A DISCRETE TIME MODEL ON DEPLETION OF FOREST RESOURCES BY HUMAN POPULATION: EFFECT OF TECHNOLOGY ON ITS CONSERVATION

ELIZABETH SEBASTIAN, PREETHI VICTOR

Abstract: In this paper, a nonlinear mathematical model is proposed and analyzed to study the deforestation due to human population and its effect on farm fields and role of technology in its conservation. We consider the variables namely, density of human population, density of trees , Farm fields and measure of effort of technology applied for conservation of Trees. Both the growth rate and carrying capacity of trees, which follows logistic model, are assumed to be simultaneously depleted by density of human population but it is conserved by technological effort. Further this affects the growth rate of crops in farm fields. We discretize the model by applying Backward Euler scheme and analyse the stability of the model. Finally, we provide some numerical simulations using MATLAB.

Keywords: Difference Equations, Forests, Human Population, Technology.

1.Introduction: Human ecology has been defined as a type of analysis applied to the relations in human beings that was traditionally applied to plants and animals in ecology. Deforestation, is the removal of a forest or stand of trees where the land is thereafter converted to a non-forest use. Deforestation includes conversion of forestland to farms, ranches, or urban use. Since the industrial age, about half of world's forests have been destroyed and millions of animals and living things have been endangered. Forests cover almost a third of the earth's land surface. Forests provides many environmental benefits including its major role in the hydrologic cycle, soil conservation, prevention of climate change and preservation of biodiversity.

Forest produce vital oxygen and provide homes for people and wildlife. Many of the world's most threatened and endangered animals live in forests, and 1.6 billion people rely on benefits forests offer, including food, fresh water, clothing, traditional medicine and shelter. But forests around the world are under threat from deforestation, jeopardizing these benefits. Deforestation comes in many forms, including fires, clear-cutting for agriculture, ranching and development, unsustainable logging for timber, and degradation due to climate change. Forests in India faces heavy pressure of human and livestock population. The total forest cover in the country is only about 69 million hectares whereas human population is 1210 million, hence per capita forests are as low as 0.06 hectares. About 69 percent of population in India i.e. 833 million live in rural areas and most of them have land based economy and use forest resources one way or the other . It is estimated that about 200 million people live in and around forests, and fully depend for their livelihood on forest resources. Further, of the 530 million livestock population in India, about 190 million fully depends on forests either by direct grazing or by harvesting of fodder causing additional burdens on the forests.

Deforestation can result to watersheds that are no longer able to sustain and regulate water flows from rivers to streams. Trees are highly effective in absorbing water quantities, keeping the amount of water in watersheds to a manageable level. The forest also serves as cover against erosion. Once they are gone, too much water can results to downstream flooding. Forests perform a valuable function by capturing rainwater and releasing it to streams and rivers that provide water for cities and agriculture. Forest soils with a carpet of decomposing leaves absorb rainwater like a sponge, holding the water for gradual release to streams throughout the year. When watersheds lose their forest, the soil can lose its capacity to absorb rainwater as it did before. Rainwater flows quickly off the watershed, causing floods during the rainy season and a diminished supply of water during the dry season.

In addition, the flow of water from the hills irrigate the farm fields. And the quality of the water is worse because deforested hills no longer have trees to protect the ground from heavy rain, so soil erosion is greater, and the irrigation water contains large quantities of mud that settles in irrigation canals and clogs the canals. This decline in the quantity and quality of irrigation water reduces food production even further. The result is poor nutrition and health for people. This chain of effects involving human population growth, deforestation and lower food production is a vicious cycle that is difficult to escape. Forests benefits all who live downstream by reducing erosion and flooding. Floods and erosion will cause severe constraints on food production throughout the world. Farmers in villages depend on the sponge-effect of forests to absorb and slowly release water. Many agricultural exports, are dependent upon forest-generated soils and water. In some cases, water flow, due to rapid runoff kills many high-yielding rice varieties. It is therefore very essential to study the effect of technology on conservation of trees and farm fields.

J.B. Shukla, Kusum Lata and A.K. Misra analyzed on Modeling the depletion of a renewable resource by population and Industrialization: Effect of Technology on its conservation. In this paper, we have proposed mathematical model to study the effect of technological effort on the conservation of trees, which is depleted due to population. In section 2, A mathematical model is proposed and discretized. In section 3, We list the equilibrium points of the model. We analyze the stability of the model in section 4. Section 5 consists of numerical simulations through MATLAB. Conclusion is given in section 6.

2. The Mathematical Model: The Mathematical model of deforestation due to human population is given below

$$\frac{dH}{dt} = r(1-\frac{H}{L})H + r_0HF + r_1HB - r_2H$$
$$\frac{dB}{dt} = s(1-\frac{B}{K})B - r_1HB - s_2B + \theta BT \qquad (1)$$
$$\frac{dF}{dt} = (s_2 + s_3B)F - r_0HF$$
$$\frac{dT}{dt} = \phi(L-B) - \phi_0 T$$

where

H - Density of the Human population

B - Density of the Trees.

F - Density of the Farm fields.

T - the measure of effort of technology applied for conservation of Trees

r - Intrinsic growth rate the human Population.

r_0 - Growth rate of Human population due to the Farm fields.

r_1 - Growth rate of Human populationdue to Trees.

r_2 - Natural death rate of Human population.

s - Intrinsic growth rate of the trees.

s_2 - Natural death rate of the trees.

s_3 - Growth rate of Farm fields due to the irrigation formed by Trees.

ϕ - Growth-rate coefficient of technological efforts.

ϕ_0 - Depletion-rate coefficient of technological effort.

θ - Growth-rate coefficient of resource biomass due to technological effort.

L - Carrying Capacity of the Forest biomass.

K - Carrying Capacity of the human population.

Applying the Backward Euler Scheme to the system of equations (1), we obtain the discrete time model:

$$H_{n+1} = H_n + rH_{n+1}(1-\frac{H_{n+1}}{K}) + r_0H_{n+1}F_{n+1} + r_1H_{n+1}B_{n+1} - r_2H_{n+1}$$
$$B_{n+1} = B_n + sB_{n+1}(1-\frac{B_{n+1}}{K}) - r_1H_{n+1}B_{n+1} - s_2B_{n+1} + \theta B_{n+1}T_{n+1}$$
$$F_{n+1} = F_n + (s_2 + s_3B_{n+1})F_{n+1} - r_0H_{n+1}F_{n+1}$$
$$T_{n+1} = T_n + \phi(L - B_{n+1}) - \phi_0 T_{n+1}$$

(2)

3. Equilibrium Points:

i. $E_1 = (\hat{H}, 0, 0, 0)$

Where $\hat{H} = \frac{K(r-r_2)}{r}, r > r_2$

ii. $E_2 = (\bar{H}, \bar{B}, 0, 0)$

Where

$$\bar{H} = \frac{K\{Lr_1(s-s_1) + s(r-r_2)\}}{(KLr_1^2 + sr)}, \bar{B} = \frac{L(s-s_1)}{s} - \frac{Lr_1}{s}\bar{H} \quad s > s_1$$

iii. $E_3 = (\tilde{H}, \tilde{B}, \tilde{F}, 0)$

Where

$$H^* = \frac{(s-s_2)}{r_1} - \frac{sB^*}{Lr_1}, B^* = \frac{Lr_0r_1}{(Lr_1s_3 + r_0s)}\left[\frac{(s-s_2)}{r_1} - \frac{s_2}{r_0}\right],$$
$$F^* = \frac{K(r_2 - r_1B^* - r) + rf(B^*)}{Kr_0}$$

iv. $E_4 = (H^*, B^*, F^*, T^*)$

Where,

$$rH^*(1-\frac{H^*}{K}) + r_0H^*F_{n+1} + r_1H^*B^* - r_2H^* = 0$$
$$sB^*(1-\frac{B^*}{K}) - r_1H^*B^* - s_2B^* + \theta B^*T^* = 0$$
$$(s_2 + s_3B^*)F^* - r_0H^*F^* = 0, \phi(L-B^*) - \phi_0T^* = 0$$

The jacobian matrix of the system (2) is given by

$$J = \begin{pmatrix} 1 + r - \frac{2rH}{K} + r_0F + r_1B - r_2 & r_1H & r_0H & 0 \\ -r_1B & 1 + s - \frac{2sB}{L} - r_1H - s_2 & 0 & \theta \\ -r_0F & s_3F & 1 + (s_2 + s_3B) - r_0H & 0 \\ 0 & -\phi & 0 & 1 - \phi_0 \end{pmatrix}$$

(3)

4. Stability of the Mathematical Model:

Theorem 1:

The fixed point $E_1 = (\hat{H}, 0, 0, 0)$ is stable if

$$r < 2 + r_2, s_2 < 2 + s - r_1\left(\frac{K(r-r_2)}{r}\right)$$
$$r_0 < \frac{(2+s_2)r}{K(r-r_2)}, \phi_0 < 2 \qquad (4)$$

Otherwise unstable.

4.1. Local Stability of the fixed point $E_2 = (\bar{H}, \bar{B}, 0, 0)$: Consider the jacobian matrix of the system (2) with respect to the fixed point E_2.

The Jury conditions are satisfied when the following conditions hold[3].

$$1+\Omega_1\Omega_2+\Omega_2\Omega_3+\Omega_1\Omega_3+\Omega_4+\Omega_5$$
$$>\Omega_1\left[\Omega_2\Omega_3+\Omega_5+1\right]+\left[\Omega_4+1\right]\Omega_3+\Omega_2$$
$$1+\Omega_1\Omega_2+\Omega_2\Omega_3+\Omega_1\Omega_3+\Omega_4+\Omega_5$$
$$<\Omega_1\Omega_2\Omega_3-\left[\Omega_1+\Omega_2+\Omega_3\right]-\Omega_1\Omega_5-\Omega_3\Omega_4$$
$$\Omega_1\Omega_2\Omega_3<1-\Omega_1\Omega_5-\Omega_3\Omega_4 \tag{5}$$

Where

$$\Omega_1=1+r-\frac{2r\bar{H}}{L}+r_1\bar{B}-r_2,\Omega_2=1+s-\frac{2s\bar{B}}{K}-r_1\bar{H}-s_2$$
$$\Omega_3=1-\phi_0,\Omega_4=r_1^2\bar{H}\bar{B},\Omega_5=\theta\phi\bar{B}$$

Theorem 2: The fixed point $E_2=(\bar{H},\bar{B},0,0)$ is stable if the conditions given in (5) holds. Otherwise it is unstable.

4.2. Local Stability of the fixed point $E_3=(\tilde{H},\tilde{B},\tilde{F},0)$: Consider the jacobian matrix of the system (2) with respect to the fixed point E_3. The Jury conditions are satisfied when the following conditions hold[3].

$$1+\left[3+\sigma_2(\sigma_3+\sigma_4)+\sigma_3(\sigma_4+\sigma_1)+\sigma_1(\sigma_2+\sigma_4)\right]$$
$$+\left[\sigma_4(\sigma_1\sigma_2\sigma_3+\sigma_3\sigma_5-\sigma_6+\sigma_2\sigma_7)+\sigma_1\sigma_3\sigma_8\right]>$$
$$\left[\sigma_1+\sigma_2+\sigma_3+\sigma_4\right]+\left[(\sigma_1\sigma_2-\sigma_5)(\sigma_3+\sigma_4)+\sigma_1(\sigma_3\sigma_4+\sigma_8)-\sigma_6-\sigma_7(\sigma_2+\sigma_4)+\sigma_3\sigma_8\right]$$
$$1+\left[3+\sigma_2(\sigma_3+\sigma_4)+\sigma_3(\sigma_4+\sigma_1)+\sigma_1(\sigma_2+\sigma_4)\right]$$
$$+\left[\sigma_4(\sigma_1\sigma_2\sigma_3+\sigma_3\sigma_5-\sigma_6+\sigma_2\sigma_7)+\sigma_1\sigma_3\sigma_8\right]>$$
$$\left[\sigma_6+\sigma_7(\sigma_2+\sigma_4)\right]-\left[\sigma_1+\sigma_2+\sigma_3+\sigma_4\right]+\left[(\sigma_5-\sigma_1\sigma_2)(\sigma_3+\sigma_4)-\sigma_1(\sigma_3\sigma_4+\sigma_8)-\sigma_3\sigma_8\right]$$
$$\sigma_1\sigma_2\sigma_3\sigma_4<1-\sigma_1\sigma_3\sigma_8-\sigma_3\sigma_4\sigma_5-\sigma_2\sigma_4\sigma_7+\sigma_4\sigma_6$$

(6)

Where

$$\sigma_1=1+r-\frac{2r\tilde{H}}{L}+r_0F^*+r_1\tilde{B}-r_2,\sigma_2=1+s-\frac{2s\tilde{B}}{K}-r_1\tilde{H}-s_2$$
$$\sigma_3=1+(s_2+s_3\tilde{B})-r_0\tilde{H},\sigma_4=1-\phi_0,\sigma_5=r_1^2\tilde{H}\tilde{B}$$
$$\sigma_6=r_0r_1s_3\tilde{H}\tilde{B}\tilde{F},\sigma_7=r_0\tilde{F},\sigma_8=\phi$$

Theorem 3: The fixed point $E_3=(\tilde{H},\tilde{B},\tilde{F},0)$ is stable if the conditions given in (6) holds. Otherwise it is unstable.

4.3. Local Stability of the fixed point $E_4=(H^*,B^*,F^*,T^*)$: Consider the jacobian matrix of the system (2) with respect to the fixed point E_4. The Jury conditions are satisfied when the following conditions hold[3].

$$1+\left[3+\eta_2(\eta_3+\eta_4)+\eta_3(\eta_4+\eta_1)+\eta_1(\eta_2+\eta_4)\right]$$
$$+\left[\eta_4(\eta_1\eta_2\eta_3+\eta_3\eta_5-\eta_6+\eta_2\eta_7)+\eta_1\eta_3\eta_8\right]>$$
$$\left[\eta_1+\eta_2+\eta_3+\eta_4\right]+\left[(\eta_1\eta_2-\eta_5)(\eta_3+\eta_4)+\eta_1(\eta_3\eta_4+\eta_8)-\eta_6-\eta_7(\eta_2+\eta_4)+\eta_3\eta_8\right]$$

$$1+\left[3+\eta_2(\eta_3+\eta_4)+\eta_3(\eta_4+\eta_1)+\eta_1(\eta_2+\eta_4)\right]$$
$$+\left[\eta_4(\eta_1\eta_2\eta_3+\eta_3\eta_5-\eta_6+\eta_2\eta_7)+\eta_1\eta_3\eta_8\right]>$$
$$\left[\eta_6+\eta_7(\eta_2+\eta_4)\right]-\left[\eta_1+\eta_2+\eta_3+\eta_4\right]+\left[(\eta_5-\eta_1\eta_2)(\eta_3+\eta_4)-\eta_1(\eta_3\eta_4+\eta_8)-\eta_3\eta_8\right]$$

$$\eta_1\eta_2\eta_3\eta_4<1-\eta_1\eta_3\eta_8-\eta_3\eta_4\eta_5-\eta_2\eta_4\eta_7+\eta_4\eta_6 \tag{7}$$

where

$$\eta_1=1+r-\frac{2rH^*}{L}+r_0F^*+r_1B^*-r_2,\eta_2=1+s-\frac{2sB^*}{K}-r_1H^*-s_1+\theta T^*$$
$$\eta_3=1+(s_2+s_3B^*)-r_0H^*,\eta_4=1-\phi_0,\eta_5=r_1^2H^*B^*$$
$$\eta_6=r_0r_1s_3H^*B^*F^*,\eta_7=r_0F^*,\eta_8=\phi$$

Theorem 4: The fixed point $E_4=(H^*,B^*,F^*,T^*)$ is stable if the conditions given in (7) holds. Otherwise it is unstable.

5. Numerical Simulations: The simulations have been performed using MATLAB to explain our theoretical results.

Taking the following set of parametric values

$$r=1.5,s=0.5,L=5,K=5,r_0=0.4,r_1=0.1,r_2=0.07,$$
$$s_1=0.01,s_2=1.6,s_3=0.3,\phi=0.5,\phi_0=0.3$$

And considering different values for θ, we obtain the following results.

6. Conclusion: In this paper, we have considered a discrete-time model on deforestation due to human population and its effect on farm fields and role of technology in its conservation. We have considered the growth rate of human population and the trees to be logistic. And also the growth rate of human population depend on density of trees and farm fields. We list the equilibrium points of the model and analyze the stability around each equilibrium point. We have proved our theoretical results using numerical simulations through MATLAB. And we have discussed different values for Growth-rate coefficient of trees due to technological effort in the numerical simulations.

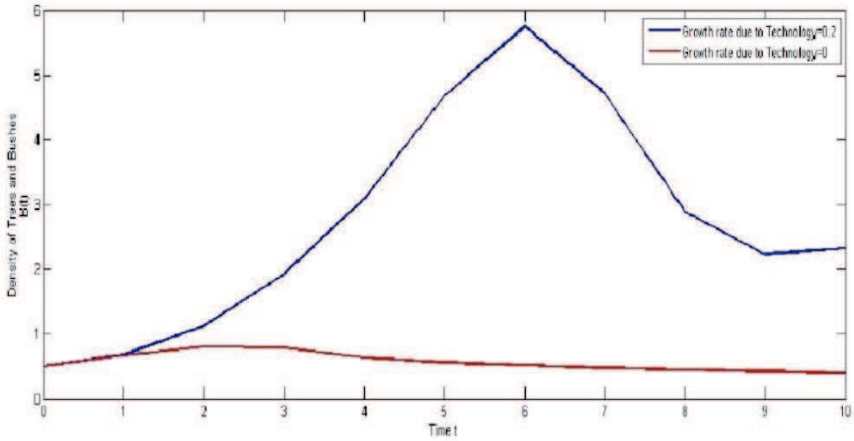

Fig.1. Density of Trees and Bushes for different values of θ

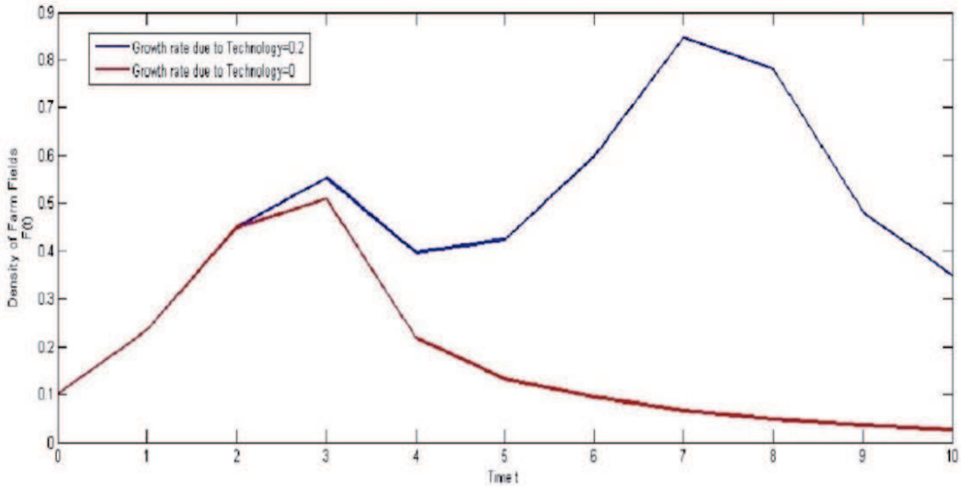

Fig.2. Density of Farm fields for different values of θ

References:

1. Canan Celik and Oktay Duman: Allee effect in a discrete-time predator–prey system, Chaos, Solitons and Fractals , 40, 1956-1962, (2009).
2. G. Britto Antony Xavier, P. Rajiniganth ,V. Chandrasekar, Finite Series and Complete Solution of Third Order; Mathematical Sciences International Research Journal ISSN 2278 – 8697 Vol 3 Issue 1 (2014), Pg 187-192
3. Unal Ufuktepe, Sinan Kapcak and Olcay Akman: Stability Analysis of the Beddington
4. Model with Allee Effect , Appl. Math. Inf. Sci. 9, No. 2, 603-608, (2015).
5. J.D. Murray, Mathematical biology. New York: Springer-Verlag, (1993).
6. Vivi Ramdhani, Jaharuddin and Nugrahani.E.H. : Dynamical System of Modelling the Depletion of Forestry Resources Due to Crowding by Industrialization, Applied Mathematical Sciences, Vol. 9, no. 82, 4067 - 4079 , (2015).
7. N. R. Dhawade, Generalizing the Ideas of Numerical Analysis; Mathematical Sciences international Research Journal ISSN 2278 – 8697 Vol 3 Issue 2 (2014), Pg 701-705
8. Manju Agarwal and Rachana Pathak: Conservation of Forestry Biomass and wildlife population: A Mathematical Model, Asian Journal of Mathematics and Computer Research, 4(1): 1-15, (2015).
9. Dubey, B., Sharma, S., Sinha, P., and Shukla.J.B :Modelling the depletion of forestry resources by population and population pressure augmented industrialization, Applied Mathematical Sciences, 33, 3002–3014, (2009).

Dr.(Sr.)Elizabeth Sebastian, HOD Department of Mathematics, Auxilium College, Gandhi Nagar, Vellore
Preethi Victor, Research Scholar, Auxilium College, Gandhi Nagar, Vellore.