A STUDY ON SEMI $^{\#}$GENERALIZED α-CLOSED SETS IN BI-$\check{C}$ECH CLOSURE SPACES

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Abstract: The purpose of this paper is to introduce the notion of semi $^{\#}$gα-closed sets in bi-$\check{C}$ech closure spaces. We also introduce the concepts of ${}_{s^{\#}g\alpha}C_0$- bi-$\check{C}$ech spaces and ${}_{s^{\#}g\alpha}C_1$- bi-$\check{C}$ech spaces and investigate their basic properties.

Keywords: semi $^{\#}$gα-closed sets, ${}_{s^{\#}g\alpha}C_0$- bi-$\check{C}$ech spaces, ${}_{s^{\#}g\alpha}C_1$- bi-$\check{C}$ech spaces.

1. Introduction : In 2006, K. Chandrasekhara Rao and R. Gowri ([], []) introduced the concepts of bi-$\check{C}$ech closure operators, k_i-semi open set and $k_i\alpha$-open set, where i = 1,2. Later in 2012, M.Vigneshwaran [] introduced the concepts of semi $^{\#}$gα-closed and semi $^{\#}$gα-open sets in bi-$\check{C}$ech closure spaces. In this paper we introduce the notion of semi $^{\#}$gα-closed sets in bi-$\check{C}$ech closure spaces. We also introduce the concepts of ${}_{s^{\#}g\alpha}C_0$ bi-$\check{C}$ech spaces and ${}_{s^{\#}g\alpha}C_1$ bi-$\check{C}$ech spaces and investigate their basic properties.

2. preliminaries: Definition 2.1 Two functions k_1 and k_2 from power set of X to itself are called bi-$\check{C}$ech closure operators [] (briefly biclosure operator) for X if they satisfy the following properties:

(i) $k_1(\phi) = \phi$ and $k_2(\phi) = \phi$.

(ii) $A \subset k_1(A)$ and $A \subset k_2(A)$ for any set $A \subset X$.

(iii) $k_1(A\cup B) = k_1(A) \cup k_1(B)$ and $k_2(A\cup B) = k_2(A) \cup k_2(B)$ for any A, B $\subset$ X.

(X,k_1,k_2) is called bi-$\check{C}$ech closure space.

Definition 2.2 A subset A in a bi-$\check{C}$ech closure space (X,k_1,k_2) is said to be

(i) k_i-semi open if $A \subseteq k_i[int_{k_i}(A)]$, i = 1,2.

(ii) $k_i\alpha$-open if $A \subseteq int_{k_i}(k_i[int_{k_i}(A)])$,i = 1,2.

The smallest k_i-semi closed set containing A is called k_i-semi closure of A and it is denoted by $k_{si}(A)$. The largest k_i-semi open set contained in A is called k_i-semi interior of A and it is denoted by $int_{k_{si}}(A)$.

The smallest k_i-α closed set containing A is called k_i-α closure of A and it is denoted by $k_{\alpha i}(A)$. The largest k_i-α open set contained in A is called k_i- α interior of A and it is denoted by $int_{k_{\alpha i}}(A)$.

Definition 2.3 A subset A in a bi-$\check{C}$ech closure space (X,k_1,k_2) is said to be

(i) (k_1,k_2)-g closed [] if $k_2(A) \subseteq U$, whenever $A \subseteq U$ and U is k_1-open set in X. The complement of (k_1,k_2)-g closed set is called (k_1,k_2)-g open.

(ii) (k_1,k_2)-gα closed [] if $k_{\alpha 2}(A) \subseteq U$, whenever $A \subseteq U$ and U is k_1-α open set in X. The complement of (k_1,k_2)-gα closed set is called (k_1,k_2)-gα open.

(iii) (k_1,k_2)-*gα closed [] if $k_2(A) \subseteq U$, whenever $A \subseteq U$ and U is k_1-gα open set in X. The complement of (k_1,k_2)-*gα closed set is called (k_1,k_2)-*gα open.

(iv) (k_1,k_2)-g$^{\#}\alpha$ closed [] if $k_{\alpha 2}(A) \subseteq U$, whenever $A \subseteq U$ and U is k_1-g open set in X. The complement of (k_1,k_2)-g$^{\#}\alpha$ closed set is called (k_1,k_2)-g$^{\#}\alpha$ open.

(v) (k_1,k_2)-$^{\#}$gα closed [] if $k_{\alpha 2}(A) \subseteq U$, whenever $A \subseteq U$ and U is k_1-g$^{\#}\alpha$ open set in X. The complement of (k_1,k_2)-$^{\#}$gα closed set is called (k_1,k_2)-$^{\#}$gα open.

Definition 2.4 A bi-$\check{C}$ech closure space (X,k_1,k_2) is said to be a ${}_{*g\alpha}C_0$ bi-$\check{C}$ech space [] if for every *gα-open subset U of (X,k_1), x $\in$ U implies $k_2(\{x\}) \subseteq U$.

Definition 2.5 A bi-$\check{C}$ech closure space (X,k_1,k_2) is said to be a ${}_{*g\alpha}C_1$ bi-$\check{C}$ech space [] if for each x,y $\in$ X such that $k_1(\{x\}) \neq k_2(\{y\})$, there exist a disjoint *gα-open subset U of (X,k_2) and a *gα-open subset V of (X,k_1) such that $k_1(\{x\}) \subseteq U$ and $k_2(\{y\}) \subseteq V$.

Definition 2.6 A bi-$\check{C}$ech closure space (X,k_1,k_2) is said to be a ${}_{g^{\#}\alpha}C_0$ bi-$\check{C}$ech space [] if for every g$^{\#}\alpha$-open subset U of (X,k_1), x $\in$ U implies $k_2(\{x\}) \subseteq U$.

Definition 2.7 A bi-$\check{C}$ech closure space (X,k_1,k_2) is said to be a ${}_{g^{\#}\alpha}C_1$ bi-$\check{C}$ech space [] if for each x,y $\in$ X such that $k_1(\{x\}) \neq k_2(\{y\})$, there exist a disjoint g$^{\#}\alpha$-open subset U of (X,k_2) and a g$^{\#}\alpha$-open subset V of (X,k_1) such that $k_1(\{x\}) \subseteq U$ and $k_2(\{y\}) \subseteq V$.

Definition 2.8 A bi-$\check{C}$ech closure space (X,k_1,k_2) is said to be a ${}_{\#g\alpha}C_0$ bi-$\check{C}$ech space [] if for every $^{\#}$gα-open subset U of (X,k_1), x $\in$ U implies $k_2(\{x\}) \subseteq U$.

Definition 2.9 A bi-$\check{C}$ech closure space (X,k_1,k_2) is said to be a ${}_{\#g\alpha}C_1$ bi-$\check{C}$ech space [] if for each x,y $\in$ X such that $k_1(\{x\}) \neq k_2(\{y\})$, there exist a disjoint $^{\#}$gα-open subset U of (X,k_2) and a $^{\#}$gα-open subset V of (X,k_1) such that $k_1(\{x\}) \subseteq U$ and $k_2(\{y\}) \subseteq V$.

3. (k_1,k_2)-Semi $^{\#}$gα Closed Sets

Definition 3.1 A subset A in a bi-$\check{C}$ech closure space (X,k_1,k_2) is said to be (k_1,k_2)-semi $^{\#}$gα closed [] if $k_{s2}(A) \subseteq U$, whenever $A \subseteq U$ and U is (k_1,k_2)-$^{\#}$gα open set in X.

Example 3.2 Let X = {a,b,c} and let k_1 and k_2 be two functions from power set of X to itself is defined as:
$k_1(\{a\}) = \{a\}$, $k_1(\{b\}) = k_1(\{c\}) = k_1(\{b,c\}) = \{b,c\}$, $k_1(\{a,b\}) = k_1(\{a,c\}) = k_1(X) = X$ and $k_1(\phi) = \phi$.
$k_2(\{a\})$, $k_2(\{a,c\}) = \{a,c\}$, $k_2(\{b\})$, $k_2(\{b,c\}) = \{b,c\}$, $k_2(\{c\}) = \{c\}$, $k_2(\{a,b\}) = k_2(X) = X$ and $k_2(\phi) = \phi$.
(k_1,k_2)-semi $^{\#}g\alpha$ closed sets in (X,k_1,k_2) = { ϕ, X, {a}, {b}, {c}, {b,c}, {a,c}}

Remark 3.3 Every (k_1,k_2)-closed set is (k_1,k_2)-semi $^{\#}g\alpha$ closed.

The following example shows that the converse need not be true.

Example 3.4 In Example 3.2, we have (k_1,k_2)-closed sets in (X,k_1,k_2) = { ϕ, X,{c},{b,c},{a,c}};
(k_1,k_2)-semi $^{\#}g\alpha$ closed sets in (X,k_1,k_2) = { ϕ, X, {a}, {b}, {c}, {b,c}, {a,c}}. Here {a} is (k_1,k_2)-semi $^{\#}g\alpha$ closed but not (k_1,k_2)-closed.

Theorem 3.5 If A and B are (k_1,k_2)-semi $^{\#}g\alpha$ closed sets, then so is A ∪ B.

Proof. Let A and B be two (k_1,k_2)-semi $^{\#}g\alpha$ closed sets. Let (A ∪ B) ⊆ U and U be a (k_1,k_2)-$^{\#}g\alpha$ open set in X. So we have A ⊆ U and B ⊆ U. Then k_{s2}(A) ⊆ U and k_{s2}(B) ⊆ U implies [k_{s2}(A)∪ k_{s2}(B)] ⊆ U. Hence k_{s2}(A ∪ B) ⊆ U. Thus A ∪ B is also (k_1,k_2)-semi $^{\#}g\alpha$ closed.

Theorem 3.6 If A is (k_1,k_2)-semi $^{\#}g\alpha$ closed set, then k_{s2}(A) \ A contains no non-empty (k_1,k_2)-$^{\#}g\alpha$ closed sets.

Proof. Let A be (k_1,k_2)-semi $^{\#}g\alpha$ closed. Let U be (k_1,k_2)-$^{\#}g\alpha$ closed contained in k_{s2}(A) \ A. Now, U ⊆ k_{s2}(A) and U ⊆ A^c, i.e., A ⊆ U^c. Since U is k_1-$^{\#}g\alpha$ closed, U^c is k_1-$^{\#}g\alpha$ open. So k_{s2}(A) ⊆ U^c. That is U ⊆ $[k_{s2}(A)]^c$. Hence U ⊆ $k_{s2}(A) \cap [k_{s2}(A)]^c = \phi$. Therefore U = ϕ. Thus k_{s2}(A) \ A contains no non-empty (k_1,k_2)-$^{\#}g\alpha$ closed sets.

Theorem 3.7 Let (X,k_1,k_2) be bi-$\check{C}$ech closure space. For each x in X, {x} is (k_1,k_2)-$^{\#}g\alpha$ closed or $\{x\}^c$ is a (k_1,k_2)-semi $^{\#}g\alpha$ closed set.

Proof. Let (X,k_1,k_2) be bi-$\check{C}$ech closure space. Suppose that {x} is not (k_1,k_2)-$^{\#}g\alpha$ closed, $\{x\}^c$ is not (k_1,k_2)-$^{\#}g\alpha$ open. Therefore the only (k_1,k_2)-$^{\#}g\alpha$ open set containing $\{x\}^c$ is X. Thus $\{x\}^c$ ⊆ X. Moreover $k_{s2}[(x)^c] \subseteq k_{s2}(x)$ = x. Hence $\{x\}^c$ is (k_1,k_2)-semi $^{\#}g\alpha$ closed set.

Theorem 3.8 Let A be (k_1,k_2)-semi $^{\#}g\alpha$ closed set and if A is (k_1,k_2)-$^{\#}g\alpha$ open, then A = k_{s2}(A).

Proof. Let A be (k_1,k_2)-semi $^{\#}g\alpha$ closed and (k_1,k_2) -$^{\#}g\alpha$ open sets in (X,k_1,k_2). Then k_{s2}(A) ⊆ U, whenever A ⊆ U and U is (k_1,k_2)-$^{\#}g\alpha$ open in X. Since A is (k_1,k_2)-$^{\#}g\alpha$ open, we have k_{s2}(A) ⊆ A. But always A ⊆ k_{s2}(A). Thus A = k_{s2}(A).

Theorem 3.9 Let A ⊆ Y ⊆ X and suppose that A is (k_1,k_2)-semi $^{\#}g\alpha$ closed in (X, k_1,k_2). Then A is (k_1,k_2)-$^{\#}g\alpha$ closed relative to Y.

Proof. Let S be any (k_1,k_2)-$^{\#}g\alpha$ open set in Y such that A ⊆ S. Then S = U ∩ Y for some U is (k_1,k_2)-$^{\#}g\alpha$ open in X. Therefore A⊂ U ∩ Y implies A ⊆ U. Since A is (k_1,k_2)-semi $^{\#}g\alpha$ closed in X, we have k_{s2}(A) ⊆ U. Hence Y ∩ k_{s2}(A) ⊆ Y ∩ U = S. Thus A is a (k_1,k_2)-semi $^{\#}g\alpha$ closed set relative to Y.

Theorem 3.10 If A is a (k_1,k_2)-semi $^{\#}g\alpha$ closed set in (X, k_1,k_2) such that A ⊆ B ⊆ k_{s2}(A), then B is also a (k_1,k_2)-semi $^{\#}g\alpha$ closed set in (X, k_1,k_2).

Proof. Let U be (k_1,k_2)-$^{\#}g\alpha$ open such that B ⊆ U. Since A is (k_1,k_2)-semi $^{\#}g\alpha$ closed, we have k_{s2}(A) ⊆ U and k_{s2}(B) ⊆ k_{s2}(A) ⊆ U. Therefore B is a (k_1,k_2)-semi $^{\#}g\alpha$ closed set in (X, k_1,k_2).

Theorem 3.11 If A is (k_1,k_2)-$^{\#}g\alpha$-open and (k_1,k_2)-semi $^{\#}g\alpha$ closed sets of (X, k_1,k_2), then A is a (k_1,k_2)-semi closed set.

Proof. It is obvious.

Theorem 3.12 Every subset is (k_1,k_2)-semi $^{\#}g\alpha$ closed if and only if every (k_1,k_2)-$^{\#}g\alpha$-open set is (k_1,k_2)-semi closed.

Proof. Let U be a (k_1,k_2)-$^{\#}g\alpha$-open set. Then we have k_{s2}(U) ⊆ U and hence U is (k_1,k_2)-semi closed.

Conversely, let A be a subset and U be a (k_1,k_2)-$^{\#}g\alpha$-open set such that A ⊆ U. Then k_{s2}(A) ⊆ k_{s2}(U) = U and hence A is (k_1,k_2)-semi $^{\#}g\alpha$ closed.

4. $_{s^{\#}g\alpha}C_0$- bi-$\check{C}$ech Spaces and $_{s^{\#}g\alpha}C_1$- bi-$\check{C}$ech Spaces

Definition 4.1 A bi-$\check{C}$ech closure space (X,k_1,k_2) is said to be a $_{s^{\#}g\alpha}C_0$- bi-$\check{C}$ech space if for every $^{\#}g\alpha$ open subset U of (X,k_1), x ∈ U implies $k_{s2}(\{x\})$ ⊆ U.

Example 4.2 In Example 3.2, U = P(X) and hence x ∈ U implies $k_{s2}(\{x\})$ ⊆ U. Therefore bi-$\check{C}$ech closure space (X,k_1,k_2) is a $_{s^{\#}g\alpha}C_0$- bi-$\check{C}$ech space.

Theorem 4.3 A bi-$\check{C}$ech closure space (X,k_1,k_2) is a $_{s^{\#}g\alpha}C_0$- bi-$\check{C}$ech space if and only if for every semi $^{\#}g\alpha$ closed subset F of (X,k_1) such that x ∉ F, $k_{s2}(\{x\}) \cap F = \phi$.

Proof. Let F be a semi $^{\#}g\alpha$ closed subset F of (X,k_1) and let x ∉ F. Since x ∈ X \ F and X \ F is a semi $^{\#}g\alpha$ closed subset of (X,k_1), $k_{s2}(\{x\})$ ⊆ X \ F. Hence $k_{s2}(\{x\}) \cap F = \phi$.

Conversely suppose that U be a semi $^{\#}g\alpha$ closed subset of (X,k_1) and let x ∈ U. Since X \ U is a semi $^{\#}g\alpha$ closed subset F of (X,k_1) and x ∉ X \ U, $k_{s2}(\{x\})$ ∩ (X \ U) = ϕ. Therefore $k_{s2}(\{x\})$ ⊆ U. Hence (X,k_1,k_2) is a $_{s^{\#}g\alpha}C_0$- bi-$\check{C}$ech space.

Definition 4.4 A bi-$\check{C}$ech closure space (X,k_1,k_2) is said to be $_{s^{\#}g\alpha}C_1$- bi-$\check{C}$ech spaces if for each x, y ∈ X such that $k_1(\{x\}) \neq k_2(\{y\})$, there exists a disjoint semi $^{\#}g\alpha$ open subset U of (X,k_2) and a semi $^{\#}g\alpha$ open subset V of (X,k_1) such that $k_1(\{x\})$ ⊆ U and $k_2(\{y\})$ ⊆ V.

Example 4.5 In Example 3.2, {a,c}, {b} ∈ X such that $k_1(\{a,c\}) \neq k_2(\{b\})$, there exists a disjoint semi $^{\#}g\alpha$ open subset U = X of (X,k_2) and a semi $^{\#}g\alpha$

open subset V = {b,c} of (X,k_1) Therefore bi-$\check{C}$ech closure space (X,k_1,k_2) is a $_{s^{\#}g\alpha}C_1$- bi-$\check{C}$ech space.

Theorem 4.6 Every $_{s^{\#}g\alpha}C_1$- bi-$\check{C}$ech space is a $_{s^{\#}g\alpha}C_0$- bi-$\check{C}$ech space.

Proof. Let (X,k_1,k_2) be a $_{s^{\#}g\alpha}C_1$- bi-$\check{C}$ech space. Also let U be a semi $^{\#}g\alpha$ open subset of (X,k_1) and let x ∈ U. If y ∉ U, then k_2({x}) ≠ k_1({y}) since x ∉ k_1({y}). Then there exists a semi $^{\#}g\alpha$ open subset V_y of (X,k_2) such that k_1({y}) ⊆ V_y and x ∉ V_y. Thus y ∉ k_2({x}). Hence k_2({y}) ⊆ U. Therefore (X,k_1,k_2) is a $_{s^{\#}g\alpha}C_0$- bi-$\check{C}$ech space.

The converse of the above theorem need not be true which can be seen from the following example.

Example 4.7 From Example 3.2, (X,k_1,k_2) is a $_{s^{\#}g\alpha}C_0$- bi-$\check{C}$ech space but not a $_{s^{\#}g\alpha}C_1$- bi-$\check{C}$ech space.

Theorem 4.8 A bi-$\check{C}$ech space (X,k_1,k_2) is a $_{s^{\#}g\alpha}C_1$- bi-$\check{C}$ech space if and only if every pair of points x, y of (X,k_1,k_2) such that k_1({x}) ≠ k_2({y}), there exists a semi $^{\#}g\alpha$ open subset U of (X,k_1) and semi $^{\#}g\alpha$ open subset V of (X,k_2) such that x ∈ V, y ∈ U and U ∩ V = ϕ.

Proof. Suppose that (X,k_1,k_2) is a $_{s^{\#}g\alpha}C_1$- bi-$\check{C}$ech space. Let x, y be the points of (X,k_1,k_2) such that k_1({x}) ≠ k_2({y}), there exists a semi $^{\#}g\alpha$ open subset U of (X,k_1) and semi $^{\#}g\alpha$ open subset V of (X,k_2) such that x ∈ k_1({x}) ⊆ V and y ∈ k_2({y}) ⊆ U.

Conversely suppose that there exists a semi $^{\#}g\alpha$ open subset U of (X,k_1) and semi $^{\#}g\alpha$ open subset V of (X,k_2) such that x ∈ V, y ∈ U and U ∩ V = ϕ. Since every $_{s^{\#}g\alpha}C_1$- bi-$\check{C}$ech space is a $_{s^{\#}g\alpha}C_0$- bi-$\check{C}$ech space [by Theorem 4.6], k_1({x}) ⊆ V and k_2({y}) ⊆ U.

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