AN APPROPRIATE SOLUTION FOR MULTI-OBJECTIVE FUZZY GAME MATRIX USING MAX-MIN SOLUTION

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Abstract: In this paper, we consider a multi-objective two person zero-sum matrix game with fuzzy goals, assuming that each player has a fuzzy goal for each of the payoffs. The max-min solution is formulated for this multi-objective game model, in which the optimization problem for each player is a linear programming problem. Every developed model for each player is demonstrated through a numerical example.

Keywords: Fuzzy goal, Interval numbers, Max-min solution, Multi-objective matrix game.

Introduction: Game theory is concerned with decision making problem where two or more autonomous decision makers have conflicting interests. They are usually referred to as players who act strategically to find out a compromise solution. On the other hand, in multi-objective optimization problems, a single decision maker optimizes the solution among the conflicting objectives. Multi-objective matrix games are capable of dealing with both types of conflicts. When interest of one player is completely against the interest of others, matrix game is determined as two person zero-sum matrix game.

1. An introduction to fuzzy sets: Fuzzy sets were introduced by **Zadeh** [4] in 1965 to represent/manipulate data and information possessing non statistical uncertainties. It was specifically designed to mathematically represent uncertainty and vagueness and to provide formalized tools for dealing with the imprecision intrinsic to many problems.

1.1. Fuzzy Set: A fuzzy subsetof a universe X (a fuzzy set) is a mathematical object A described by its (generalized) characteristic function (membership function)

$\mu_A: X \to [0, 1]$

The classical membership degrees are represented by 1 (is a member) and 0

(Not a member) Alternative notation: A(x)

$\mathcal{F}(X)$ denotes the set of all fuzzy subsets of a universe X.

2. Some basic definition:

2.1. Matrix game with fuzzy goals: [3]

A two person zero-sum matrix game with fuzzy goals (FG) is defined as

$FG = (S^m, S^n, A, \overline{v}, \gtrsim, \underline{v}, \lesssim)$

Where, $S^m, S^n$are strategy space for Player I and II, respectively and is a payoff matrix for Player I. Here $\overline{v}$ and $\underline{v}$ are scalars representing the aspiration levels of Player I and Player II, and symbols "$\gtrsim$" and "$\lesssim$" are fuzzified versions of usual "$\geq$ " and "$\leq$ " respectively. A two person zero-sum multi-objective matrix game with fuzzy goals (MOFG) is represented by multi-payoff matrices $A^1, A^2, \ldots . A^r$where it is assumed that each player has the same r objectives.

2.2. Fuzzy goal: Let $D^k \subseteq \mathbb{R}$ be the domain of k^{th}payoff for Player I, then a fuzzy goal $\tilde{g}_I^k$ with respect to k^{th} payoff for Player I is a fuzzy set onD^k , characterized by the membership function

$\mu_{\tilde{g}_I^k}: D^k \to [0,1]$

Similarly, a fuzzy goal $\tilde{g}_{II}^k$ for Player II is also a fuzzy set on D^k, characterized by the membership function

$\mu_{\tilde{g}_{II}^k}: D^k \to [0,1]$

A value of membership function for a fuzzy goal can be interpreted as the degree of attainment of fuzzy goal for the payoff. Therefore, when a player has two different payoffs, he prefers the payoff possessing higher membership function value. It means that Player I aims to maximize his degree of attainment.

2.3. Max-min value: [2]: The max-min value with respect to the degree of attainment of an aggregated fuzzy goal to Player I is

$$\max_{x \in S^m} \min_{y \in S^n} \min_k \{\mu_{\tilde{g}_I^k}(x^T A^k y)\}$$

Similarly, the max-min value with respect to degree of attainment of an aggregated fuzzy goal to Player II is

$$\max_{y \in S^n} \min_{x \in S^m} \min_k \{\mu_{\tilde{g}_{II}^k}(x^T A^k y)\}$$

2.4. Fuzzy Numbers and Fuzzy Intervals:

A fuzzy number is a fuzzy set on $\mathbb{R}$

$A: \mathbb{R} \longrightarrow [0,1]$ such that

(i) A is normal

(ii) A is a closed interval for all $\alpha \in (0,1]$

(iii) The support of A, supp (A) = A is bounded.

Since all α-cuts are closed intervals every fuzzy numbers is a convex fuzzy set.

3. Multi-Objective Matrix Game With Fuzzy Goals: [3]: A MOFG is represented by fuzzy multiple payoff matrices, $A^k, k = 1,2, \ldots, r$ and fuzzy goals to each objective is $\overline{v}^k(\underline{v}^k)$, k=1,2,…,r to Player I (Player II). In this section, we proposed linear models for optimization problem to Player I and Player II, respectively as follows:

3.1. Optimization problem for Player I: Let the membership function of the fuzzy goal fork^{th} objective of Player I be denoted by $\mu_{\tilde{g}_I}(x^T Ay)$.Assuming the membership function $\mu_{\tilde{g}_I}(x^T Ay)$ to be linear, it can be represented as,

$$\mu_{\tilde{g}^k{}_I}(x^TA^ky)=\begin{cases} 0 & ,x^TA^ky \le \underline{v}^k, \\ 1-\dfrac{\overline{v}^k - x^TA^ky}{\overline{v}^k-\underline{v}^k} & ,\underline{v}^k < x^TA^ky \le \overline{v}^k, \\ 1 & ,\overline{v}^k < x^TA^ky \end{cases}$$

Where $\underline{v}$ and $\overline{v}$are the payoffs for which degree of attainment to Player I is 0 and 1. The values of $\underline{v}$ and $\overline{v}$can be obtained as $\underline{v} = \min_x \min_y x^TAy = \min_i \min_j a_{ij}(1)\overline{v}=\max_x \max_y x^TAy = \max_i \max_j a_{ij}$(2)

The membership function for aggregated fuzzy goal $\tilde{g}_I$ to Player I can be constructed by using decision making principle [1] for fuzzy environment. Accordingly, the membership function for aggregated fuzzy goal i

$$\min_k \{ \mu_{\tilde{g}^k{}_I}(x^TA^ky) \}$$

The degree of attainment of an aggregated fuzzy goal $\tilde{g}_I$ to Player I is

$$\max_{x\in S^m} \min_{y \in S^n} \min_k \{ \mu_{\tilde{g}^k{}_I}(x^TA^ky) \}$$

The optimal solution for the above game problem can be obtained the following linear programming problem.

Max λ

Subject to,

$$\sum_{i=1}^{m} \frac{a_{i1}^k}{\overline{v}^k-\underline{v}^k} x_i - \frac{\underline{v}^k}{\overline{v}^k-\underline{v}^k} \ge \lambda, \quad (k=1,\ldots\ldots,r),$$

$$\sum_{i=1}^{m} \frac{a_{i2}^k}{\overline{v}^k-\underline{v}^k} x_i - \frac{\underline{v}^k}{\overline{v}^k-\underline{v}^k} \ge \lambda, \quad (k=1,\ldots\ldots,r),$$

....

$$\sum_{i=1}^{m} \frac{a_{in}^k}{\overline{v}^k-\underline{v}^k} x_i - \frac{\underline{v}^k}{\overline{v}^k-\underline{v}^k} \ge \lambda, \quad (k=1,\ldots\ldots,r),$$

$\sum_{i=1}^{m} x_i = 1,\quad \lambda \le 1,\quad x,\ \lambda \ge 0$(3)

3.2. Optimization problem for Player II:

Similarly for Player II, the membership function $\mu_{\tilde{g}^k{}_{II}}(x^TA^ky)$ for k^{th}fuzzy goal is linear and can be represented as

$$\mu_{\tilde{g}^k{}_{II}}(x^TA^ky)=\begin{cases} 1 & ,x^TA^ky \le \underline{v}^k, \\ 1-\dfrac{x^TA^ky-\underline{v}^k}{\overline{v}^k-\underline{v}^k} & ,\underline{v}^k < x^TA^ky \le \overline{v}^k, \\ 0 & ,\overline{v}^k < x^TA^ky \end{cases}$$

The membership function of an aggregated fuzzy goal $\tilde{g}_{II}$ to Player II can be obtained as

$$min_k\{\mu_{\tilde{g}^k{}_{II}}(x^TA^ky)\}$$

The degree of attainment of the aggregated fuzzy goal to Player II is

$$max_{y\in S^n} min_{x\in S^m} min_k \ \{ \mu_{\tilde{g}^k{}_{II}}(x^TA^ky) \}$$

Equivalent LP, FLP for the above game problem can be written as

Max η

Subject to,

$$\sum_{j=1}^{n} \frac{a_{1j}^k}{\overline{v}^k-\underline{v}^k} y_i - \frac{\underline{v}^k}{\overline{v}^k-\underline{v}^k} \le 1-\eta, \quad (k=1,\ldots\ldots,r),$$

$$\sum_{j=1}^{n} \frac{a_{2j}^k}{\overline{v}^k-\underline{v}^k} y_i - \frac{\underline{v}^k}{\overline{v}^k-\underline{v}^k} \le 1-\eta, \quad (k=1,\ldots\ldots,r),$$

....

$$\sum_{j=1}^{n} \frac{a_{mj}^k}{\overline{v}^k-\underline{v}^k} y_i - \frac{\underline{v}^k}{\overline{v}^k-\underline{v}^k} \le 1-\eta, \quad (k=1,\ldots\ldots,r),$$

$\sum_{j=1}^{n} y_i = 1,\quad \eta \le 1,\quad y,\ \eta \ge 0$.....(4)

4. Numerical Example: Suppose there are two companies, I and II, aiming to enhance the sales amount and market share of a product in a targeted market. Under the circumstance that the demand amount of the product in the targeted market is basically fixed, the sales amount and market share of one company increases, following the decrease of the sales amount and market share of another company, but the sales amount is not certain to be proportional to the market share. The two companies are considering the three strategies to increase the sales amount and market share:

x_1 : Advertisement; x_2: reduce the price; x_3 : improve the package.

This problem is a multi-objective two person zero-sum matrix game. Further, let Company I be Player I, adopting the strategy (x_1, x_2, x_3), Company II be Player II, adopting strategy (y_1, y_2, y_3). Under the three strategies, the payoff matrices A^1, A^2, of targeted sales quantity and market share with interval numbers are separately indicated as

$$A^1 = \begin{pmatrix} (179,181) & (349,351) & (574,576) \\ (254,256) & (429,431) & (179,181) \\ (89,91) & (155,157) & (124,126) \end{pmatrix}$$

$$A^2 = \begin{pmatrix} (24,26) & (34,36) & (41,43) \\ (31,33) & (21,23) & (28,30) \\ (14,16) & (9,11) & (24,26) \end{pmatrix}$$

Here, using (1) & (2), we get

$\underline{v}^1 = 90, \overline{v}^1 = 575$ and $\overline{v}^1 - \underline{v}^1 = 485$

$\underline{v}^2 = 10, \overline{v}^2 = 42$ and $\overline{v}^2 - \underline{v}^2 = 32$

In this case, using (3) we get

The optimal solution of the problems is obtained for player I is

(x^* = (0.1194, 0.2369, 0.9260), λ^*=0.0903)

Similar results for Player II are obtained by using (4) we get

The optimal solution to Player II is

($y^* = (0.0739, 0.3564, 0.049)$,

η^* =0.0127)

Conclusion: A solution of a multi-objective two person zero sum matrix game with fuzzy goals and fuzzy interval numbers is obtained using max-min solution approach. In this method, a solution of multi objective matrix game is generalized. A numerical example is demonstrated to show the improvement in value of the game for the winning player.

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