DIFFERENTIAL TRANSFORMATION METHOD FOR SOLVING KOLMOGROVE-PETROVSKII-PISKUNOV EQUATION AND POROUS MEDIUM EQUATION

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Abstract: In this paper, Differential Transform Method is used to find the solution of Kolmogroveç- Petrovskii-Piskunov equation and Porous Medium equation generally arising in heat and mass transfer and Oil Recovery problems respectively. Finally the obtained solution is compared with the exact solution for Kolmogrove-Petrovskii-Piskunov equation and with VIM for porous medium equation. Interpretation of the results has been done by using maple software.

Keywords: Differential Transform Method, Kolmogrove-Petrovskii-Piskunov Equation, Porous medium equation

Introduction: Mathematical modeling of many physical systems generally arising in various fields of heat and mass transfer, combustion theory, biology, ecology & oil recovery leads to ordinary/partial differential equations. Now a days many effective analytical/approximate methods i.e Adomian decomposition method, Homotopy perturbation method, Variation iteration method, Homotopy analysis method and Differential transform method (DTM) has been proposed for finding the analytical solutions of these type of ordinary/partial differential equations. The Homotopy perturbation method (HPM) and Variation iteration method (VIM) is proposed by Ji-Huan He in 1997. On the other hand homotopy analysis method (HAM) is developed and proposed by ShijunLiao in 1992. These methods provides an immediate and visible symbolic terms of analytic solutions, as well as numerical approximate solutions of differential equations.

The Differential transform method (DTM) was first introduced by Zhou [7] in 1986 to solve linear and nonlinear initial/boundary value problems generally arising in electrical circuit analysis. This method constructs an analytical solution in the form of series solution with x and t.

Infect differential transform method has been developed for solving various types of differential and integral equations. For example in [4] this method has been used for solving a system of differential equations and in [5] for differential–algebraic equations. In [3, 6], this method has been applied to partial differential equations and in [1, 9] to one dimensional Volterra integral and integro-differential equations. Also in [2] the DTM has been developed for solving two-dimensional Volterra integral equations.

Here in this paper Kolmogrove-Petrovskii-Piskunov equation and porous medium equation is solved by using Differential Transform Method and compared the obtained solution with the exact solution and VIM solution respectively.

Basic idea of Differential Transform Method.

Definition: Differential Transform of function w(x,y) can be defined as[7] :

$$W(k,h)=\frac{1}{k!h!}\left[\frac{\partial^{k+h}}{\partial^k x\partial^h y}[w(x,y)]_{(0,0)}\right]$$

Where $w(x,y)$ is the original function and $W(k,h)$ is the transformed function, which is known as as the T-function.

The inverse differential transform of $w(x,y)$ can be defined as [7]:

$$w(x,y)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}W(k,h)x^k y^h$$

$$=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\frac{1}{k!h!}\left[\frac{\partial^{k+h}}{\partial^k x\partial^h y}[w(x,y)]_{(0,0)}\right]x^k y^h$$

Properties of Differential Transform Method.[6]

1. If $w(x,y)=u(x,y)+v(x,y)$ then
$$W(\mathrm{k,h})=U(\mathrm{k,h})+V(\mathrm{k,h}).$$
2. If $w(x,y)=C.u(x,y)$ then
$$W(\mathrm{k,h})=C.U(\mathrm{k,h}).$$
3. If $w(x,y)=\frac{\partial u(x,y)}{\partial x}$ then
$$W(k,h)=(k+1)U(k+1,h)$$
4. If
$$w(x,y)=\frac{\partial u(x,y)}{\partial y}$$
then $W(k,h)=(h+1)U(k,h+1)$.
5. If $w(x,y)=u(x,y)\square v(x,y)$ then
$$W(k,h)=\sum_{r=0}^{k}\sum_{s=o}^{h}U(r,h-s)V(k-r,s)$$

6. If $w(x,y)=\frac{\partial u(x,y)}{\partial x}\frac{\partial u(x,y)}{\partial y}$ then

$$W(k,h)=\sum_{r=0}^{k}\sum_{s=0}^{k}(r+1)(k-r+1)U(r+1,h-s)V(k-r+1,s)$$

7. If $w(x,y)=\frac{\partial^{r+s}u(x,y)}{\partial x^r\partial y^s}$ then

$$W(\mathrm{k},\mathrm{h})=(\mathrm{k}+1)(\mathrm{k}+2)\ldots(\mathrm{k}+\mathrm{r}).(\mathrm{h}+1)(\mathrm{h}+2)\ldots(\mathrm{h}+\mathrm{s})U(\mathrm{k}+\mathrm{r},\mathrm{h}+\mathrm{s}).$$

8. If w(x,y)= $w(x,y)=x^m y^n$ then

$$W(k,h)=\delta(k-m.h-n)=\delta(k-m)\delta(h-n)$$

Where $\delta(k-m)=\begin{cases}1 & k=m\\ 0 & k\neq m\end{cases}$ and

$$\delta(h-n)=\begin{cases}1 & h=n\\ 0 & h\neq n\end{cases}$$

Application of DTM to Kolmogrove-Petrovskii-Piskunov Equation: Consider a Kolmogrove-Petrovskii-Piskunov Equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial t^2}+u-u^2$

with initial condition $u(x,0)=(1+e^{\sqrt{1/6}x})^{-2}$ (1)

The exact solution of eq.(1) is given by

$$u(x,t)=(1+e^{\frac{-5t}{6}+\sqrt{1/6}x})^{-2} \quad (2)$$

The Differential transform of eq.(1) can be written as

$$(h+1)U(k,h+1)=(k+1)(k+2)U(k+2,h)+U(k,h)-\sum_{r=0}^{k}\sum_{s=0}^{h}U(r,h-s)U(k-r,s) \quad (3)$$

Using the initial conditions (1) in eq. (3), we have,

$$U(0,0)=\frac{1}{4},U(1,0)=\frac{-\sqrt{6}}{24},U(2,0)=\frac{1}{96},$$
$$U(3,0)=\frac{\sqrt{6}}{1728}U(4,0)=\frac{-1}{3456}, \quad (4)$$
$$U(5,0)=\frac{-\sqrt{6}}{103680},U(6,0)=\frac{17}{2488320}$$

Now by substituting the results (4) in eq. (3), it obtains

$U(0,1)=\frac{5}{24}$, $U(1,1)=\frac{-5\sqrt{6}}{288}$, $U(2,1)=\frac{-5}{576}$, $U(3,1)=\frac{-5\sqrt{6}}{5184}$, $U(4,1)=\frac{5}{20736}$, $U(0,2)=\frac{25}{576}$, $U(1,2)=\frac{25\sqrt{6}}{3456}$, $U(2,2)=\frac{25}{3456}$, $U(0,3)=\frac{-125}{10368}$

Hence the approximate solution of eq.(1) up to 3 degree terms can be written as

$$u(x,t)=1/4-(1/24)\sqrt{6}x+(1/96)x^2+(1/1728)\sqrt{6}x^3-(1/3456)x^4-(1/103680)\sqrt{6}x^5-(17/2488320)\sqrt{6}x^6+(5/24)t-(5/288)\sqrt{6}xt-(5/576)x^2t+(5/5184)\sqrt{6}x^3t+(5/20736)x^4t+(25/576)t^2+(25/3456)\sqrt{6}xt^2+(25/3456)x^2t^2-(125/10368)t^3+\ldots$$

Application of DTM to Porous Medium Equation: Consider a porous medium equation [8]

$$\frac{\partial u}{\partial t}=A+\left(\frac{\partial u}{\partial x}\right)^2+u\frac{\partial^2 u}{\partial x^2} \quad (5)$$

with initial condition $u(x,0)=0.01x^2$ (6)

Where A=0.68

The Differential transform of eq. (5) can be written as

$$(h+1)U(k,h+1)=\delta(h)\delta(k)A+\sum_{r=0}^{k}\sum_{s=0}^{h}(r+1)(k-r+1)U(r+1,h-s)U(k-r+1,s)+\sum_{r=0}^{k}\sum_{s=0}^{h}(k-r+1)(k-r+2)U(r,h-s)U(k-r+2,s) \quad (7)$$

Using the initial conditions (6) in eq.(7) , we have,

$$\mathrm{U}(0,0)=0\quad \mathrm{U}(1,0)=0\quad \mathrm{U}(2,0)=0.01$$
$$\mathrm{U}(3,0)=0\quad \mathrm{U}(4,0)=0\quad \mathrm{U}(5,0)=0\quad \mathrm{U}(6,0)=0 \quad (8)$$

Now by substituting the results (8) in eq.(7), it obtains

$$\mathrm{U}(0,1)=0.68\quad \mathrm{U}(1,1)=0\quad \mathrm{U}(2,1)=0.0006$$
$$\mathrm{U}(0,2)=0.0068\quad \mathrm{U}(1,2)=0\quad \mathrm{U}(2,2)=0.000036$$
$$\mathrm{U}(0,3)=0.00031733\quad \mathrm{U}(1,3)=0$$
$$\mathrm{U}(2,3)=0.00000216\quad \mathrm{U}(0,4)=0.00001587$$

Hence the approximate solution of eq. (5) up to 4 degree terms can be written as

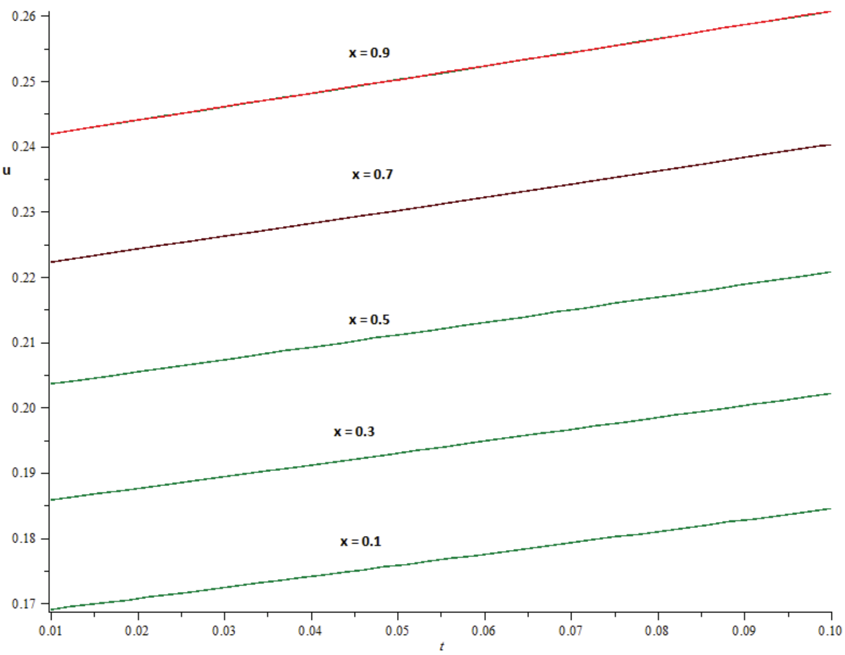
$$u(x,t)=0.01x^2+0.68t+0.0006x^2t+0.0068t^2+0.000036x^2t^2+0.00031733t^3+0.00000216x^2t^3+0.00001587t^4$$

Numerical Results and discussion: The following Table-I show the comparisons of the absolute errors of Kolmogrove-Petrovskii-Piskunov equation obtained by using three terms approximation for DTM at different values of x and t. To show the effectiveness and accuracy of proposed schemes, we compared our results with the results obtained by exact solution. Fig-I shows the comparison graphically between the approximate solutions obtained by DTM and the exact solutions for different values of x and for t = 0.01 to 0.1which shows the reliability and efficiency of DTM.

Table-I

t/x		0.1	Error	0.3	Error	0.5	Error	0.7	Error	0.9	Error
DTM	**0.01**	0.2419438	1.4E-08	0.2460596	1E-07	0.2502108	2.7E-07	0.2543967	5E-07	0.258617	8E-07
EXACT		0.2419438		0.2460595		0.2502105		0.2543962		0.258616	
DTM	**0.02**	0.2320148	5.5E-08	0.2360423	5E-07	0.2401067	1.2E-06	0.2442075	2E-06	0.248344	4E-06
EXACT		0.2320148		0.2360418		0.2401055		0.2442052		0.24834	
DTM	**0.03**	0.2223083	1.1E-07	0.2262446	1E-06	0.2302197	2.9E-06	0.2342327	6E-06	0.238283	9E-06
EXACT		0.2223081		0.2262435		0.2302167		0.2342272		0.238274	
DTM	**0.04**	0.2128309	1.2E-07	0.2166737	2E-06	0.220557	5.3E-06	0.2244802	1E-05	0.228443	2E-05
EXACT		0.2128308		0.2166718		0.2205517		0.2244699		0.228426	
DTM	**0.05**	0.2035886	-9E-08	0.2073358	3E-06	0.2111253	8.3E-06	0.2149567	2E-05	0.218829	3E-05
EXACT		0.2035887		0.207333		0.211117		0.2149405		0.218803	
DTM	**0.06**	0.1945865	-8E-07	0.1982362	3E-06	0.2019303	1.2E-05	0.2056683	2E-05	0.20945	4E-05
EXACT		0.1945873		0.1982328		0.2019188		0.205645		0.209411	
DTM	**0.07**	0.1858284	-3E-06	0.1893792	3E-06	0.1929766	1.5E-05	0.19662	3E-05	0.200309	5E-05
EXACT		0.1858312		0.189376		0.1929621		0.1965891		0.200257	
DTM	**0.08**	0.1773172	-7E-06	0.180768	1E-06	0.1842676	1.6E-05	0.1878156	4E-05	0.191411	7E-05
EXACT		0.177324		0.1807667		0.1842513		0.1877775		0.191345	
DTM	**0.09**	0.1690543	-3E-06	0.1724043	-4E-06	0.1758056	1.5E-05	0.1792577	4E-05	0.18276	8E-05
EXACT		0.1690577		0.1724084		0.1757902		0.1792141		0.18268	

Fiq-I



Similarly the Table-II show the comparisons results for VIM and DTM of the porous medium equation obtained by using four terms approximation at different values of x and t .To show the effectiveness and accuracy of proposed schemes, we compared our results with the results obtained by VIM.

Table-II

x\t	DTM	VIM	DTM	VIM	DTM	VIM	DTM	VIM	DTM	VIM	DTM	VIM
	0	0	0.2	0.2	0.4	0.4	0.6	0.6	0.8	0.8	1	1
0	0	0	0.136275	0.13627	0.273108	0.273109	0.410517	0.410519	0.548515	0.548521	0.6871	0.6871
0.2	0.0004	0.0004	0.136679	0.136679	0.273518	0.273519	0.410932	0.410934	0.548935	0.548941	0.6875	0.6876
0.4	0.0016	0.0016	0.137894	0.137894	0.274748	0.274748	0.412176	0.412178	0.550196	0.550202	0.6888	0.6888
0.6	0.0036	0.0036	0.139918	0.139918	0.276797	0.276797	0.414251	0.414253	0.552297	0.552303	0.6909	0.691
0.8	0.0064	0.0064	0.142752	0.142752	0.279666	0.279666	0.417156	0.417158	0.555238	0.555244	0.6939	0.6939
1	0.01	0.01	0.146396	0.146396	0.283354	0.283355	0.42089	0.420892	0.559019	0,559025	0.6978	0.6978

Conclusion: In this paper, the Kolmogrove-Petrovskii-Piskunov equation and the porous medium equation has been solved by using DTM. The obtained results are then compared with exact solutions for Kolmogrove-Petrovskii-Piskunov equation and with VIM solution for the porous medium equation. The accuracy and efficiency of DTM has been demonstrated graphically as well as by tables to check the efficiency of the method which is very useful for solving highly nonlinear nonlinear PDE generally arising in real world problems.

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