THE ACYCLIC CHROMATIC NUMBER OF CORONA GRAPHS

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Abstract: An acyclic coloring of a graph G is a proper coloring of the vertices of G such that every cycle in the graph uses at least three colors. The a-chromatic number of a graph G, denoted by $\chi_A(G)$, is the minimum k such that G admits an acyclic k- coloring. In this paper, we find that the a-chromatic number on corona graph of path P_n with cycle C_n, star $K_{1,n}$ and wheel W_n.

Keywords: Acyclic coloring, Corona graph, Cycle grap, Path graph, Star graph, Wheel graph,.

1.Introduction: The notion acyclic coloring is introduced by Grunbaum in 1973 [6], he conjectured that 5 colors are sufficient to acyclically color any planar graph. It is known that $\chi_A(G) \leq 4$ when $\Delta(G) = 3$, and $\chi_A(G) \leq 5$ when $\Delta(G) = 4$ [4]. This was confirmed by Borodin in 1979 [2]. Checking whether $\chi_A(G) \leq 3$ is a NP complete problem [8]. Alon, McDiarmid and Reed have proven that $\chi_A(G) = O(\Delta(G)^{4/3})$, $\Delta\rightarrow\infty$ [9].

Two efficient acyclic graph coloring algorithms for graphs with maximum vertex degree 3 were presented by Skulrattanakulchai[11].

2. Preliminaries: An acyclic coloring of a graph G is a proper coloring of the vertices of G such that every cycle in the graph uses at least three colors. The acyclic chromatic number $\chi_a(G)$ of a graph G is the least number of colors needed in any acyclic coloring of G.

The corona of two graphs G_1 and G_2 is the graph $G = G_1{}^{O}\, G_2$ formed from one copy of G_1 and $|V(G_1)|$ copies of G_2 where the *i*th vertex of G_1 is adjacent to every vertex in the *i*th copy of G_2.

3. Acyclic chromatic number on corona graph of path with star:

Theorem: 3.1: Let $n \geq 3$ be a positive integer. Then $\chi_A (P_n \circ K_{1,n}) = 3$.

Proof: Let $V(P_n) = \{v_1, v_2, \ldots\ldots, v_n\}$, $V(K_{1,n}) = \{u_i, u_{ij} : 1\leq i \leq n\ ; 1\leq j \leq n\}$, and

$V(P_n \circ K_{1,n}) = \{ v_i : 1\leq i \leq n \} \cup \{ u_i : 1\leq i \leq n \} \cup \{ u_{ij} : 1\leq i \leq n\ ; 1\leq j \leq n \}$.

By the definition of corona graph, each vertex of P_n is adjacent to every vertex of a copy of $K_{1,n}$. i.e., every vertex from the set $V(P_n)$ is adjacent to every vertex from the set $V(K_{1,n})$.

Assign following n- coloring for $P_n \circ K_{1,n}$ as a-chromatic:

- For $1\leq i \leq n$, color the vertices v_i with colors C_1, C_2 alternatively.
- For $1\leq i \leq n$, color the vertices u_i with colors C_2, C_1, alternatively.
- For $1\leq i \leq n$; $1\leq j \leq n$ color the vertices u_{ij} with colors C_3 alternatively.

Therefore $\chi_A (P_n \circ K_{1,n}) \leq 3$................ (1).

To prove $\chi_A (P_n \circ K_{1,n}) \geq 3$,

let us assume that $\chi_A (P_n \circ K_{1,n})$ is less than 3, i.e., $\chi_A (P_n \circ K_{1,n}) = 2$.we must assign 2 colors for $\{v_1, u_1, u_{1i} : 1\leq i \leq n\}$, Since $\{ u_1, u_{1i} : 1\leq i \leq n\}$ is adjacent to v_1 shows that v_1 needs one more distinct color. If we use only two colors then it is an easy check shows that cycle in the graph is bicolored. This is a contradiction, acyclic coloring with two colors is impossible, so we have to use the third color.

Thus, $\chi_A (P_n \circ K_{1,n}) \geq 3$. .... (2).

From (1) and (2), we have $\chi_A (P_n \circ K_{1,n}) = 3$.

This completes the proof of the theorem.

4. Acyclic chromatic number on corona graph of path with cycle:

Theorem: 4.1.

For any $n \geq 3$, $\chi_A (P_n \circ C_n) = 4$.

Proof: Let $V(P_n) = \{ v_1, v_2, \ldots\ldots, v_n\}$, $V(C_n) = \{u_1, u_2, \ldots\ldots, u_n \}$, and

$V(P_n \circ C_n) = \{ v_i : 1\leq i \leq n \} \cup \{ u_{ij} : 1\leq i \leq n\ ; 1\leq j \leq n \}$.

By the definition of corona graph, each vertex of P_n is adjacent to every vertex of a copy of C_n. i.e., every vertex from the set $V(P_n)$ is adjacent to every vertex from the set $V(C_n)$.

Assign following n- coloring for $P_n \circ C_n$ as acyclic-chromatic:

- For $1\leq i \leq n$, assign the colors C_1, C_2 alternatively to v_i.
- For $v_i \in C_1$, color the vertices of $V(C_n^{(1)})$, $V(C_n^{(2)})$, $V(C_n^{(3)})$,........., $V(C_n^{(n-1)})$ with colors C_2, C_3, alternatively.
- For $v_i \in C_1, C_2$,

Color the vertex $V(C_n^{(n)})$ with color C_4.

Therefore $\chi_A (P_n \circ C_n) \leq 4$.

To prove $\chi_A (P_n \circ K_{1,n}) \geq 4$,

let us suppose that $\chi_A (P_n \circ C_n)$ is less than 4 say $\chi_A (P_n \circ C_n) = 3$.we must assign 3 colors for $\{v_1, u_{1i} : 1\leq i \leq n\}$, Since $\{u_{1i} : 1\leq i \leq n\}$ is a cycle, it needs 3 colors for proper acyclic coloring and v_1 is adjacent to each $\{u_{1i} : 1\leq i \leq n\}$. If we assign only 3 colors, then an easy check shows that cycle in the graph $P_n \circ C_n$ is bicolored. This is a contradiction, acyclic coloring for this graph with 3 colors is impossible.

Thus, $\chi_A (P_n \circ K_{1,n}) \geq 4$.

Hence $\chi_A (P_n \circ C_n) = 4$.

This completes the proof of the theorem.

5. Acyclic chromatic number on corona graph of path with wheel.

Theorem: 5.1

For any $n \geq 3$, $\chi_A (P_n \circ W_n) = 5$.

Proof:

Let $V (P_n) = \{ v_1, v_2, \ldots\ldots, v_n\}$, $V (W_n) = \{u_1, u_2, \ldots\ldots, u_n \}$, and

$V (P_n \circ W_n) = \{ v_i : 1 \leq i \leq n \} \cup \{ u_{ij} : 1 \leq i \leq n ; 1 \leq j \leq n \}$.

By the definition of corona graph, each vertex of P_n is adjacent to every vertex of a copy of W_n. i.e., every vertex from the set $V (P_n)$ is adjacent to every vertex from the set $V(W_n)$.

Assign following n- coloring for $P_n \circ W_n$ as acyclic-chromatic:

- For $1 \leq i \leq n$, assign the colors C_1, C_2 alternatively to v_i.
- For $v_i \in C_1$, color the vertices of $V(W_n^{(1)})$, $V(W_n^{(2)})$, $V(W_n^{(3)})$,........., $V(W_n^{(n-2)})$ with colors C_2, C_3, alternatively.
- For $v_i \in C_2$, color the vertices of $V(W_n^{(1)})$, $V(W_n^{(2)})$, $V(W_n^{(3)})$,........., $V(W_n^{(n-2)})$ with colors C_1, C_3, alternatively.
- Color the vertex $V(W_n^{(n-1)})$ with color C_4.
- Color the vertex $V(W_n^{(n)})$ with color C_5.

Therefore $\chi_A (P_n \circ W_n) \leq 5$.

To prove $\chi_A (P_n \circ W_n) \geq 5$, let us suppose that $\chi_A (P_n \circ W_n)$ is less than 5 say $\chi_A (P_n \circ W_n) = 4$.

we must assign 4 colors for $\{v_1, u_{1i} : 1 \leq i \leq n\}$, Since $\{u_{1i} : 1 \leq i \leq n\}$ is a wheel, it needs 5 colors for proper acyclic coloring and v_1 is adjacent to each $\{u_{1i} : 1 \leq i \leq n\}$. If we assign only 4 colors, then an easy check shows that wheel in the graph $P_n \circ W_n$ is not acyclically colored. This gives a contradiction, acyclic coloring for this graph with 4 colors is impossible.

Thus, $\chi_A (P_n \circ W_n) \geq 5$.

Hence $\chi_A (P_n \circ W_n) = 5$.

This completes the proof of the theorem.

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