DOMINATOR COLORING OF FAN GRAPH AND LADDER GRAPH FAMILIES

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Abstract: A dominator coloring is a coloring of the vertices of a graph such that every vertex is either alone in its color class or adjacent to all vertices of at least one other class. In this paper , we obtain the dominator chromatic number for the Middle Graph of Fan Graph M(Fn), Line Graph of Fan Graph L(Fn), Total Graph of Fan Graph T(Fn), Middle Graph of Ladder Graph M(Ln), TotalGraph of Ladder Graph T(Ln), Central Graph of Ladder Graph C(Ln).

Keywords: Coloring, Domination, Dominator Coloring, Middle, Total, Line Graph, Fan graph and Ladder graph.

1 Introduction: All graphs considered here are finite, undirected, simple graphs. Let G be a graph, with vertex set V(G) and edge set E(G). The neighbors of a vertex $v \in V(G)$ are all the vertices u such that $uv \in E(G)$. Any vertex of G is said to dominate itself and all its neighbors. A set $D \subseteq V(G)$ is a dominating set if every vertex of $V(G) \setminus D$ has a neighbor in D.

A proper coloring of a graph G is a function from the set of vertices of a graph to a set of colors such that any two adjacent vertices have different colors. A subset of vertices colored with the same color is called a color class. The chromatic number is the minimum number of colors needed in a proper coloring of a graph and is denoted by χ (G).

A dominator coloring of a graph G is a proper coloring of graph such that every vertex of V dominates all vertices of at least one color class (possibly its own class). i.e., it is coloring of the vertices of a graph such that every vertex is either alone in its color class or adjacent to all vertices of at least one other class and this concept was introduced by Ralucca Michelle Gera in 2006 [2]. The dominator chromatic number $\chi_d(G)$ is the minimum number of color classes in a dominator coloring of G. The relation between dominator chromatic number, chromatic number and domination number of some classes of graphs were studied in [3]. The dominator coloring of bipartite graph, central and middle graph of path and cycle graph were also studied in various papers [4] [5].

2 Preliminaries: The middle graph [1] of G, is defined with the vertex set $V(G) \cup E(G)$ where two vertices are adjacent if they are either adjacent edges of G or one is the vertex and the other is an edge incident with it and it is denoted by M(G).The total graph [1] of G has vertex set $V(G) \cup E(G)$, and edges joining all elements of this vertex set which are adjacent or incident in G.

The central graph [7] C(G) of a graph G is obtained from G by adding an extra vertex on each edge of G, and then joining each pair of vertices of the original graph which were previously non-adjacent.

The line graph [6] of G, denoted by L(G), is a graph whose vertices are the edges of G, and if $u, v \in E(G)$ then $uv \in E(L(G))$ if u and v share a vertex in G.

A fan graph $F_{m;n}$ [6] is defined as the graph join $k_m + P_n$, where k_m is the empty graph on m vertices and P_n is the path graph on n vertices. In paricular, when m =1 the graph $F_{1;n}$ is called the fan graph of order n.

A dominator coloring of a graph G is a proper coloring in which every vertex of G dominates every vertex of at least one color class. The convention is that if v is a color class, then v dominates the color class v. The dominator chromatic number $\chi_d(G)$ is the minimum number of colors required for a dominator coloring of G.

3 Main Results:

Theorem 1 For any $n \geq 5$ the dominator chromatic number of middle graph of ladder is n+3. i.e, $\chi_d(M(L_n)) = n + 3$

Proof: Let L_n be the ladder graph with 2n vertices and 3n-2 edges

.Let $\{v_1, v_2, v_3, \ldots v_n\} \cup \{u_1, u_2, u_3, u_n\}$ be the vertices of ladder graph L_n. i.e, $V(L_n) = \{v_1, v_2, v_3, \ldots v_n \cup u_1, u_2, u_3, \ldots u_n\}$ Now by definition of middle graph each edge of graph is subdivided by a new vertex therefore assume that each (v_i, v_{i+1}); (u_i, u_{i+1}) and $v_j u_j$ is subdivided by the vertex $(v_{i,i+1})$, $(u_{i,i+1})$ and e_j for $(1 \leq i \leq n - 1)$ and $(1 \leq j \leq n - 1)$ respectively.

Let $V(M(Ln)) = (v_{i,i+1}, u_{i,i+1}$ and $e_j : 1 \leq i \leq n - 1$ and $1 \leq j \leq n - 1)$.

Case 1 if n is odd: Assign color $c_{\lceil i/2 \rceil}$ to $(u_{i,i+1})$ where i=1,3,5,... assign color $c_{\lceil i/2 \rceil+2}$ colors to $(V_{i,i+1})$ where i = 1,3,5, ... Assign C_{n+2} colors to u_i and v_i. vertices $(u_{i,i+1})(v_{i,i+1})$ where i=2,4,6,... colored by color c_{n+3}. The vertices e_j for $1 \leq j \leq n - 1$ are colored by color c_{n+1} , the n^{th} vertex e_n is colored by color c_n. Vertices $(u_{i,i+1})(v_{i,i+1})(i = 1,3,5\ldots)$ dominate itself.

u_i, u_{i+1} and v_i, v_{i+1} $1 \leq i \leq n - 1$ dominate the color class of $(u_{i,i+1})(v_{i,i+1})$ where (i = 1,3, 5,....). u_n, v_n and $(u_{n-1,n})$; $(V_{n-1,n})$ dominate the color class of e_n. e_n dominate itself. Vertices $(u_{i,i+1})(v_{i,i+1})$ where i = 2, 4, 6, ... n - 3 dominate the color class of $(u_{i+1,i+2})$ and $(v_{i+1,i+2})$.

Case 2 if n is even: vertex $u_{i;i+1}$ (i = 1, 3, ,...) is colored by color $c_{\lceil i/2 \rceil}$. vertex $v_{i;i+1}$ (i = 1,3,5...) is colored by color $c_{\lceil i/2 \rceil+3}$. Assign C_{n+1} color to u_i and v_i (1 ≤ j ≤ n). vertices $(u_{i,i+1})(v_{i,i+1})$ where i = 2, 4, 6,... colored by color c_{n+2}. The vertex e_j (1 ≤ j ≤ n) are colored by color c_{n+3},. Vertices $u_{i;i+1}$ $v_{i;i+1}$(i = 1, 3, ,...) dominate itself. u_i, u_{i+1} and v_i, v_{i+1} dominate the color class of $(u_{,;i+1})(v_{i,i+1})$ where (i = 1, 3, ,...) .Vertices $(u_{i;i+1})(v_{i;i+1})$ where i = 2, 4, 6,... dominate the color class of $(u_{i+1;i+2})$ and $(v_{i+1;i+2})$ (i = 1,3,5...) Vertex e_j and e_{j+1} dominate at least one of the color class of $(u_{i,i+1})(v_{i,i+1})$ where (i = 1,3,5...). Hence $\chi_d(M(L_n)) = n + 3$.

Theorem: 2: For any n ≥ 5 the dominator chromatic number of central graph of ladder is 2n. i.e, $\chi_d(C(L_n)) = 2n$.

Proof: Let L_n be the ladder graph with 2n vertices and 3n-2 edges

.Let $\{v_1, v_2, v_3, \ldots v_n\} \cup \{u_1, u_2, u_3, u_n\}$ be the vertices of ladder graph L_n. i.e, $V(L_n) = \{v_1, v_2, v_3, \ldots v_n \cup u_1, u_2, u_3, \ldots u_n\}$ Now by definition of Central graph each edge of graph is subdivided by a new vertex therefore assume that each (v_i, v_{i+1}); (u_i, u_{i+1}) and $v_j u_j$ is subdivided by the vertex

$(v_{i,i+1})$, $(u_{i,i+1})$ and e_j for (1 ≤ i ≤ n - 1) and (1≤ j ≤ n - 1) respectively.

Let V (C(Ln)) = ($v_{i,i+1}$, $u_{i,i+1}$ and e_j : 1 ≤ i ≤ n - 1 and 1 ≤ j ≤ n - 1).

Assign color ci to u_i (1 ≤ i ≤ n) vertex vi is colored by color c_{n+i}(1 ≤ i ≤ n). Assign c_{2n} color to all the vertices of (v_i, v_{i+1}), (u_i, u_{i+1}) (1 ≤ i ≤ n - 2).the n^{th} vertex $u_{n-1,n}$,$v_{n-1,n}$ is colored by color c_2. The vertex e_j(1 ≤ i ≤ n-1) are colored by color c_{2n}. The n^{th} vertex e_n is colored by color c_2. All vertices of v_i and u_i dominate at least one of the color class of v_i and u_i,$u_{i,i+1}$(1 ≤ i ≤ n) dominate the color class of v_i. e_j (1 ≤ j ≤ n) dominate the color class of v_i and u_i .Hence $\chi_d(C(L_n)) = 2n$.

Theorem: 3: For any n ≥ 5 the dominator chromatic number of Total graph of ladder is n+4. i.e, $\chi_d(T(L_n)) = n+4$.

Proof: Let L_n be the ladder graph with 2n vertices and 3n-2 edges

.Let $\{v_1, v_2, v_3, \ldots v_n\} \cup \{u_1, u_2, u_3, u_n\}$ be the vertices of ladder graph L_n. i.e, $V(L_n) = \{v_1, v_2, v_3, \ldots v_n \cup u_1, u_2, u_3, \ldots u_n\}$ Now by definition of total graph each edge of graph is subdivided by a new vertex therefore assume that each (v_i, v_{i+1}); (u_i, u_{i+1}) and $v_j u_j$ is subdivided by the vertex

$(v_{i,i+1})$, $(u_{i,i+1})$ and e_j for (1 ≤ i ≤ n - 1) and (1≤ j ≤ n - 1) respectively.

Let V (T(Ln)) = ($v_{i,i+1}$, $u_{i,i+1}$ and e_j : 1 ≤ i ≤ n - 1 and 1 ≤ j ≤ n - 1).

case if 1 n is odd: vertices u_i(i = 2, 4, 6,...) assign color $c_{\lceil i/2 \rceil}$ and vertices v_i (i = 1, 3, 5,...) are colored by color $c_{\lceil i/2 \rceil + \lfloor n/2 \rfloor}$. Assign color c_{n+1} to remaining vertices of v_i and u_i . vertices $(u_{i,i+1})(v_{i,i+1})$ where (i = 1,3, 5,....). are colored by color c_{n+2}. Assign c_{n+3} colors to $(u_{i,i+1})(v_{i,i+1})$ (i =2,4,6,...) vertex e_j colored by color c_{n+4}. Vertex

u_i (i = 1,3, 5,.... n - 1) dominate the color class of u_{i+1}. The n^{th} vertex u_n dominate the color class of u_{n-1}. Vertex v_i((i =2,4,6,...) n - 1) dominate the color class of v_{i+1}. Vertices e_j (j =1,3, 5,....) dominate the color class of v_i . e_j (j =2,4,6,...) dominate the color class of u_i. Vertices $u_{i;i+1}$ and $u_{i+1,i+2}$) (i =1, 3, 5,...) dominate the color class of u_{i+1}. Vertex $v_{i;i+1}$ (i =1, 3, 5,...) dominate the color class of v_i,$v_{i,i+1}$(i= i =2,4,6,...) dominate the color class of $v_{i+1.}$

case if 2 n is even vertices u_i(i =2,4,6,...) assign color $c_{\lceil i/2 \rceil}$ and for v_i (i =1, 3, 5,...) are colored by color $c_{\lceil i/2 \rceil}$ +n . Assign color c_{n+1} to remaining vertices of v_i and u_i . vertices $(u_{i,i+1})(v_{i,i+1})$ where (i = 1,3, 5,....) are colored by color c_{n+2}. Assign c_{n+3} colors to $(u_{i,i+1})(v_{i,i+1})$ where (i =2,4,6,...)) vertex e_i colored by color c_{n+4}:u_i(i =1,3, 5,....) dominate the color class of u_{i+1}. Vertex v_i (i = 2,4,6,... n - 1) dominate the color class of v_{i+1}. The nth vertex v_n dominate the color class of v_{n-1}. Hence $\chi_d(T(L_n)) = n + 4$.

Theorem 4: For any n ≥ 5 the dominator chromatic number of middle graph of fan is n+2. i.e, $\chi_d(M(F_n)) = n + 2$

Proof

$V(F_{1,n}) = \{v, v_1, v_2, \ldots v_n\}$ and let $V(M(F_{1,n})) = \{v, v_1, v_2, \ldots v_n \cup e_1, e_2, \ldots e_n \cup e'_1, e'_2, \ldots e'_{n-1}\}$

where e_i is the vertex of $M(F_{1,n})$ corresponding to $F_{1,n}$(1≤ i ≤ n). Consider the following n + 2-coloring of $M(F_{1,n})$, For (1≤ i ≤ n) Assign the c_i colors to e_i. v and (1 ≤ i ≤ n-1) Assign c_n colors to v_i for v_n assign the color c_{n+2}. Which already assigned to the vertex v.

Case 1 if n is odd: Assign the color cn+2 to the vertex e'_i (i =1, 3, 5,... n - 2). Assign the color cn-1 to the vertices e'_i (i = 2,4,6,....n - 3). Assign the color cn+1 to the vertex e'_{n-1}

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Case 2 n is even: Assign the color c_{n-1} to the vertex e'_i (i = 1; 3; 5; : : : n - 3). Assign the color cn+2 to the vertices e'_i (i = 2, 4, 6,... n - 2). Assign the color c_{n+1} to the vertex e'_{n-1}. The following procedure gives the dominator coloring of $\chi_d(M(F_n))$ the vertex v dominate the color class of e_i. The vertices ei dominate itself. e'_I and vi dominate the color class of e_i((1≤ i ≤ n). Hence $\chi_d(M(F_n)) = n + 2$.

Theorem 5: For any n ≥ 5 the dominator chromatic number of total graph of fan is n+4. i.e, $\chi_d(T(F_n)) = n + 4$

Proof:

$V(F_{1,n}) = \{v, v_1, v_2, \ldots v_n\}$ and let $V(M(F_{1,n})) = \{v, v_1, v_2, \ldots v_n \cup e_1, e_2, \ldots e_n \cup e'_1, e'_2, \ldots e'_{n-1}\}$

, where e'_i is the vertex of T(F1;n) corresponding to the edge $v_i v_{i+1}$ of $F_{1;n}$(1≤ i ≤ n - 1) and ei is the vertex of T(F1;n) corresponding to the edge vv_i of F1;n(1≤ i ≤ n). For (1≤ i ≤ n) Assign the c_i color to e_i. Assign the color c_{n+1} and c_{n+2} colors alternatively to v_i(1≤ i ≤ n). For (1≤ i ≤ n) assign c_{n+3} and c_{n+4} color alternatively to e'_i assign color c_{n+3} to v.vertices v and v_i and e'_i dominate the color class of e_i. All the vertices of e_i dominate at least one of the color class of e_i. hence $\chi_d(T(F_n)) = n + 4$.

Theorem 6: For any $n \geq 5$ the dominator chromatic number of line graph of fan is n. i.e, $\chi_d(L(F_n)) = n$

V (L(F1;n)) = {e1, e2,... en ∪ u1, u2, ... un-1} where u_i is the vertex corresponding to the edge v_iv_{i+1} of F1;n(1≤ i ≤ n-1) and e_i is the vertex corresponding to the edge

vv_i of F1;n(1≤ i ≤ n). Proof: For (1≤ i ≤ n) Assign the ci color to ei. For (1≤ i ≤ n) assign colors c3c5c1 alternatively to ui. Vertices ei(i = 2, 4, 6, ... n) dominate itself. Vertex e_i(i = 1, 3, 5,...) dominate the color class of e_{i+1}: Vertex u_i and u_{i+1} i = 1,3;,5,... dominate the color class of e_{i+1} hence $\chi_d(L(F_n)) = n$.

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