Phi_{\mathcal{A}}$and we prove later (1.15), $\Phi_{\mathcal{A}}$ is the least Fs-subset among all Fs-subsets of $\mathcal{A}$.

Definition: Let $\mathcal{B}_1 = (B_{11}, B_1, \overline{B}_1(\mu_{1B_{11}}, \mu_{2B_1}), L_{B_1})$ and $\mathcal{B}_2 = (B_{12}, B_2, \overline{B}_2(\mu_{1B_{12}}, \mu_{2B_2}), L_{B_2})$ be a pair of Fs-subsets.

1. We say that $\mathcal{B}_1$and $\mathcal{B}_2$ are (1,5)-equal, if $B_{11} = B_{12}$ and $L_{B_1} = L_{B_2}$
2. We say that $\mathcal{B}_1$and $\mathcal{B}_2$ are (2,5)-equal, if $B_1 = B_2$ and $L_{B_1} = L_{B_2}$
3. We say that $\mathcal{B}_1$and $\mathcal{B}_2$ are 3-equal, if $\mathcal{B}_1$and $\mathcal{B}_2$ are (1,5)-equal and $\mu_{1B_{11}} = \mu_{1B_{12}}$
4. We say that $\mathcal{B}_1$and $\mathcal{B}_2$ are 4-equal, if $\mathcal{B}_1$and $\mathcal{B}_2$ are (2,5)-equal and $\mu_{2B_1} = \mu_{2B_2}$
5. We say that $\mathcal{B}_1$and $\mathcal{B}_2$ are Total equal denoted $\mathcal{B}_1 = \mathcal{B}_2(T)$, if $\mathcal{B}_1$and $\mathcal{B}_2$ are (2,5)-equal and $\overline{B}_1 = \overline{B}_2$
6. We say that$\mathcal{B}_1$, $\mathcal{B}_2$ are Full-equal, denoted $\mathcal{B}_1 = \mathcal{B}_2$, if $\mathcal{B}_1$and $\mathcal{B}_2$ are 3-equal and 4-equal.

Proposition: $\mathcal{B}_1 = (B_{11}, B_1, \overline{B}_1(\mu_{1B_{11}}, \mu_{B_1}), L_{B_1})$ and $\mathcal{B}_2 = (B_{12}, B_2, \overline{B}_2(\mu_{1B_{12}}, \mu_{B_2}), L_{B_2})$ are equal if, only if $\mathcal{B}_1 \subseteq \mathcal{B}_2$ and $\mathcal{B}_2 \subseteq \mathcal{B}_1$

Definition of Fs-union for a given pair of Fs-subsets of $\mathcal{A}$: Let $\mathcal{B}$=$(B_1, B, \overline{B}(\mu_{1B_1,}\mu_{2B}), L_B)$and $\mathcal{C}$= $(C_1, C, \overline{C}(\mu_{1C_1}, \mu_{2C}), L_C)$, be a pair of Fs-subsets of$\mathcal{A}$. Then,
the Fs-union of $\mathcal{B}$ and $\mathcal{C}$,denoted by $\mathcal{B}$∪$\mathcal{C}$ is defined as
$\mathcal{B}\cup\mathcal{C}=\mathcal{D}=(D_1, D, \overline{D}(\mu_{1D_1}, \mu_{2D}), L_D)$,where

1. $D_1 = B_1 \cup C_1$, $D = B \cap C$
2. $L_D = L_B \vee L_C$=complete subalgebra generated by $L_B \cup L_C$
3. $\mu_{1D_1}: D_1 \longrightarrow L_D$ is defined by $\mu_{1D_1}x = (\mu_{1B_1} \vee \mu_{1C_1})x$

$\mu_{2D} : D \longrightarrow L_D$ is defined by $\mu_{2D}x = \mu_{2B}x \wedge \mu_{2C}x$
$\overline{D}: D \longrightarrow L_D$ is defined by $\overline{D}x = \mu_{1D_1}x \wedge (\mu_{2D}x)^c$

Proposition: $\mathcal{B}\cup\mathcal{C}$ is an Fs-subset of $\mathcal{A}$.

Definition of Fs-intersection for a given pair of Fs-subsets of $\mathcal{A}$: Let $\mathcal{B}$=$(B_1, B, \overline{B}(\mu_{1B_1,}\mu_{2B}), L_B)$ and $\mathcal{C}$= $(C_1, C, \overline{C}(\mu_{1C_1}, \mu_{2C}), L_C)$ be a pair of Fs-subsets of$\mathcal{A}$ satisfying the following conditions:

1. $B_1 \cap C_1 \supseteq B \cup C$
2. $\mu_{1B_1}x \wedge \mu_{1C_1}x \geq (\mu_{2B} \vee \mu_{2C})x$, for each $x \in A$

Then, the Fs-intersection of $\mathcal{B}$ and $\mathcal{C}$, denoted by $\mathcal{B}\cap\mathcal{C}$ is defined as $\mathcal{B}\cap\mathcal{C}$ =$\mathcal{E}$=$(E_1, E, \overline{E}(\mu_{1E_1}, \mu_{2E}), L_E)$,where

1. $E_1 = B_1 \cap C_1$, $E = B \cup C$
2. $L_E = L_B \wedge L_C$=$L_B \cap L_C$
3. $\mu_{1E_1}: E_1 \longrightarrow L_E$ is defined by $\mu_{1E_1}x = \mu_{1B_1}x \wedge \mu_{1C_1}x$
   $\mu_{2E} : E \longrightarrow L_E$ is defined by $\mu_{2E}x = (\mu_{2B} \vee \mu_{2C})x$
   $\overline{E}: E \longrightarrow L_E$ is defined by
   $\overline{E}x = \mu_{1E_1}x \wedge (\mu_{2E}x)^c$.

Remark: If (i) or (ii) fails we define $\mathcal{B}\cap\mathcal{C}$ as $\mathcal{B}\cap\mathcal{C}=\Phi_{\mathcal{A}}$, which is the Fs-empty set of first kind.

Proposition: For any pair of Fs-subsets $\mathcal{B}$=$(B_1, B, \overline{B}(\mu_{1B_1}, \mu_{2B}), L_B)$and $\mathcal{C}$=$(C_1, C, \overline{C}(\mu_{1C_1}, \mu_{2C}), L_C)$ of$\mathcal{A}$, the following results are true

1. $\mathcal{B}\subseteq\mathcal{B}\cup\mathcal{C}$ and $\mathcal{C}\subseteq\mathcal{B}\cup\mathcal{C}$
2. $\mathcal{B}\cap\mathcal{C}\subseteq\mathcal{B}$ and $\mathcal{B}\cap\mathcal{C}\subseteq\mathcal{C}$ provided $\mathcal{B}\cap\mathcal{C}$ exists
3. $\mathcal{B}\subseteq\mathcal{C}$ implies $\mathcal{B}\cup\mathcal{C}=\mathcal{C}$
4. $\mathcal{B}\cap\mathcal{C}=\mathcal{B}$ when $\mathcal{B}\neq \Phi_{\mathcal{A}}$ and $\mathcal{B}\subseteq\mathcal{C}$ and $\Phi_{\mathcal{A}} \cap\mathcal{C}=\Phi_{\mathcal{A}}$
5. $\mathcal{B}\cup\mathcal{C}= \mathcal{C} \cup \mathcal{B}$ (commutative law of Fs-union)
6. $\mathcal{B}\cap\mathcal{C}= \mathcal{C} \cap \mathcal{B}$ provided $\mathcal{B}\cap\mathcal{C}$ exists. (commutative law of Fs-intersection)
7. $\mathcal{B}\cup\mathcal{B}=\mathcal{B}$
8. $\mathcal{B}\cap\mathcal{B}=\mathcal{B}$

((7)and (8) are Idempotent laws of Fs-union and Fs-intersection respectively)

Proposition: For any Fs-subsets $\mathcal{B}$, $\mathcal{C}$and $\mathcal{D}$ of $\mathcal{A} = (A_1, A, \overline{A}(\mu_{1A_1}, \mu_{2A}), L_A)$, the following associative laws are true:

1. $\mathcal{B} \cup (\mathcal{C} \cup \mathcal{D}) = (\mathcal{B} \cup \mathcal{C}) \cup \mathcal{D}$
2. $\mathcal{B} \cap (\mathcal{C} \cap \mathcal{D}) = (\mathcal{B} \cap \mathcal{C}) \cap \mathcal{D}$, whenever Fs-intersections exist.

Arbitrary Fs-unions and arbitrary Fs-intersections: Given a family $(\mathcal{B}_i)_{i\in I}$ of Fs-subsets of $\mathcal{A}$=$(A_1, A, \overline{A}(\mu_{1A_1}, \mu_{2A}), L_A)$, where
$\mathcal{B}_i = (B_{1i}, B_i, \overline{B}_i(\mu_{1B_{1i}}, \mu_{2B_i}), L_{B_i})$,for any $i \in I$

Definition of Fs-union is as follows: Case (1): For I=Φ, define Fs-union of $(\mathcal{B}_i)_{i\in I}$,denoted by $\bigcup_{i\in I}\mathcal{B}_i$ as$\bigcup_{i\in I}\mathcal{B}_i = \Phi_{\mathcal{A}}$, which is the Fs-empty set
Case (2): Define for I≠Φ, Fs-union of $(\mathcal{B}_i)_{i\in I}$ denoted by $\bigcup_{i\in I}\mathcal{B}_i$ as follow
$\bigcup_{i\in I}\mathcal{B}_i = \mathcal{B} = (B_1, B, \overline{B}(\mu_{1B_1,}\mu_{2B}), L_B)$, where

1. $B_1 = \bigcup_{i\in I} B_{1i}$,$B = \bigcap_{i\in I} B_i$
2. $L_B = \bigvee_{i\in I} L_{B_i}$ = complete subalgebra generated by $\bigcup L_i(L_i = L_{B_i})$
3. $\mu_{1B_1}: B_1 \to L_B$ is defined by $\mu_{1B_1}x = (\bigvee_{i\in I} \mu_{1B_{1i}})x = \bigvee_{i\in I_x} \mu_{1B_{1i}}x$, where $I_x = \{i \in I | x \in B_i\}$

$\mu_{2B}: B \longrightarrow L_B$ is defined by $\mu_{2B}x = (\bigwedge_{i\in I} \mu_{2B_i})x$ =$\bigwedge_{i\in I} \mu_{2B_i}x$
$\overline{B}: B \longrightarrow L_B$ is defined by $\overline{B}x = \mu_{1B_1}x\wedge(\mu_{2B}\ x)^c$

Remark: We can easily show that (d) $B_1 \supseteq B$ and $\mu_{1B_1}|B \geq \mu_{2B}$.

Definition of Fs-intersection: Case (1): For I=Φ, we define Fs-intersection of $(\mathcal{B}_i)_{i\in I}$, denoted by $\bigcap_{i\in I}\mathcal{B}_i$ as $\bigcap_{i\in I}\mathcal{B}_i = \mathcal{A}$
Case (2): Suppose $\bigcap_{i\in I} B_{1i} \supseteq \bigcup_{i\in I} B_i$ and $\bigwedge_{i\in I} \mu_{1B_{1i}}|(\bigcup_{i\in I} B_i) \geq \bigvee_{i\in I} \mu_{2B_i}$
Then, we define Fs-intersection of $(\mathcal{B}_i)_{i\in I}$, denoted by $\bigcap_{i\in I}\mathcal{B}_i$ as follows
$\bigcap_{i\in I}\mathcal{B}_i = \mathcal{C} = (C_1, C, \overline{C}(\mu_{1C_1}, \mu_{2C}), L_C)$

1. $C_1 = \bigcap_{i\in I} B_{1i}$, $C = \bigcup_{i\in I} B_i$
2. $L_C = \bigwedge_{i\in I} L_{B_i}$
3. $\mu_{1C_1}: C_1 \longrightarrow L_C$ is defined by $\mu_{1C_1}x = (\bigwedge_{i\in I} \mu_{1B_{1i}})x = \bigwedge_{i\in I} \mu_{1B_{1i}}x$
   $\mu_{2C}: C \longrightarrow L_C$ is defined by $\mu_{2C}x = (\bigvee_{i\in I} \mu_{2B_i})x = \bigvee_{i\in I_x} \mu_{2B_i}x$,where,$I_x = \{i \in I | x \in B_i\}$
   $\overline{C}: C \longrightarrow L_C$ is defined by $\overline{C}x = \mu_{1C_1}x\wedge(\mu_{2C}\ x)^c$

Case (3): $\bigcap_{i\in I} B_{1i} \not\supseteq \bigcup_{i\in I} B_i$ or $\bigwedge_{i\in I} \mu_{1B_{1i}}|(\bigcup_{i\in I} B_i) \not\geq \bigvee_{i\in I} \mu_{2B_i}$ We define $\bigcap_{i\in I}\mathcal{B}_i = \Phi_{\mathcal{A}}$

Lemma: For any Fs-subset $\mathcal{B}$=$(B_1, B, \overline{B}(\mu_{1B_1}, \mu_{2B}), L_A)$ and $\mathcal{B} \subseteq \mathcal{B}_i = (B_{1i}, B_i, \overline{B}_i(\mu_{1B_{1i}}, \mu_{2B_i}), L_{B_i})$

Proposition: $(\mathcal{L}(\mathcal{A}), \cap)$ is $\wedge$-complete lattics.

Corollary: For any Fs-subset $\mathcal{B}$ of $\mathcal{A}$, the following results are true

1. $\Phi_{\mathcal{A}} \cup \mathcal{B} = \mathcal{B}$
2. $\Phi_{\mathcal{A}} \cap \mathcal{B} = \Phi_{\mathcal{A}}$.

Proposition: $(\mathcal{L}(\mathcal{A}), \cup)$ is $\vee$-complete lattics.

Corollary: $(\mathcal{L}(\mathcal{A}), \cup, \cap)$ is a complete lattice with $\vee$ and $\wedge$

Proposition: Let $\mathcal{B}$=$(B_1, B, \overline{B}(\mu_{1B_1}, \mu_{2B}), L_B)$, $\mathcal{C}$=$(C_1, C, \overline{C}(\mu_{1C_1}, \mu_{2C}), L_C)$and $\mathcal{D}$=$(D_1, D, \overline{D}(\mu_{1D_1}, \mu_{2D}), L_D)$.Then $\mathcal{B} \cup (\mathcal{C} \cap \mathcal{D})$=$(\mathcal{B} \cup \mathcal{C}) \cap (\mathcal{B} \cup \mathcal{D})$ provided $\mathcal{C} \cap \mathcal{D}$ exists.

Proposition: Let $\mathcal{B}$=$(B_1, B, \overline{B}(\mu_{1B_1}, \mu_{2B}), L_B)$, $\mathcal{C}$=$(C_1, C, \overline{C}(\mu_{1C_1}, \mu_{2C}), L_C)$and $\mathcal{D}$=$(D_1, D, \overline{D}(\mu_{1D_1}, \mu_{2D}), L_D)$.Then $\mathcal{B} \cap (\mathcal{C} \cup \mathcal{D})$=$(\mathcal{B} \cap \mathcal{C}) \cup (\mathcal{B} \cap \mathcal{D})$ provided in R.H.S $(\mathcal{B} \cap \mathcal{C})$ and $(\mathcal{B} \cap \mathcal{D})$ exist.

Proposition: Let $\mathcal{B}$=$(B_1, B, \overline{B}(\mu_{1B_1}, \mu_{2B}), L_B)$ and $\mathcal{C}_i = (C_{1i}, C_i, \overline{C}_i(\mu_{1C_{1i}}, \mu_{2C_i}), L_{C_i})$ be Fs-subsets of $\mathcal{A}$ for $i \in I$, then the following statements are true

1. $\mathcal{B} \cap (\cup_{i\in I} \mathcal{C}_i) = \cup_{i\in I}(\mathcal{B} \cap \mathcal{C}_i)$, provided $\mathcal{B} \cap \mathcal{C}_i$ exists for $i \in I$
2. $\mathcal{B} \cup (\cap_{i\in I} \mathcal{C}_i) = \cap_{i\in I}(\mathcal{B} \cup \mathcal{C}_i)$, provided $\cap_{i\in I} \mathcal{C}_i$ exists for $i \in I$

Definition: Consider a particular Fs-set $\mathcal{A} = (A_1, A, \overline{A}(\mu_{1A_1}, \mu_{2A}), L_A)$, $A \neq \Phi$,where

1. $A \subseteq A_1$
2. $L_A = [0, M_A]$, M_A is the largest element of L_A
3. , $\mu_{2A} = 0$

$\overline{A}x = \mu_{1A_1}x \wedge (\mu_{2A}x)^c = M_A$ for each $x \in A$

Given $\mathcal{B}$=$(B_1, B, \overline{B}(\mu_{1B_1}, \mu_{2B}), L_B)$. We define Fs-complement of $\mathcal{B}$ in $\mathcal{A}$, denoted by $\mathcal{B}^{C_{\mathcal{A}}}$ for B=A and $L_B = L_A$ as

$\mathcal{B}^{C_{\mathcal{A}}} = \mathcal{D}$=$(D_1, D, \overline{D}(\mu_{1D_1}, \mu_{2D}), L_D)$, where

1. $D_1 = C_A B_1 = B_1^c \cup A, D = B = A$ where $B_1^c = A_1 - B_1$
2. $L_D = L_A$
3. $\mu_{1D_1}: D_1 \longrightarrow L_A$ is defined by $\mu_{1D_1}x = M_A$

$\mu_{2D}: A \longrightarrow L_A$ is defined by $\mu_{2D}x = \overline{B}x = \mu_{1B_1}x \wedge (\mu_{2B}x)^c$

$\overline{D}: A \longrightarrow L_A$ is defined by

$$\overline{D}x = \mu_{1D_1}x \wedge (\mu_{2D}x)^c = M_A \wedge (\overline{B}x)^c = (\overline{B}x)^c.$$

Fs-Cartesian Product:

Definition: Let $(\mathcal{A}_i)_{i\in I}$ be a non empty family of non empty Fs-sets.

Define Fs-Cartesian Product of $(\mathcal{A}_i)_{i\in I}$, denoted by $\prod_{i\in I} \mathcal{A}_i$ as follows.

Here $\mathcal{A}_i = (A_{1i}, A_i, \overline{A}_i(\mu_{1A_{1i}}, \mu_{2A_i}), L_{A_i})$, L_{A_i} is a non-degenerating complete Boolean algebra and $\overline{A}_i a_i \neq 0$ for at least one $a_i \in A_i$

Here $\prod_{i\in I} \mathcal{A}_i = \mathcal{X} = (X_1, X, \overline{X}(\mu_{1X_1}, \mu_{2X}), L_X)$, where

$X_1 = \prod_{i\in I} A_{1i}$ such that $(\prod_{i\in I} A_{1i}, (P_{1i})_{i\in I})$ is the product of $(A_{1i})_{i\in I}$ in $\mathbb{SET}$, the category of sets with usual maps between crisp sets.

$X = \prod_{i\in I} A_i$ such that $(\prod_{i\in I} A_i, (P_i)_{i\in I})$ is the product of $(A_i)_{i\in I}$ in $\mathbb{SET}$, the category of sets with usual maps between crisp sets.

$L_X = \prod_{i\in I} L_{A_i}$ such that $(\prod_{i\in I} L_{A_i}, (\pi_i)_{i\in I})$ is the product of $(L_{A_i})_{i\in I}$ in $\mathbb{CBOO}$, the category of complete Boolean algebras with complete homomorphism between complete Boolean algebras.

$$\mu_{1X_1} = \prod_{i\in I} \mu_{1A_{1i}} : \prod_{i\in I} A_{1i} \longrightarrow \prod_{i\in I} L_{A_i}$$

$$(a_i)_{i\in I} \longmapsto (\mu_{1A_{1i}} P_{1i}(a_i)_{i\in I}) = (\mu_{1A_{1i}} a_i)_{i\in I}$$

$$\mu_{2X} = \prod_{i\in I} \mu_{2A_i} : \prod_{i\in I} A_i \longrightarrow \prod_{i\in I} L_{A_i}$$

$$(a_i)_{i\in I} \longmapsto (\mu_{2A_i} P_i(a_i)_{i\in I}) = (\mu_{2A_i} a_i)_{i\in I}$$

$$\overline{X} = \prod_{i\in I} \overline{A}_i : \prod_{i\in I} A_i \longrightarrow \prod_{i\in I} L_{A_i}$$

$(a_i)_{i\in I} \longmapsto (\overline{A}_i P_i(a_i)_{i\in I}) = (\overline{A}_i a_i)_{i\in I} = [\mu_{1A_{1i}} a_i \wedge (\mu_{2A_i} a_i)^c]_{i\in I}$ is an Fs-set

The Fs-function $(P_{1i}, P_i, \pi_i): \mathcal{X} \longrightarrow \mathcal{A}_i$ are Fs-projections

In particular $\prod_{i\in I} \mathcal{A}_i = \mathcal{X} = \mathcal{A}^I$ where $\mathcal{A}_i = \mathcal{A}, \forall i \in I$

Fs-Cartesian product of non empty family of non-empty Fs-subsets of $\mathcal{A}$: Let $(\mathcal{B}_i)_{i\in I}$ be a non empty family of non empty Fs-subsets of $\mathcal{A}$.

Define Fs-Cartesian Product of $(\mathcal{B}_i)_{i\in I}$, denoted by $\prod_{i\in I} \mathcal{B}_i$ as follows.

Here $\mathcal{B}_i = (B_{1i}, B_i, \overline{B}_i(\mu_{1B_{1i}}, \mu_{2B_i}), L_{B_i})$, L_{B_i} is a non-degenerating complete Boolean algebra and $\overline{B}_i b_i \neq 0$ for at least one $b_i \in B_i$

Let $\prod_{i\in I} \mathcal{B}_i = \mathcal{C} = (C_1, C, \overline{C}(\mu_{1C_1}, \mu_{2C}), L_C)$, where

$C_1 = \prod_{i\in I} B_{1i}$ such that $(\prod_{i\in I} L_{B_{1i}}, (P_{1i})_{i\in I})$ is the product of $(B_{1i})_{i\in I}$ in $\mathbb{SET}$, the category of sets with usual maps.

$C = \prod_{i\in I} B_i$ such that $(\prod_{i\in I} B_i, (P_i)_{i\in I})$ is the product of $(B_i)_{i\in I}$ in $\mathbb{SET}$, the category of sets with usual maps.

$L_C = \prod_{i\in I} L_{B_i}$ such that $(\prod_{i\in I} B_i, (\pi_i)_{i\in I})$ is the product of $(L_{B_i})_{i\in I}$ in $\mathbb{CBOO}$, the category of complete Boolean algebras.

$$\mu_{1C_1} = \prod_{i\in I} \mu_{1B_{1i}} : \prod_{i\in I} B_{1i} \longrightarrow \prod_{i\in I} L_{B_i}$$

$$(b_i)_{i\in I} \longmapsto (\mu_{1B_{1i}} P_{1i}(b_i)_{i\in I}) = (\mu_{1B_{1i}} b_i)_{i\in I}$$

$$\mu_{2C} = \prod_{i\in I} \mu_{2B_i} : \prod_{i\in I} B_i \longrightarrow \prod_{i\in I} L_{B_i}$$

$$(b_i)_{i\in I} \longmapsto (\mu_{2B_i} P_i(b_i)_{i\in I}) = (\mu_{2B_i} b_i)_{i\in I}$$

$$\overline{C} = \prod_{i\in I} \overline{B}_i : \prod_{i\in I} B_i \longrightarrow \prod_{i\in I} L_{B_i}$$

$(b_i)_{i\in I} \longmapsto (\overline{B}_i, P_i(b_i)_{i\in I}) = (\overline{B}_i b_i)_{i\in I} = [\mu_{1B_{1i}} b_i \wedge (\mu_{2B_i} b_i)^c]_{i\in I}$.

Then, $\mathcal{C}$ is an Fs-subset of $\mathcal{A}^I$

Here $(P_{1i}, P_i, \pi_i): \mathcal{C} \longrightarrow \mathcal{B}_i$ is a preserving Fs-function, where $P_{1i}|_C = P_i$. i.e following diagrams are commutative.

$$\begin{array}{ccc} \prod_{i\in I} B_{1i} & \xrightarrow{P_{1i}} & B_{1i} \\ (\mu_{1B_{1i}})_{i\in I} \downarrow & & \downarrow \mu_{1B_{1i}} \\ \prod_{i\in I} B_i & \xrightarrow{P_i} & B_i \end{array}$$

$$\begin{array}{ccc} \prod_{i\in I} L_{B_i} & \xrightarrow{\pi_i} & L_{B_i} \\ (\mu_{2B_i})_{i\in I} \downarrow & & \downarrow \mu_{2B_i} \\ \prod_{i\in I} L_{B_i} & \xrightarrow{\pi_i} & L_{B_i} \end{array}$$

(Dig-3.1)

$$(\mu_{1B_{1i}} \circ P_{1i})(b_i)_{i\in I} = \mu_{1B_{1i}} b_i$$
$$\pi_i \circ (\mu_{1B_{1i}})_{i\in I}(b_i)_{i\in I} = \mu_{1B_{1i}} b_i$$
$$(\mu_{2B_i} \circ P_i)(b_i)_{i\in I} = \mu_{2B_i} b_i$$
$$\pi_i \circ (\mu_{2B_i})_{i\in I}(b_i)_{i\in I} = \mu_{2B_i} b_i$$

Proposition: $\Pi_i: \prod_{i\in I} L_{B_i} \longrightarrow L_{B_i}$ is a complete homomorphism.

Axiom of Choice for Fs-Sets:

Theorem Let $(\mathcal{B}_i)_{i\in I}$ be a nonempty family of non empty Fs-subsets of $\mathcal{A}$, then Fs-Cartesian Product of $(\mathcal{B}_i)_{i\in I}$, namely $\prod_{i\in I} \mathcal{B}_i$ is a non-empty Fs-subset, that is, $\prod_{i\in I} \mathcal{B}_i \neq \Phi_{\mathcal{A}}$

Converse of the above theorem is true conditionally:

Proposition Let $(\mathcal{B}_i)_{i\in I}$ be a family of Fs-subsets such that $\prod_{i\in I} \mathcal{B}_i \neq \Phi_{\mathcal{A}}$. Then, for each $i \in I$, $\mathcal{B}_i$ are non-Fs-empty set of first kind.

FsB-topology:

Definition: $\mathcal{A} = (A_1, A, \overline{A}(\mu_{1A_1}, \mu_{2A}), L_A)$, $L_A =$ a complete Boolean algebra, $\mu_{1A_1} = 1, \mu_{2A} = 0, \overline{A} = 1$

$\mathfrak{T} \subseteq \mathcal{L}(\mathcal{A})$ is said to be FsB –topology if, and only if

1. $(\mathcal{B}_i)_{i\in I} \subseteq \mathfrak{T} \Rightarrow \bigcup_{i\in I} \mathcal{B}_i \in \mathfrak{T}$
2. $(\mathcal{B}_i)_{i\in I}$, I is finite set $\Rightarrow \bigcap_{i\in I} \mathcal{B}_i \in \mathfrak{T}$

The following are important remarks:

1. Elements of $\mathfrak{T}$ are called FsB-open sets or FsB-open subsets of $\mathcal{A}$.
2. Union over empty set, that is, for $I = \Phi, \bigcup_{i\in I} \mathcal{B}_i = \Phi_{\mathcal{A}}$ -Fs-empty set is in $\mathfrak{T}$.
3. Intersection over empty set, that is, for $I = \Phi, \bigcap_{i\in I} \mathcal{B}_i = \mathcal{A}$ is in $\mathfrak{T}$.
4. $\mathcal{B} \in \mathfrak{T} \Leftrightarrow \mathcal{B}^{C_{\mathcal{A}}}$ is FsB-closed set of $\mathcal{A}$.
5. $(\mathcal{A}, \mathfrak{T})$ is called FsB- topological space
6. We write $\mathcal{A}$ for $(\mathcal{A}, \mathfrak{T})$ itself.
7. $\mathcal{A}$ and $\Phi_{\mathcal{A}}$ are always FsB-open sets of the FsB-topological space $\mathcal{A}$.

Definition: FsB-closure of $\mathcal{B}$ is denoted by $\overline{\mathcal{B}}$ and is defined as $\overline{\mathcal{B}} = \bigcap\{\mathcal{C} | \mathcal{C} \text{ is closed}, \mathcal{B} \subseteq \mathcal{C}\}$

Remark: Clearly $\overline{\mathcal{B}}$ is an FsB-closed subset of $\mathcal{A}$.

Theorem: $\mathcal{B}$ is FsB-closed set if, and only, if $\mathcal{B} = \overline{\mathcal{B}}$

Definition: FsB-interior of $\mathcal{B}$- is denoted by $\mathcal{B}^{\circ}$ and is defined as $\mathcal{B}^{\circ} = \bigcup\{\theta | \theta \text{ is FsB} - \text{open}, \theta \subseteq \mathcal{B}\}$

Remark: Clearly $\mathcal{B}^{\circ}$ is an FsB-open subset of $\mathcal{A}$.

Theorem: $\mathcal{B}$ is FsB-open set if, and only, if $\mathcal{B} = \mathcal{B}^{\circ}$

Definition: An Fs-subset $\mathcal{G}$ of is said to be compact if, and only if, $\mathcal{G} \in \bigcup_{i\in I} \mathcal{B}_i$ implies $\mathcal{G} \subseteq \mathcal{B}_{i_1} \cup \mathcal{B}_{i_2} \cup \ldots \cup \mathcal{B}_{i_k}$ for some $i_1, i_2, \ldots, i_k \in I$, where $\mathcal{B}_i$ is FsB-open for each $i \in I$.We call $(\mathcal{B}_i)_{i\in I}$ is an FsB-open cover of $\mathcal{G}$ and the family $\{\mathcal{B}_{i_1}, \mathcal{B}_{i_2}, \ldots, \mathcal{B}_{i_k}\}$ is a finite FsB-open cover of $\mathcal{G}$.

Definition: FsB-open Base or FsB-open Basis:

Let $(\mathcal{A}, \mathfrak{T})$ be an FsB-topological space, $\mathcal{G}$ be an FsB-open set. $\mathfrak{B} \subseteq \mathfrak{T}$ is said to be an FsB-open base or FsB-open basis of $\mathfrak{T} \Leftrightarrow$ for some $\mathfrak{C} \subseteq \mathfrak{B}$, $\mathcal{G} = \bigcup_{\mathcal{B}\in\mathfrak{C}} \mathcal{B}$.

Definition: FsB-open subbase or FsB-open subbasis:

Let $(\mathcal{A}, \mathfrak{T})$ be an FsB-topological space, let $\mathfrak{B}$ be an FsB-open base of $\mathfrak{T}$. $\mathfrak{S} \subseteq \mathfrak{B}$ is said to be an FsB-open subbase or FsB-open subbasis for $\mathfrak{T} \Leftrightarrow \mathcal{B} = \mathcal{S}_1 \cap \mathcal{S}_2 \cap \ldots \cap \mathcal{S}_n$, for some $\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_n \in \mathfrak{S}$ for any $\mathcal{B} \in \mathfrak{B}$

FsB-topological subspace: Let $(\mathcal{A}, \mathfrak{T})$ be a given FsB-topological space. Let $\mathcal{B} \subseteq \mathcal{A}$ (i.e. $\mathcal{B}$ is an Fs-subset of $\mathcal{A}$). We define $\mathfrak{T}_{\mathcal{B}}$ as $\mathfrak{T}_{\mathcal{B}} = \{\mathcal{B} \cap \mathcal{G} | \mathcal{G} \in \mathfrak{T}\}$

Theorem: $\mathfrak{T}_{\mathcal{B}}$ is an FsB-topology on $\mathcal{B}$.

Proof: $\mathfrak{T}_{\mathcal{B}} = \{\mathcal{B} \cap \mathcal{G} | \mathcal{G} \in \mathfrak{T}\}$

Case(1): $\mathcal{G} = \Phi_{\mathcal{A}} \Rightarrow \mathcal{B} \cap \mathcal{G} = \Phi_{\mathcal{A}} \in \mathfrak{T}_{\mathcal{B}}$.

Case(2): $\mathcal{G} = \mathcal{A} \Rightarrow \mathcal{B} \cap \mathcal{G} = \mathcal{B} \in \mathfrak{T}_{\mathcal{B}}$

Case(3): $\mathcal{B} \cap \mathcal{G}_1, \mathcal{B} \cap \mathcal{G}_2 \in \mathfrak{T}_{\mathcal{B}}$ and $\mathcal{B} \cap \mathcal{G}_1, \mathcal{B} \cap \mathcal{G}_2$ both are non-Fs-empty sets. then

$(\mathcal{B} \cap \mathcal{G}_1) \cap (\mathcal{B} \cap \mathcal{G}_2) = \mathcal{B} \cap (\mathcal{G}_1 \cap \mathcal{G}_2) \in \mathfrak{T}_{\mathcal{B}}$

If $\mathcal{B} \cap \mathcal{G}_1 = \Phi_{\mathcal{A}}$ or $\mathcal{B} \cap \mathcal{G}_2 = \Phi_{\mathcal{A}}$, then

$(\mathcal{B} \cap \mathcal{G}_1) \cap (\mathcal{B} \cap \mathcal{G}_2) = \Phi_{\mathcal{A}} \in \mathfrak{T}_{\mathcal{B}}$

Let $\{\mathcal{B} \cap \mathcal{G}_i\}_{i\in I} \subseteq \mathfrak{T}_{\mathcal{B}}$

Let $I_1 = \{i \in I | \mathcal{G}_i = \Phi_{\mathcal{A}}\}$, $I_2 = \{i \in I | \mathcal{G}_i \neq \Phi_{\mathcal{A}}\}$

$\bigcup_{i\in I}(\mathcal{B} \cap \mathcal{G}_i) = [\bigcup_{i\in I_1}(\mathcal{B} \cap \mathcal{G}_i)] \cup [\bigcup_{i\in I_2}(\mathcal{B} \cap \mathcal{G}_i)]$

$= \Phi_{\mathcal{A}} \cup [\bigcup_{i\in I_2}(\mathcal{B} \cap \mathcal{G}_i)] = [\bigcup_{i\in I_2}(\mathcal{B} \cap \mathcal{G}_i)]$

$= \mathcal{B} \cap \bigcup_{i\in I_2}(\mathcal{G}_i) = \mathcal{B} \cap \bigcup_{i\in I}(\mathcal{G}_i) \in \mathfrak{T}_{\mathcal{B}}$ $(\because \bigcup_{i\in I} \mathcal{G}_i \in \mathfrak{T}_{\mathcal{B}})$

FsB-Product Topology: $\mathcal{S} = \prod_{i\in I} \mathcal{G}_i$ with $\mathcal{G}_i$ is open in $\mathcal{A}_i$

Every component of RHS is $\mathcal{A}_i$ for each $j \neq i$ and at the j^{th} place $\mathcal{G}_j$ is there.

$\mathfrak{S} = \{\mathcal{S}\}$ is called defining FsB-open subbase for FsB-Product topology on $\prod_{i\in I} \mathcal{A}_i$

The FsB-open base $\mathfrak{B}$ generated by $\mathcal{S}$ is described as

$$\mathfrak{B} = \left\{\prod_{i\in I} \mathcal{G}_i \middle| \mathcal{G}_i = \mathcal{A}_i \ \forall i \in I - \{i_1, i_2, \ldots, i_n\}, i_1, i_2, \ldots, i_n \in I\right\}$$

$\mathfrak{B}$ is called defining FsB-open base for the FsB-topology generated by $\mathfrak{B}$

The FsB-topology on $\prod_{i\in I} \mathcal{A}_i$ generated by $\mathfrak{B}$ is called FsB-Product topology

Let $\mathcal{A}_i = (A_{1i}, A_i, \overline{A}_i(\mu_{1A_{1i}}, \mu_{2A_i}), L_{A_i})$ be a family of FsB-topological spaces

Let $\prod_{i\in I} \mathcal{A}_i$ be Fs-Cartesian product of the family $\{\mathcal{A}_i\}_{i\in I}$

Let $\mathfrak{S} = \{\mathcal{S}\}$ where $\mathcal{S} = \prod_{i\in I} \mathcal{B}_i$, where $\mathcal{B}_i = \begin{cases} \mathcal{A}_i, & i \neq i_1 \\ \mathcal{G}_i, & i = i_1 \end{cases}$

and $\mathcal{G}_{i_1}$ be FsB-open in $\mathcal{A}_{i_1}$, $i_1 \in I$

Where $\mathcal{G}_{i_1} = \left(G_{1i_1}, G_{i_1}, \overline{G}_{i_1}\left(\mu_{1G_{1i_1}}, \mu_{2G_{i_1}}\right), L_{G_{i_1}}\right)$

$\mathfrak{B} = \{\mathcal{S}_1 \cap \mathcal{S}_2 \cap \dots \cap \mathcal{S}_n | \mathcal{S}_1, \mathcal{S}_2, \dots, \mathcal{S}_n \in \mathfrak{S}\}$

Theorem: $\mathfrak{B}$ is an FsB-open base for some FsB-topology on $\mathcal{A}$

Proof: Let $\mathfrak{T} = \{\bigcup_{j\in J} \mathcal{B}_j \,|\mathcal{B}_j \in \mathfrak{B}\}$

Clearly $\Phi_{\mathcal{A}} \in \mathfrak{T}$

$\mathcal{G}_1 \cap \mathcal{G}_2 = \mathcal{G}_1 \cap \left(\bigcup_{j_2\in J_2} \mathcal{B}'_{j_2}\right) = \bigcup_{j_2\in J_2}\left(\mathcal{G}_1 \cap \mathcal{B}'_{j_2}\right)$

$= \bigcup_{j_2\in J_2}\left\{\left(\bigcup_{j_1\in J_1} \mathcal{B}_{j_1}\right) \cap \mathcal{B}'_{j_2}\right\} = \bigcup_{j_2\in J_2}\left\{\bigcup_{j_1\in J_1}\left(\mathcal{B}_{j_1} \cap \mathcal{B}'_{j_2}\right)\right\}$

$= \bigcup_{(j_1,j_2)\in J_1\times J_2}\left(\mathcal{B}_{j_1} \cap \mathcal{B}'_{j_2}\right)$

Observe that $\mathcal{B}_{j_1} \cap \mathcal{B}'_{j_2} \in \mathfrak{B}$ $\left(\because \mathcal{B}_{j_1}, \mathcal{B}'_{j_2} \in \mathfrak{B} \Rightarrow \mathcal{B}_{j_1} = \mathcal{S}_1 \cap \mathcal{S}_2 \cap \dots \cap \mathcal{S}_{n_1}, \mathcal{B}'_{j_2} = \mathcal{S}'_1 \cap \mathcal{S}'_2 \cap \dots \cap \mathcal{S}'_{n_2} \Rightarrow \mathcal{B}_{j_1} \cap \mathcal{B}'_{j_2} = \mathcal{S}_1 \cap \mathcal{S}_2 \cap \dots \cap \mathcal{S}_{n_1} \cap \mathcal{S}'_1 \cap \mathcal{S}'_2 \cap \dots \cap \mathcal{S}'_{n_2},\ \text{where } \mathcal{S}_1, \mathcal{S}_2, \dots, \mathcal{S}_n, \mathcal{S}'_1, \mathcal{S}'_2, \dots, \mathcal{S}'_{n_2} \in \mathfrak{S}\right)$

Therefore $\mathcal{G}_1 \cap \mathcal{G}_2 \in \mathfrak{T}$

If $\mathcal{G}_1 = \Phi_{\mathcal{A}}$ or $\mathcal{G}_2 = \Phi_{\mathcal{A}}$, then $\mathcal{G}_1 \cap \mathcal{G}_2 = \Phi_{\mathcal{A}} \in \mathfrak{T}$

Let $\mathcal{G}_j \in \mathfrak{T}$ for each $j \in s\mathfrak{T}$

$\mathcal{G}_j = \bigcup_{j_k\in J} \mathcal{B}_{j_k}$

$\therefore \bigcup_{j\in J} \mathcal{G}_j = \bigcup_{j\in J}\left(\bigcup_{j_k\in J} \mathcal{B}_{j_k}\right)$

Define a choice function $\gamma: J \longrightarrow J$ such that

$\bigcup_{j\in J} \mathcal{G}_j = \bigcup_{\gamma: J\to J}\left(\bigcup_{\gamma(k)\in J} \mathcal{B}_{k\gamma(k)}\right) \in \mathfrak{T}\ \left(\because \mathcal{B}_{k\gamma(k)} \in \mathfrak{B}\right)$

Suppose $\mathfrak{T}_1$ is some other FsB-topology containing $\mathfrak{S}$, then clearly $\mathcal{B} \in \mathfrak{T}_1$. So that, $\mathfrak{T} \in \mathfrak{T}_1$

Hence $\mathfrak{T}$ is the smallest topology containing $\mathfrak{S}$, that is, $\mathfrak{S}$ is the FsB-open subbase for FsB-topology $\mathfrak{T}$

This $\mathfrak{T}$ we call Fs-Cartesian product FsB-topology or simply FsB-product topology.

Conclusion: Tychonoff Theorem on FsB-product topology and neighborhood system of a given Fs-point of a given FsB-topological space are under study.

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