A STUDY ON MCPHERSON NUMBER OF J_n (1)

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Abstract: The first study related to this concept was that on paper A Note on McPherson Number of Graphs by Susanth. C and S. J. Kalayathankal in JIMS. The McPherson Number is indeed the study on explosion of a vertex *v* in a graph *G*, we mean drawing edges from *v* to all other vertices in *G* that are not already adjacent to it. The recursive concept of vertex explosions is called the McPherson recursion. The McPherson number of a graph is the minimum number of vertex explosions required in *G* so that the resultant graph becomes a complete graph. In this paper, we determine the McPherson number of Jaco graph.

Keywords: Jaco graph, McPherson number, McPherson recursion, Vertex explosion.

1. Introduction: For all terms and definitions, we refer to [1], [2], [9]. Unless mentioned otherwise, all graphs considered here are simple, finite and connected.

By the term explosion of a vertex *v* in a graph G, we mean drawing edges from *v* to all other vertices in G that are not adjacent to it [8]. The concept of the McPherson number of a simple connected graph G on n vertices, denoted by $\gamma(G)$ is introduced in [6] as the minimum number of vertex explosions required in G so that the resultant graph becomes a complete graph.

An initial study on McPherson number of certain fundamental graph classes has been done in [6]. This parameter has been determined for empty graphs, paths, cycles, bipartite graphs, n-partite graphs etc. Motivated by this study, in this paper, we determine the McPherson number of Jaco graph $J_{n+1}(1)$.

The linear Jaco graph is an infinite directed graph called the *f*(*x*) root diagraph satisfying four fundamental properties which are :

- $V(J_\infty(f(x))) = \{v_i : i \in \mathrm{N}\}$
- If v_i is the head of an arc then the tail is always a vertex v_i , $i < j$
- If v_k for smallest $k \in \mathrm{N}$ is a tail vertex then all vertices $v_l, k < l < j$ are tails of the arcs to v_j
- The degree of vertex *k* is $d(v_k) = mk + c$.

Definition 1.1 [7]: Let $f(x) = mx + c$; $x, m \in \mathrm{N}, c \in N_\circ$. A **linear Jaco graph**, denoted by $J_\infty(mx + c)$, $n, x \in \mathrm{N}$, $m, c \in \mathrm{N}_0$, is defined as a directed graph whose vertex set is $V(J_\infty(mx+c)) = \{v_i : i \in N\}$ and the arc set is $A(J_\infty(mx+c)) \subseteq \{(v_i, v_j) : i, j \in N, i < j\}$ such that $(v_i, v_j) \in A(J_\infty(x))$ if and only if $(m+1)i + c - d^-(v_i) \geq j$.

A **finite linear Jaco graph**, denoted by $J_n(mx+c)$, is a finite subgraph of order *n* of the infinite linear Jaco graph $J_\infty(mx+c)$, where $n, x \in \mathrm{N}_1$ and $m, c \in \mathrm{N}_0$.

A Jaco graph $J_n(f(1))$ as defined in the earlier work of Kok, Fisher, Wilkens, Mabula and Mukungunugwa [3], [7] is exactly a linear Jaco graph $J_n(f(x))$ with $f(x) = x$

Definition 1.2 [4], [5]: The **infinite Jaco graph**, denoted by $J_\infty(1)$, is defined as a directed graph whose vertex set is $V(J_\infty(1)) = \{v_i : i \in N\}$ and the arc set is $A(J_\infty(1)) \subseteq \{(v_i, v_j) : i, j \in N, i < j\}$ such that $(v_i, v_j) \in A(J_\infty(x))$ if and only if $2i - d^-(v_i) \geq j$.

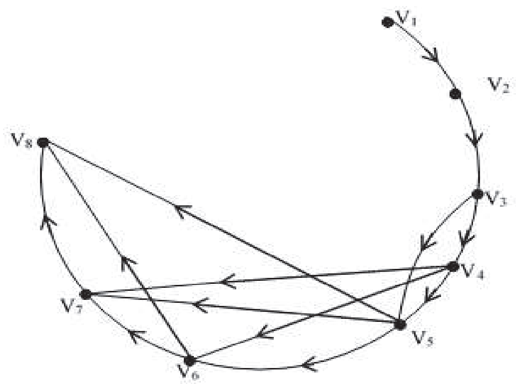

Fig. (a) J_8(1)

A **finite Jaco graph**, denoted by $J_n(1)$, is defined to be a finite subgraph of order *n* of the infinite Jaco graph $J_\infty(1)$. A vertex of the Jaco graph $J_n(1)$, which attains the maximum possible degree $\Delta(J_n(1))$, is called a **Jaconian vertex** of $J_n(1)$. The set of Jaconian vertices of a Jaco- graph $J_n(1)$ is denoted by $\mathrm{J}(J_n(1))$. The prime Jaconian vertex of

$J_n(1)$ is its lowest subscripted Jaconian vertex. If v_p is the prime Jaconian vertex of $J_n(1)$, the complete subgraph induced by the vertices $v_{p+1}, v_{p+2}, v_{p+3}, ...,$ v_n is called the **Hope sub graph** or Hope graph of $J_n(1)$, denoted by $H_n(1)$.

2. Results on McPherson number of Jaco graphs

Theorem 2.1 [6]: Consider, the Jaco graph $J_n(1)$. If v_i is the prime Jaconian vertex we have:

$$\gamma(J_n(1)) = \begin{cases} i & \text{if edge } v_i v_n \notin E(J_n(1)) \\ i-1 & \text{otherwise} \end{cases}$$

Example 2.2 : Let $J_8^*(1)$ be the underlying finite Jaco graph with vertex set

$V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$

$$E(G) = \begin{Bmatrix} (v_1,v_2),(v_2,v_3),(v_3,v_4),(v_3,v_5), \\ (v_4,v_5),(v_4,v_6),(v_4,v_7),(v_5,v_6), \\ (v_5,v_7),(v_5,v_8),(v_6,v_7),(v_6,v_8), \\ (v_7,v_8) \end{Bmatrix}$$

In the first iteration vertex v_1 explode to edges (v_1,v_3), (v_1,v_4), (v_1,v_5), (v_1,v_6), (v_1,v_7), (v_1,v_8). On the second iteration vertex v_2 explode to edges (v_2,v_4), (v_2,v_5), (v_2,v_6), (v_2,v_7), (v_2,v_8). On the third iteration vertex v_3 explode to edges (v_3,v_6), (v_3,v_7), (v_3,v_8). Finally, vertex v_4 explode to edges (v_4,v_8). Since, three explosions are minimum needed to ensure that the underlying Jaco graph $J_8^*(1) \cong K_8$, It follows that $\gamma(J_8^*(1)) = 4$. The figure below demonstrates the example.

In general, the underlying Jaco graphs $J_n^*(1)$ after explosions satisfy $J_n^*(1) \cong K_n$ and the resultant graph will be regular graph with degree (n-1). Since, after explosions the resultant graph is always a Complete graph and in a Complete graph K_n there exists n vertices with all having (n-1) as degree.

Corollary 2.3 : If v_k is the prime Jaconian vertex of J_n (1) then $\gamma(J_{n+1}(1)) = \deg(v_k)$

(i.e)., $\gamma(J_{n+1}(1)) = \Delta(J_n(1)) : n \in \mathrm{N}$

Proof: From the theorem 2.1, McPherson Number of any Jaco graph can be obtained by.

$$\gamma(J_n(1)) = \begin{cases} i & \text{if edge } v_i v_n \notin E(J_n(1)) \\ i-1 & \text{otherwise} \end{cases}$$

By the definition of Jaco graph, the prime Jaconian vertex, its degree and the maximum degree of the graphs can also be determined.

Since, the minimum number of vertex explosions that occur in $J^*_{n+1}(1)$ stops at the vertex v_i, *where* v_i is the prime Jaconian vertex of J^*_n (1), the result is obvious. And the following table proves the result. Hence the Proof.

□

Table 1

$n \in \mathrm{N}$	v_j, Prime Jaconian Vertex	$\gamma(J_n^*(1))$	$\Delta(J_n^*(1))$
3	v_2	1	2
4	v_2	2	2
5	v_3	2	3
6	v_3	3	3
7	v_4	3	4
8	v_5	4	5
9	v_5	5	5
10	v_6	5	6
11	v_7	6	7
12	v_7	7	7

3. Conclusion: In this paper, the McPherson number of $J_{n+1}(1)$ is obtained as $\Delta(J_n(1))$, $n \in N$ and is concluded by an open problem.

Problem 1: Determine the McPherson number for Linear Jaco graphs [Definition 1.2]

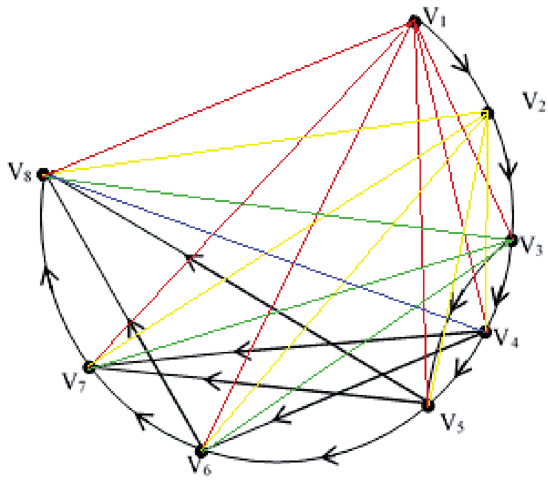

Fig. Vertex exploded $J_8(1)$

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