REVERSE JORDAN*_ GENERALIZED DERIVATIONS ON SEMI PRIME RINGS

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Abstract: This paper proved that if R is a semiprime *-ring of char. $\neq 6$ and $G: R \to R$ is an additive mapping satisfying the relation $G(xyx) = x^*y^*G(x) + x^*D(y)x + D(x)yx$ for all x,y in R, where D is some reverse Jordan *-derivation of R, then G is a Reverse Jordan *-generalized derivation.

Keywords: semiprime*-ring, reverse Jordan *-derivation, additive mapping, zero *-derivation

Introduction Bresar and Vukman [1],On some additive mappings in rings with involution, Aequationes Math. 38 (1989), 178-185.] first studied the *-derivations and Jordan *-derivations and proved that there are no non-zero *-derivations on non-commutative prime *-rings. The study of Jordan *-derivations has been motivated by the problem of the representability of quadratic forms by bilinear forms. Vukman and Kosi – Ulbl [2] proved that if R is a semiprime *-ring of char. $\neq 2$ such that R has a commutator which is not a zero divisor and G is an additive mapping that satisfies the identity, $G(xx^*) = G(x)x^* + xD(x^*)$ for all $x \in R$ for some derivation D of R, then G is a generalized derivation on R.

This paper proves that if R is a semiprime *-ring of char. $\neq 6$ and $G: R \to R$ is an additive mapping satisfying the relation $G(xyx) = x^*y^*G(x) + x^*D(y)x + D(x)yx$ for all x,y in R, where D is some reverse Jordan *-derivation of R, then G is a Reverse Jordan *-generalized derivation.

Preliminaries: An additive mapping $D: R \to R$, where R is a *-ring, is a *-derivation if $D(xy) = D(x)y^* + xD(y)$ hold for all $x,y \in R$ and is a Jordan *-derivation if $D(x^2) = D(x)x^* + xD(x)$ hold for x in R. An additive mapping $G: R \to R$ is a *-generalized derivation if there exists a *-derivation $D: R \to R$ such that $G(xy) = G(x)y^* + xD(y)$ for all x,y in R and an additive mapping $G: R \to R$ is Jordan *-generalized derivation if there exists a *-derivation $D: R \to R$ such that $G(x^2) = G(x)x^* + xD(x)$ for all $x \in R$. An additive mapping $D: R \to R$, where R is a *-ring, is a Reverse *-derivation if $D(xy) = y^*D(x) + D(y)x$ hold for all $x,y \in R$ and is a Reverse Jordan *-derivation if $D(x^2) = x^*D(x) + D(x)x$ hold for all x in R. An additive mapping $G: R \to R$ is a Reverse *-generalized derivation if there exists a reverse *-derivation $D: R \to R$ such that $G(xy) = y^*G(x) + D(y)x$ for all x,y in R and an additive mapping $G: R \to R$ is a Reverse Jordan *-generalized derivation if there exists a reverse *-derivation $D: R \to R$ such that $G(x^2) = x^* G(x) + D(x)x$ for all x in R.

Throughout this section consider R will be a semiprime *-ring.

Main Results: To prove the main results we require the following Lemma:

Lemma 1: Let R be a semiprime *-ring of char. $\neq 2$. Suppose that the relation $ax^*b^* + bxa = 0$ holds for all $x \in R$ and some $a,b \in R$. In this case $ab = ba = 0$ is fulfilled. In case R is prime, then either $a = 0$ or $b = 0$.

Proof: We have the relation

$ax^*b^* + bxa = 0$, for all $x \in R$. (1)

By putting in the above relation ybx for x and applying equation (1), we obtain

$0 = a(ybx)^* b^* + bybxa = (ax^*b^*)y^* b^* + bybxa.$

$= -bx (a y^* b^*) + bybxa.$

$= bxbya + bybxa.$

It proves that

$bxbya + bybxa = 0$, for all $x,y \in R$. (2)

In particular for $y=x$ the above relation reduces to $bxbxa = 0$, for all $x \in R$, since R is char. $\neq 2$.

By applying equation (1), we obtain from the above relation

$bxax^*b^* = 0$, for all $x \in R$. (3)

Now by substituting in equation (2) xay for y and applying equation (1) and (3) we obtain

$0 = bx (bxa)ya + bxaybxa.$

$= -(bx\, ax^*b^*)ya + bxaybxa.$

$= bxaybxa.$

We have therefore proved that $(bxa)y (bxa) = 0$ holds for all $x,y \in R$. Hence it follows that

$bxa = 0$, for all $x \in R$. (4)

It gives relation $(ab) x (ab) = 0$, for all $x \in R$ which gives $ab = 0$.

Similarly, $ba = 0$. In case R is prime it follows from equation (4) that either $a = 0$ or $b = 0$. Hence the proof is complete.

Theorem 1: Let R be a semiprime *-ring of char. $\neq 2$ and let D: $R \rightarrow R$ be an additive mapping satisfying the relation $D(xyx) = D(x)y^*x^* + xD(y)\, x^* + xyD(x)$, for all $x,y \in R$. In this case D is a Jordan *-derivation.

Proof: Consider the relation

$D(xyx) = D(x)\, y^*x^* + xD(y)\, x^* + xyD(x)$, for all $x,y \in R$. (5)

By substituting xyx for y in the above relation gives

$D(x^2yx^2) = D(x)x^*y^*\, x^{*^2} + xD(xyx)x^* + x^2yxD(x)$.

Which implies

$D(x^2yx^2)=D(x)x^*y^*\, x^{*^2} +xD(x)y^*\, x^{*^2} +x^2D(y)\, x^{*^2} +x^2yD(x)x^*+x^2yxD(x)$, for all $x,y \in R$.(.6)

On the other hand, by substituting x^2 for x in equation (5) gives

$D(x^2yx^2) =D(x^2)y^*\, x^{*^2} +x^2D(y)\, x^{*^2} + x^2yD(x^2)$, for all $x,y \in R$. (7)

From (6) from (7),

$A(x)\, y^*\, x^{*^2} + x^2yA(x) = 0$,for all $x,y \in R$ (8)

where $A(x)$ stands for $D(x^2) - D(x)x^* - xD(x)$.From the relation above it follows from Lemma 1 that $A(x)\, x^2 = 0$, (9)

and$x^2A(x) = 0$,for all $x \in R$. (10)

By replacing x by $x+y$ in the equation (9)

$A(x)y^2 + A(y)\, x^2 + B(x,y)x^2 + B(x,y)y^2 + A(x)(xy + yx) + A(y)(xy + yx)+ B(x,y)(xy + yx) = 0$, for all $x,y \in R$, (11)

where $B(x,y)$ stands for $D(xy + yx) - D(x)y^* - xD(y) - yD(x)$. By putting $-x$ for x in the above relation and comparing the relation obtained with the relation (11),Which gives $B(x,y)x^2 + B(x,y)\, y^2 +A(x)\, (xy + yx) + A(y)\, (xy + yx) = 0$, for all $x,y \in R$, since R is of char. $\neq 2$.

By substituting $2x$ for x in the above relation gives

$4B(x,y)x^2 + B(x,y)y^2 + 4A(x)\, (xy + yx) + A(x)\, (xy + yx) = 0$,for all $x,y \in R$. (12)

Now (11) and (12) gives

$3A(x)\, (xy + yx) + 3B(x,y)x^2 = 0$,for all $x,y \in R$.

Since R is of char. $\neq 3$,

$A(x)\, (xy + yx) + B(x,y)x^2 = 0$, for all $x,y \in R$. (13)

By right multiplication of the above equation with $A(x)x$ and using (10) gives

$A(x)xy\, A(x)x + A(x)yx\, A(x)x = 0$,for all $x,y \in R$. (14)

By replacing y by yx, multiplying the equation (14) from the left side by x and using (10), we obtain $(xA(x)x)y\, (xA(x)x) = 0$, for all $x,y \in R$. Hence it follows that $xA(x)x = 0$, $x \in R$. Now the equation (14) reduces to $(A(x)x)y(A(x)x) = 0$, for all $x,y \in R$ which gives $A(x)x = 0$,for all $x \in R$. (15)

Now the equation (13) reduces to $A(x)yx + B(x,y)x^2 = 0$, for all $x,y \in R$. By right multiplication of this equation with $A(x)$ and left multiplication with x gives $(xA(x))\, y\, (xA(x)) = 0$, for all $x,y \in R$ which gives

$xA(x) = 0$, for all $x\in R$. (16)

From the equation (15), $A(x)y + B(x,y)x = 0$.

By right multiplication of the above equation with $A(x)$ gives $A(x)y\, A(x)=0$, for all $x,y \in R$, by the equation (16). Since R is semiprime, we have $A(x) = 0$, for all $x\in R$. In otherwords, $D(x^2) = D(x)x^* + xD(x)$, for all $x \in R$ which means that D is a Jordan *-derivation.

Theorem 2: Let R be a semiprime *-ring of char. $\neq 6$ and let G: $R \rightarrow R$ be an additive mapping satisfying the relation $G(xyx) = x^*y^*G(x) + x^*D(y)x + D(x)yx$, for all $x,y \in R$ and some reverse Jordan *-derivations D of R. Then G is a reverse Jordan *-generalized derivation.

Proof: Consider

$G(xyx) = x^*y^*G(x) + x^*D(y)x + D(x)yx$, for all $x,y \in R$. (17)

If replace y by xyx in equation (17), then

$G(x^2yx^2)= x^{*^2}\, y^*x^*G(x)+x^*D(xyx)x+ D(x)xyx^2$, for all $x,y \in R$. (18)

Equations (5) and (18), gives

$G(x^2yx^2) = x^{*^2}\, y^*\, x^*G(x) + x^{*^2}\, y^*D(x)x + x^{*^2}\, D(y)x^2+ x^*\, D(x)y\, x^2 + D(x)xyx^2$,for all $x,y \in R$. (19)

From replacing x by x^2 in (17),

$G(x^2yx^2) = x^{*^2}\, y^*G(x^2) + x^{*^2}\, D(y)x^2 +D(x^2)yx^2$, for all $x,y \in R$. (20)

Since D is a Reverse Jordan *-derivation, equation (20) may be rewritten as

$G(x^2yx^2) = x^{*^2}\, y^*G(x^2) + x^{*^2}\, D(y)x^2 + x^*D(x)yx^2 + D(x)xyx^2$, for all $x,y \in R$. (21)

By (19) and (21),

$x^{*^2} y^*A(x) = 0$, for all $x,y \in R$ (22)
where $A(x) = G(x^2) - x^*G(x) - D(x)x$.
It proves that

$A(x) = 0$, for all $x \in R$. (23)

If replace y by y^* in equation (22), then

$x^{*^2} yA(x) = 0$, for all $x,y \in R$. (24)

On left multiplication by $A(x)$ and right multiplication by x^{*^2} in equation (24), then $A(x)\ x^{*^2}\ yA(x)\ x^{*^2} = 0$, for all $x,y \in R$. Since R is semiprime,

$A(x)\ x^{*^2} = 0$, for all $x \in R$. (25)

Now by replacing y by $A(x)y\ x^{*^2}$ in equation (24) and by the semiprimeness of R, then

$x^{*^2} A(x) = 0$, for all $x \in R$. (26)

By linearizing equation (26), then

$y^{*^2}\ A(x) + x^{*^2} A(y) + x^{*^2} B(x,y) + y^{*^2} B(x,y) + (xy + yx)^* A(x) + (xy + yx)^* A(y) + (xy + yx)^* B(x,y) = 0$, for all $x,y \in R$, (27)

where $B(x,y)$ stands for $G(xy + yx) - y^*D(x) - x^*D(y) - D(y)x - D(x)y$.
By putting $-x$ for x in the above relation and comparing the relation obtained with the relation (27) which gives

$x^{*^2} B(x,y) + y^{*^2} B(x,y) + (xy + yx)^* A(x) + (xy + yx)^* A(y) = 0$, for all $x,y \in R$. (28)

By substituting $2x$ for x in equation (28) gives

$\dot{x}^{*^2}\ 4B(x,y) + y^{*^2} B(x,y) + 4(xy + yx)^* A(x) + (xy + yx)^* A(y) = 0$, for all $x,y \in R$. (29)

By subtracting the relation (28) from (29),

$3\,x^{*^2} B(x,y) + 3(xy + yx)^* A(x) = 0$, for all $x,y \in R$.

Since R is of char. $\neq 3$,

$x^{*^2} B(x,y) + (xy + yx)^* A(x) = 0$,
for all $x,y \in R$. (30)

By right multiplication of the above relation by $x^*A(x)$ and using (25),

$x^*A(x)\, x^*\, y^*A(x) + x^*A(x)\, y^*x^*A(x) = 0$, for all $x,y \in R$. (31)

By substitute yx for y in equation (31), multiplying the equation (31) from the right hand side by x^* and using (25) gives

$x^*\, A(x)\, x^*\, y^*\, x^*A(x)x^* = 0$, $x,y \in R$.

By replacing y by y^* and by the semiprimeness of R,

$x^*A(x)x^* = 0$, $x \in R$. Now the relation (31) reduces to $(x^*A(x))\, y^*(x^*A(x)) = 0$, $x,y \in R$, which gives

$x^*A(x) = 0$, for all $x \in R$. (32)

Now the relation (30) reduces to $x^{*^2} B(x,y) + x^*\, y^*A(x) = 0$, $x,y \in R$.
By left multiplication of this relation by $A(x)$, right multiplication by x^* and replacing y by y^* gives
$(A(x)x^*)\, y\, (A(x)x^*) = 0$, for all $x,y \in R$, which gives $A(x)x^* = 0$, for all $x \in R$. (33)
By linearizing relation (32) gives
$y^*A(x) + x^*\, A(y) + x^*B(x,y) + y^*\, B(x,y) = 0$. for all $x,y \in R$. (34)
By putting $-x$ for x in the above relation and comparing the relation obtained with the relation (34) gives
$y^*A(x) + x^*B(x,y) = 0$, for all $x,y \in R$. (35)
By left multiplication of the above relation by $A(x)$ and using (33) gives that
$A(x)\, y^*A(x) = 0$, $x,y \in R$, which implies that $A(x) = 0$, for all $x \in R$. In otherwords,
$G(x^2) = x^*G(x) + D(x)x$, for all $x \in R$ which means that G is a reverse Jordan*-generalized derivation.
It is clear that if we use the reverse *-derivation D to be zero *-derivation, in the above theorem, we get
Corollary 1: Let R be a semiprime *-ring of char. $\neq 6$ and let T: $R \to R$ be an additive mapping. If $T(xyx) = x^*y^*T(x)$ for all $x,y \in R$, then T is a right Jordan *-centralizer.

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