MULTI-OBJECTIVE TRANSPORTATION PROBLEM VIA LINEAR AND NON-LINEAR MEMBERSHIP FUNCTIONS

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Abstract: This paper is focused on the solution of multi-objective transportation problem via fuzzy programming algorithm using Interval Valued Number. The results of the problem reveal that if we use the hyperbolic membership function, then the crisp model becomes linear. For multi objective transportation problem it is not easy to find optimal solution which can optimize all objectives simultaneously. A Fuzzy compromise approach used in this paper helps in getting compromise solution.

Keywords: Fuzzy Programming Algorithm, Hyperbolic Membership Function, Interval Valued Numbers, Linear Membership Function, MOTP.

1. Introduction: The Fuzzy Transportation Problem (FTP) is one of the special kinds of fuzzy linear programming problems. A fuzzy transportation problem is a transportation problem in which the transportation costs, supply and demand quantities are fuzzy quantities. Fuzzy transportation is the transportation of fuzzy quantity from the fuzzy origin to fuzzy destination in such a way that the total fuzzy transportation cost is minimum. The objective of the fuzzy transportation problem is to determine the shipping schedule that minimizes the total fuzzy transportation cost, while satisfying fuzzy supply and demand limits.

The Fuzzy set Theory has been applied in many fields such as Management, Engineering etc.

2. Preliminaries:

2.1 Definition: Let X be a nonempty set. A fuzzy set A in X is characterized by its membership function A : X → [0, 1], where A(x) is interpreted as the degree of membership of element x in fuzzy A for each other x ∈ X.

2.2 Interval Number: Let R be the set of real numbers. Then closed interval [a, b] is said to be an interval number, where a, b ∈ R, a ≤ b.

2.3 Fuzzy Number: A Fuzzy set A of the real line R with membership function $\mu\tilde{A}$ (X) : R → [0, 1] is called fuzzy number if

1. A must be normal and convex fuzzy set;
2. The support of $\tilde{A}$, must be bounded
3. α_A must be closed interval for every α ∈ [0, 1]

3. Multi-Objective Transportation Problems [3] (MOTP): Most of the transportation problems are not single objective in real life situation. Here, the transportation problems which are characterized by multiple objective functions are considered .The MOTP is a special type of linear programming problem in which constraint are of equality type and all the objectives are conflicting with each other. Similarly a typical transportation problem, in a MOTP problem a product is to transported from m sources to n destination and their capacities are a_1, a_2,..., a_m and b_1, b_2,...,b_n respectively. Hence, there is a penalty C_{ij} associated with transporting a unit of product from i^{th} source to j^{th} destination. This penalty may be cost or delivery time or safety of delivery or etc. a variable x_{ij} represents the unknown quantity to be shipped from i^{th} source to j^{th} destination. The MOTP with r objectives, m source and n destinations are mathematically formulated as:

Minimize $Z_r = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{r} x_{ij}$,

$r = 1,2,\dots,K$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad i = 1, 2 \dots \dots m$$
$$\sum_{i=1}^{m} x_{ij} = b_j \qquad j = 1,2 \dots \dots n$$
$$x_{ij} \geq 0, \ \forall\, i \text{ and } j$$

The subscript in Z_r and superscript in C_{ij}^{r} are related to the r^{th} penalty criterion. Without loss of generality, it may be assumed that $a_i \geq 0\ \forall i$ and $b_j \geq 0$, $\forall\, j$ and the equilibrium condition $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ is satisfied.

3.1. Multi-Objective Optimization Problem: Most of the real life optimization problems are to be optimized simultaneously subject to a common set of constraints and also they are multi-objective in nature. The most general mathematical model of a multi-objective in nature is to be optimized simultaneously subject to a common set of constraints. The most general mathematical model of a multi-objective optimization problem is:

Maximize $F(X) = [f_1(X), f_2(X) \dots f_m(X)]$,

$X = (x_1, x_2, \dots, x_n)$

Subject to:

$g_j(X) \leq 0, \quad j = 1,2,\dots,k$

$h_j(X) = 0, \quad j = 1,2,\dots,m$

$l_j(X) \geq 0, \quad j = 1,2,\dots,r$

Where f_1, f_2 ,..., f_m are the objective functions, Variables x_1, x_2,...,x_n are called decision variables and X is called decision vector. This problem is also called multi-objective programming problem.

4. Fuzzy Programming Technique to Solve Multi-Objective Transportation Problems[2]

In this section, fuzzy programming technique to solve the MOTP with different type of membership functions is presented.

4.1. Fuzzy Programming Technique with Linear Membership Function: A linear membership function [1] is defined as:

$$\mu^L(Z_k) = \begin{cases} 1 & if \quad Z_k \le L_k \\ 1 - \frac{Z_k - L_k}{U_k - L_k} & if\ L_k < F^k(x) < U_k \\ 0 & if \quad Z_k \ge U_k \end{cases}$$

If we use a linear membership function, the crisp model can be simplified as:

Maximize λ

Subject to

$Z_k + \lambda(U_k - L_k) \le U_k,$

$$k = 1,2, \ldots, k$$

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad i = 1, 2 \ldots \ldots m$$

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad \mathrm{j} = 1,2 \ldots \ldots n$$

$x_{ij} \ge 0, \ ,\forall\, i \; and \; j$

$$\lambda \ge 0$$

4.2. Fuzzy Programming Technique with Hyperbolic Membership Function : A hyperbolic membership functions [4] is defined by

$$\mu^H(Z_k) = \begin{cases} \mathbf{1} & \mathbf{f} \quad \mathbf{Z_k \le L_k} \\ \frac{1}{2}\frac{e^{\{(U_k+L_k)/2 - Z_k(X)\}a_k} - e^{-\{(U_k+L_k)/2 - Z_k(X)\}a_k}}{e^{\{(U_k+L_k)/2 - Z_k(X)\}a_k} + e^{-\{(U_k+L_k)/2 - Z_k(X)\}a_k}} + \frac{1}{2} & \mathbf{f} \quad \mathbf{L_k < F^k(x) < U_k} \\ \mathbf{0} & \mathbf{if} \quad \mathbf{Z_k \ge U_k} \end{cases}$$

Where $a_k = \frac{6}{(U_k - L_k)}$

If we will use the hyperbolic membership function then an equivalent crisp model for the fuzzy model can be formulated as:

Maximize λ

Subject to

$\lambda \le \frac{1}{2}\frac{e^{\{(U_k+L_k)/2 - Z_k(X)\}a_k} - e^{-\{(U_k+L_k)/2 - Z_k(X)\}a_k}}{e^{\{(U_k+L_k)/2 - Z_k(X)\}a_k} + e^{-\{(U_k+L_k)/2 - Z_k(X)\}a_k}} + \frac{1}{2},$ k = 1,2,...k (1)

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad i = 1, 2 \ldots \ldots m$$

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad \mathrm{j} = 1,2 \ldots \ldots n$$

$x_{ij} \ge 0, \ ,\forall\, i \; and \; j$

$$\lambda \ge 0$$

Constraint (1) can further be simplified as:

$$\lambda \le \frac{1}{2}\tanh\left[\left\{\frac{U_k + L_k}{2} - Z_k(X)\right\} a_k\right] + \frac{1}{2}$$

$$a_k Z_k + tanh^{-1}(2\lambda - 1) \le \frac{(U_k + L_k)a_k}{2}$$

$$a_k Z_k(X) + X_{mn+1} \le \frac{(U_k + L_k)a_k}{2}$$

Hence, the given problem is simplified as:

Maximize X_{mn+1}

Subject to

$a_k Z_k(X) + X_{mn+1} \le \frac{(U_k + L_k)a_k}{2},$ k=1,2,....k

$$\sum_{j=1}^{n} x_{ij} = a_i \qquad i = 1, 2 \ldots \ldots m$$

$$\sum_{i=1}^{m} x_{ij} = b_j \qquad \mathrm{j} = 1,2 \ldots \ldots n$$

$x_{ij} \ge 0, \ ,\forall\, i \; and \; j$

$X_{mn+1} \ge 0,$

where $X_{mn+1} = tanh^{-1}(2\lambda - 1)$

5. Example:

Sources\Destination	D_1	D_2	D_3	Supply
S_1	(17,19)	(20,22)	(18,21)	(17,20)
S_2	(15,17)	(11,14)	(12.15)	(16,18)
S_3	(10,12)	(13,15)	(14,16)	(15,19)
Demand	(14,16)	(20,23)	(14,18)	

Sources\Destination	D_1	D_2	D_3	Supply
S_1	(11,13)	(12,14)	(15,17)	(17,20)
S_2	(12,15)	(13,15)	(16,18)	(16,18)
S_3	(13,16)	(15,17)	(10,12)	(15,19)
Demand	(14,16)	(20,23)	(14,18)	

Solution:

Sources\Destination	D_1	D_2	D_3	Supply
S_1	18	21	19.5	18.5
S_2	16	12.5	13.5	17
S_3	11	14	15	17
Demand	15	21.5	16	

Sources\Destination	D_1	D_2	D_3	Supply
S_1	12	13	16	18.5
S_2	13.5	14	17	17
S_3	14.5	16	11	17
Demand	15	21.5	16	

If a linear membership function is employed, the crisp model can be presented as follows:

Maximize λ

Subject to:

$18X_{11}+21X_{12}+19.5X_{13}+16X_{21}+12.5X_{22}+13.5X_{23}+11X_{31}+14X_{32}+15X_{33}+40\ \lambda \le 810$

$12X_{11}+13X_{12}+16X_{13}+13.5X_{21}+14X_{22}+17X_{23}+14.5X_{31}+16X_{32}+11X_{33}+121\,\lambda \le 776$

$\sum_{j=1}^{3} X_{1j} = 18.5, \sum_{j=1}^{3} X_{2j} = 17,$

$$\sum_{j=1}^{3} X_{3j} = 17$$

$\sum_{i=1}^{3} X_{i1} = 15, \sum_{i=1}^{3} X_{i2} = 21.5,$

$$\sum_{i=1}^{3} X_{i3} = 16$$

$X_{ij} \ge 0, i = 1,2,3; j = 1,2,3$

The optimal solution of the above problem is thus presented as below:

$X^* = \{X_{11} = 15, X_{13} = 3.5, X_{22} = 17, X_{32} = 4.5, X_{33} = 12.5\}$

$Z_1^* = 996.25$; $Z_1^* = 683.5$; $\lambda^* = 0.8$

If we use the hyperbolic membership functions, an equivalent crisp model can be formulated ,

Maximize X_{10}

Subject to:

$2.7X_{11}+3.15X_{12}+2.93X_{13}+2.4X_{21}+1.88X_{22}+2.01X_{23}+1.65X_{31}+2.1X_{32}+2.25X_{33}+X_{10} \leq 118.5$

$0.6X_{11}+0.65X_{12}+0.8X_{13}+0.68X_{21}+0.7X_{22}+0.85X_{23}+0.73X_{31}+0.8X_{32}+0.55X_{33}+X_{10} \leq 107.325$

$\sum_{j=1}^{3} X_{1j} = 18.5, \sum_{j=1}^{3} X_{2j} = 17, \sum_{j=1}^{3} X_{3j} = 17$

$\sum_{i=1}^{3} X_{i1} = 15, \sum_{i=1}^{3} X_{i2} = 21.5, \sum_{i=1}^{3} X_{i3} = 16$

$X_{ij} \geq 0, i = 1,2,3; j = 1,2,3$

Solving the above problem, the optimal solution is shown a follows:

$X^* = \{X_{12} = 4.5, X_{13} = 14, X_{22} = 17, X_{31} = 15, X_{33} = 2\}$

$Z_1^* = 775$; $Z_1^* = 760$; $X_{10} = 2$

6. Conclusion: In this chapter, two special types of membership functions linear and non-linear are used to solve the MOTP. It is observed that if we use the hyperbolic membership function, then the crisp model becomes linear. However, if we compare with the solution obtained by the linear membership function, it is shown that the fuzzy optimal values do not depend on the chosen membership function whether linear or non-linear membership function is used.

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