STABILITY ANALYSIS OF DISCRETE NEURAL NETWORK USING LYAPUNOV METHOD

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Abstract: This paper derives a new sufficient condition for the global robust stability of uncertain recurrent neural network with interval time varying delays for the discrete case. By employing Lyapunov krosovski functional method, linear matrix inequality technique and some new delay dependent conditions are established to ensure the robust stability of the neural network. We prove that the system is globally asymptotically stable and robust stable only if the linear matrix inequality is less than zero. After that, we show that the system is globally asymptotically robust stable using stability conditions and also give conclusive remark for the main results. A numerical example is given to show that the obtained conditions can give less conservative. Numerical Simulation are proved by MATLAB.

Keywords: Recurrent neural network, Linear Matrix inequality, Lypunov Function, asymptotically stable.

Introduction: During the last decade, there has been increasing interest in the potential application of neural network in many areas. Different classes of neural network which are Hopfield neural networks, cellular neural network, Lotka Volterra neural network, recurrent neural network, Cohen Crossberg neural network and bidirectional associative memory neural network have essentially studied [1-5]. Among the many models of neural network proposed and studied in the literature, the recurrent type neural network has been very important one due to its quality of application in associative memory, pattern recognition, optimization, model identification, signal processing, etc. In the obligation of recurrent neural network in practical application is sometimes required to introduce delays in the signal transmitted among neurons. It is well known that delays can cause systems to oscillate [1,7-8]. A source of instability and oscillations considerable attention has been focused on the stability problem of neural networks with time delay [10]. Generally speaking for delayed neural network classified into two types: Delay-Independent and Delay dependent stability. Since delay dependent criteria make use of information on the size of delay, they are less conservative than delay independent ones especially when the delay is small in size [6,9].

There are many cases in recurrent neural network have been assumed to act in a continuous time manner. However, when it comes to the implementation of continuous time neural networks for the sake of computer based simulation, experimentation or computation, it is useful to discretize the continuous time network [10]. Recently, the stability analysis problems for discrete time neural networks received considerable research interest and various stability criteria have been proposed in the literature.[1-9]

In the present paper concern about the exponential stability of neural network using LMI approach. Here we have consider the delay dependent model of recurrent neural network and proved that the given system is globally robustly stable. We have provided some numerical example; it shows that our system is less conservative.

This paper is organized as follows, in section 2, we deal with problem formulation, section 3, presents some general preliminaries which is suitable to main results. In section 4, we construct a main result of the proposed model. Here we have framed the Lyapunov and linear matrix inequality. Based on these assumption, we arrived at the system is globally exponentially stable. Finally last section deals with numerical results, which gives the desired output for the proposed model.

2. Problem formulation: Consider the following discrete time recurrent neural networks with distributed time varying delays,

$$y(k+1) = Cy(k) + Bf(W(y-h(k)+J)$$
$$y(k) = \varphi(k), -h \le k \le 0 \qquad (1)$$

where, $y(k) = (y_1(k), y_2(k), \dots, y_n(k))^T$ is the neuron state vector, $f(y(k) = [f_1 y_1(k), \dots, f_n y_n(k)]^T \in R^n$ denotes the neuron activation function , $J = [j_{1,} j_{2,}, \dots, j_{n,}]^T \in R^n$ is a exogenous constant input vector, $C = diag\{c_1, c_2, \dots, c_n\}$ a positive diagonal matrix, W is the delayed connection weight matrix $h(k)$ is a time varying delays satisfying, $h_m \le h(k) \le h_M$. For $i = 1, 2, ..., n$, the neuron activation function $f(y(k))$ are continuous and bounded and each neurons activation function in system (1) are satisfied,

$$m_i \le \frac{f_i(x)-f_i(y)}{x-y} \le l_i, \forall x,y \in R, i=1,2,...,n \quad (2)$$

where, l_i and m_i are constant matrix.

A system containing time varying structured uncertainties is described by,

$$y(k+1)=(C+\Delta C(K)y(k)+ (B+\Delta B(K))f(y(k-h(k)+J), \quad k>0 \quad (3)$$

$$y(k)=\psi(k) \quad k\in[-h,0]$$

The uncertainties are assumed to be of the form,

$$\left[\Delta C(k) \quad \Delta B(k)\right]=DF(k)\left[E_c, E_b\right]$$

where D, E_c and E_b are constant matrices with appropriate dimensions and *F(K)* is an unknown, real and possibly time varying matrix with Lebesgue measurable elements satisfying,

$$F^T(k)F(k)\le I$$

Lemma 2.1: [Schur Complements]

The linear matrix inequality $\begin{pmatrix} A(x) & B(x) \\ B^T(x) & C(x)\end{pmatrix}<0$, where $A^T(x)=A(x),\ C^T(x)=C(x)$ is equivalent to either of the following conditions:

1. $A(x)<0$ and $C(x)-B^T(x)A^{-1}(x)B(x)<0$
2. $C(x)<0$ and $A(x)-B(x)C^{-1}(x)B^T(x)<0$

3. Main Results: In this section, we will present some asymptotic stability criteria and globally exponential stable criteria for the considered neural network.

Theorem 2.1: Given integers $h_m>0$ and $h_M>0$. Then, the discrete time delay system in (1) is globally asymptotically stable for any time delay $h(k)$ satisfying $h_m\le h(k)\le h_M$ if there exist matrices, $M>0, N_1>0, N_2>0, N_3>0,$ and $R>0, S>0, L_1, L_2, P_1, P_2, Q_1, Q_2$, such that the following LMI holds,

$$\begin{bmatrix} \varphi_{11} & \varphi_{12} & P_1 & -L_1 & \sqrt{\delta}L_1 & \sqrt{\delta}P_1 & \sqrt{h_M}Q_1 \\ * & \varphi_{22} & P_2 & -L_2 & \sqrt{\delta}L_2 & \sqrt{\delta}P_2 & \sqrt{h_M}Q_2 \\ * & * & -N_1 & 0 & 0 & 0 & 0 \\ * & * & * & -N_2 & 0 & 0 & 0 \\ * & * & * & * & -R-S & 0 & 0 \\ * & * & * & * & * & -R & 0 \\ * & * & * & * & * & * & -S \end{bmatrix}<0 \quad (4)$$

where, $\delta=h_M-h_m$ and

$$\varphi_{11}=C^TMC-M+(1+\varepsilon)P_1+P_2+P_3+\varepsilon(C-I)^TR(C-I)+ h(C-I)^TS(C-I)+Q_1+Q_1^T$$

$$\varphi_{12}=C^TMC+\varepsilon(C-I)^TRB+h_M(C-I)^TSB+ L_1-M_1-N_1+N_1^{\tau}$$

$$\varphi_{22}=B^TPB-Q_1+\varepsilon B^TRB+h_MB^TSB+L_2+L_2^T -M_2-M_2^T-N_2-N_2^T$$

Proof

Case I:

First we consider the case when $h_m=h_M$. In this case $h(k)$ reduces to constant delay, Denote $h=h(k)=h_m=h_M$. Then it follows from the given LMI,

$$\begin{bmatrix} \theta_{11} & \overline{\theta_{12}} & \sqrt{h}Q_1 \\ * & \overline{\theta_{22}} & \sqrt{h}Q_2 \\ * & * & -S \end{bmatrix}<0$$

where, $\overline{\theta_{12}}=C^TMB+h(C-I)^TSB-Q_1+Q_2$

$$\overline{\theta_{22}}=B^TPB-P_1-P_2-P_3-\tau B^TSB-Q_2-Q_2^T$$

Applying the Schur complements equivalence yields,

$$\theta=\begin{bmatrix} \theta_{11} & \overline{\theta_{12}} \\ * & \overline{\theta_{22}} \end{bmatrix}+\tau\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}S^{-1}\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}<0$$

Now choose a Lyapunov Function,

$$V(k)=y(k)^TMy(k)+\sum_{i=k-h}^{k-1}y(i)^TQy(i)+ \sum_{j=-h}^{-1}\sum_{i=k+j}^{k-1}\zeta^T(i)S\zeta(i) \quad (5)$$

For $\alpha>0$, we have $\Delta V(k)\le-\alpha\|y\|^2$.

Therefore, the discrete time neural network delay system in (1) is globally asymptotically stable, when $h_m=h_M$.

Case II

Now, we consider the case when $h_m\ne h_M$. In this case, we choose Lyapunov functional candidate for the system (1) as,

$$V(k)=\sum_{i=1}^{6}V_i(k)$$

where, $V_1(k)=y(k)^TPy(k)$

$$V_2(k) = \sum_{i=k-h(k)}^{k-1} y(i)^T N_1 y(i)$$

$$V_3(k) = \sum_{i=k-h_m}^{k-1} y(i)^T N_2 y(i) + \sum_{i=k-h_M}^{k-1} y(i)^T N_3 y(i)$$

$$V_4(k) = \sum_{j=h_m+1}^{-h_M} \sum_{i=k+j}^{k-1} y(i)^T N_1 y(i)$$

$$V_5(k) = \sum_{j=-h_M}^{-h_m-1} \sum_{i=k+j}^{k-1} \zeta(i)^T R \zeta(i)$$

$$V_6(k) = \sum_{j=-h_M}^{-1} \sum_{i=k+j}^{k-1} \zeta(i)^T R \zeta(i)$$

Define, $\Delta V(k) = V(k+1) - V(k)$

$$\Delta V_1(k) = y(k)^T (C^T MC - C) y(k) + 2y^T(k) C^T MBy(k-h(k)) + y^T(k-h(k)) B^T MBy(k-h(k)))$$

$$\Delta V_2(k) \le y(k)^T N_1 y(k) - y(k-h(k))^T N_1 y(k-h(k)) + \sum_{i=k+1-h_M}^{k-h_M} y(i)^T N_1 y(i)$$

$$\Delta V_3(k) = y(k)^T N_2 y(k) - y(k-h_M)^T N_2 y(k-h_M) + y(k)^T N_3 y(k) - y(k-h_M) N_3 y(k-h_M)$$

$$\Delta V_4(k) = \varepsilon y(k)^T N_1 y(k) - \sum_{i=k+1-h_M}^{k-h_M} y(i)^T N_1 y(i)$$

$$\Delta V_5(k) = \varepsilon \zeta(k)^T R \zeta(k) - \sum_{i=k+1-h_M}^{k-h_M-1} \zeta(i)^T R \zeta(i)$$

$$\Delta V_6(k) = h_M \zeta(k)^T S \zeta(k) - \sum_{i=k-h_M}^{k-1} \zeta(i)^T S \zeta(i)$$

$$\varnothing_1(k) = \begin{cases} y(k-T(k)) - y(k-h_M) - \sum_{i=k-h_M}^{k-h(k)-1} \zeta(i) = 0, & when\ h(k) \ne h_M \\ y(k-T(k)) - y(k-h_M) = 0, & when\ h(k) = h_M \end{cases}$$

$$\varnothing_2(k) = \begin{cases} y(k-h_M) - y(k-h(k)) - \sum_{i=k-h_M}^{k-h(k)-1} \zeta(i) = 0, & when\ h(k) \ne h_M \\ y(k-h_M) - y(k-h(k)) = 0, & when\ h(k) = h_M \end{cases}$$

$$\varnothing_3(k) = y(k) - y(k-h(k)) - \sum_{i=k-h(k)}^{k-1} \zeta(i) = 0$$

This implies that $\varnothing_1(k) = 0, \varnothing_2(k) = 0\ and\ \varnothing_3(k) = 0$ By some simplification we have,

$$\Delta V(k) = \sum_{i=1}^{6} \Delta V_i(k) + \left[2y(k)^T L_1 + 2y(k-h(k))^T L_2\right] \varnothing_1(k) + \left[2y(k)^T P_1 + 2y(k-h(k))^T P_2\right] \varnothing_2(k) + \left[2y(k)^T Q_1 + 2y(k-h(k))^T Q_2\right] \varnothing_3(k) \le \eta(k)^T E \eta(k)$$

where,

$$Q(k) = \left[y(k)^T\ y(k-h(k))^T\ y(k-h_m)^T\ y(k-h_M)^T \right]^T$$

$$E = \varphi + \partial L(R+S)^{-1} + \partial PR^{-1}P^T + h_m QS^{-1}Q^T$$

$$\varphi = \begin{bmatrix} \varnothing_{11} & \varnothing_{12} & P_1 & L_1 \\ * & \varnothing_{22} & P_2 & -L_2 \\ * & * & -N_2 & 0 \\ * & * & * & -N_3 \end{bmatrix}$$

$L = [L_1^T\ L_2^T\ 0\ 0]^T$ $\quad P = [P_1^T\ \ P_2^T\ \ 0\ \ 0]^T$

$N = [N_1^T\ \ N_2^T\ \ 0\ \ 0]^T$

Applying the schur equivalence leads to $E > 0$. This together with $\Delta V(k) < 0$, Shows that there exist a sufficient small scalar $\alpha > 0$ such that,

$$\Delta V(k) \le -\alpha \|y(k)\|^2 \qquad (7)$$

Therefore, the discrete time system in (1) is globally asymptotically stable, when $h_m \le h(k) \le h_M$.

Theorem 2.1:

Consider a system (2) with delay $h(k)$, which satisfies $h_m \le h(k) \le h_M$. Given scalars $h > 0$ and $\mu > 0$, the systems is robustly stable. If there exist a matrices $P > 0, Q > 0, R > 0\ and\ X > 0,$ any appropriately dimensional matrices, N_1 and N_2 and a scalar $\lambda > 0$ such that LMI in equation (4) and the following LMI holds,

$$\begin{bmatrix} \varphi_{11} + \lambda E_a^T E & \varphi_{12} + \lambda E_a^T E_b & P_1 & -L_1 & \sqrt{\delta} L_1 & \sqrt{\delta} P_1 & \sqrt{h_M} Q_1 & PD \\ * & \varphi_{22} + \lambda E_b^T E_b & P_2 & -L_2 & \sqrt{\delta} L_2 & \sqrt{\delta} P_2 & \sqrt{h_M} Q_2 & 0 \\ * & * & -N_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -N_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & -R-S & 0 & 0 & 0 \\ * & * & * & * & * & -R & 0 & 0 \\ * & * & * & * & * & * & -S & \sqrt{h_M} SD \\ D^T P & 0 & 0 & 0 & 0 & 0 & \sqrt{h_M} D^T S & -I \end{bmatrix} < 0$$

Proof:

where, $\varphi_{11}, \varphi_{12}, and\ \varphi_{22}$ are defined in equation(4), Replacing $\lambda P, \lambda Q, \lambda S, \lambda X, \lambda Q_1, and\ \lambda Q_2$ with $P, Q, S, X, Q_1, and\ Q_2$ respectively and applying schur complements shows that the above Linear Matrix is equivalent to equation (4). This completes the proof.

5. Numerical Example:

Example 1: Consider the robust stability of system with the following parameters:

$$C = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}$$

$E_c = diag\{1.6, 0.05\}, E_b = diag\{0.1, 0.3\}$ and $D = I$

Example 2: Consider the robust stability of system (2) with the following parameter, $C = \begin{bmatrix} -0.5 & -2 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -0.5 & -1 \\ 0 & 0.6 \end{bmatrix}$ $E_c = E_b = diag\{0.2, 0.2\}$ and $D = I$

Substituting these parameters value in the above two theorems we have the following table,

Table 1. Various values of h

h	0	0.5	0.9
Example 1	1.1490	0.9247	0.6954
Example 2	0.8435	0.2433	0.2420

Conclusion: In this paper, we have investigated the globally robust stability problem of discrete recurrent neural network with time varying delay. By the introduction Lyapunov functional approach and linear matrix inequality technique, some new stability criteria are proposed for the considered neural network. This proposed model gives less conservative and is proved by numerical results. The numerical simulations are proved through MATLAB.

Figure 1

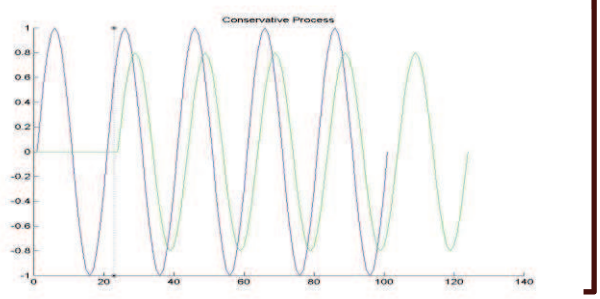
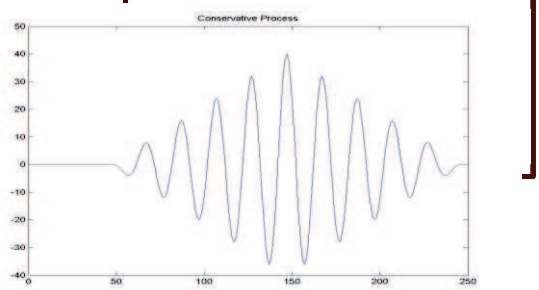


Figure 2



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