A STUDY ON ACHROMATIC NUMBER OF THE TRANSFORMATION GRAPH G^{++-}

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Abstract: The transformation graph G^{++-} of G is the graph with vertex set $V(G) \cup E(G)$ in which the vertices u and v are joined by an edge if one of the following conditions holds: (i) $u,v \in V(G)$ and they are adjacent in G, (ii) $u,v \in E(G)$ and they are adjacent in G, (iii) one of u and v is in V(G) while the other is in E(G), and they are not incident in G. In this paper, for a graph G, we determine the achromatic number of Transformation graph G^{++-} for Cycle, Path, Star and Wheel graph.

Key words: chromatic number, achromatic number, achromatic coloring, Transformation graph.

1.Introduction : In this paper, we are concerned with finite, undirected, loopless graph without multiple edges. Let G = (V(G),E(G)) be a graph. For two vertices u and v of G, if there is an edge e joining them, we say u and v are adjacent. Here, both u and v are end vertices of e, and u and e or v and e are said to be incident. A coloring of a graph G is a partitioning of the vertex set V into color classes. A proper coloring or chromatic vertex coloring of a graph G with vertex set V and edge set E is a map f : $V(G) \to S$ such that $f(u) \neq f(v)$ whenever $uv \in E(G)$. The smallest number of colors used in such a coloring is called the chromatic number of G, denoted by $\chi(G)$. Similarly, a proper achromatic coloring of a graph G assigns colors to each vertex of G such that for each color class C_i, $N[C_i]$ contains representatives of every color class. The maximum number of color classes in a proper achromatic partition of G is the achromatic number of G, and is denoted by $\psi(G)$.

The line graph L(G) of G is the graph whose vertex set is E(G) in which two vertices are adjacent if and only if they are adjacent in G. The total graph T(G) of G is the graph whose vertex set is $V(G) \cup E(G)$ in which two vertices are adjacent if and only if they are adjacent or incident in G. Since there are eight distinct 3-permutations of {+,-}. Wu and Meng[2] generalized the concept of total graphs to a transformation graph G^{xyz} with $x,y,z \in \{-,+\}$, where G^{+++} is the total graph of G, and G^{---} is its complement. Also, G^{--+}, G^{-+-} and G^{-++} are the complement of G^{++-}, G^{+-+} and G^{+--} respectively. Here we shall investigate the transformation graph G^{++-} of some graphs.

2. Achromatic Coloring on Transformation Graphs.

Theorem 2.1: The achromatic number of G^{++-} of Path graph P_n has n colors,

ie $\psi [G^{++-} (P_n)] = n$

Proof : Consider a path graph of length n-1 with vertex set V = $\{ v_1,v_2,\ldots\ldots.v_n\}$ and edge set E = $\{ e_1,e_2,\ldots\ldots.e_{n-1}\}$. In path graph P_n, each vertex v_i is adjacent with the vertices v_{i-1} and v_{i+1} for $i = 2,3,\ldots.n-1$, the vertex v_1 is adjacent with v_2 and v_n is adjacent with v_{n-1} and the lines e_1 and e_n are non - adjacent with n-3 lines and remaining e_i for $i = 2,3,\ldots n-1$ are non-adjacent with n-4 lines

By the definition of Transformation graph G^{++-}, the vertex set of $G^{++-} (P_n)$ corresponds to both vertex set and edge set of Path graph. The vertex set of $G^{++-} (P_n)$ is defined as follows :

ie $[G^{++-} (P_n)] = \{v_{i\,:}\, 1 \leq i \leq n \} \cup \{e_{i\,:}\, 1 \leq i \leq n-1 \}$

Assign the following n coloring for $G^{++-} (P_n)$ as achromatic:

Consider the color class C = $\{ c_1,c_2,\ldots\ldots.c_n\}$ to make the coloring as achromatic one, we should assign the coloring as follow:

To get the maximum number of pair of colors, assign the colors $c_1,c_2,\ldots\ldots.c_n$ to the vertices $v_1,v_2,\ldots\ldots.v_n$ and c_{n+1} to e_1 and $c_2,c_3,\ldots\ldots.c_n$ to the remaining vertices $e_2,e_3,\ldots\ldots.e_n$ respectively. Here we leave some pair of colors (c_1,c_n) and (c_1,c_{n+1}). Thus to accommodate all the missing pairs color the vertex e_1 by c_1. This indicates that we cannot introduce a new color c_{n+1} in this transformation graph. Hence, the given coloring is achromatic and by the very construction, it is maximal. Therefore $\psi [G^{++-} (P_n)] = n$ for every $n \geq 3$.

Theorem 2.2: The achromatic number of G^{++-} of the Cycle C_n has n+1 coloring, for $n \geq 7$.

ie $\psi [G^{++-} (C_n)] = n+1$

Proof: Consider a Cycle of length n with vertex set V = $\{v_1,v_2,\ldots\ldots.v_n\}$ and edge set E = $\{e_1,e_2,\ldots\ldots.e_n\}$. Here every point in the Cycle C_n is non - adjacent with n-2 lines.

Let us consider $G^{++-} (C_n)$. It is clear that there is no non - incident lines. By the definition of Transformation graph G^{++-}, the vertex set of $G^{++-} (C_n)$ corresponds to both vertex set and edge set of Cycle.

ie V $[G^{++-} (C_n)] = \{v_{i\,:}\, 1 \leq i \leq n \} \cup \{e_{i\,:}\, 1 \leq i \leq n \}$

Assign the following n+1 coloring for $G^{++-} (C_n)$ as achromatic:

Consider the color class C = $\{ c_1,c_2,\ldots\ldots.c_{n+1}\}$. Assign the color c_i to v_i for $i = 1,2,\ldots.\ n-1$ and c_{n+1} to v_n. Assign the colors c_n to e_1 and e_{n-1} and assign c_3 to e_n. For $2 \leq i \leq n-3$, assign the colors c_{i+2} to e_i and also assign the color c_{n+1} to e_{n-2}. This produces the non - adjacency condition. Now this coloring will accommodate all the pairs of the color class and by the very construction it is maximal.

Hence $\psi [G^{++-} (C_n)] = n+1$, for $n \geq 7$.

Theorem 2.3: The achromatic number of transformation graph of the Star graph is n+1

ie $\psi [G^{++-} (K_{1,n})] = n+1$

Proof : Let $v_1, v_2, \ldots\ldots v_n$ be the pendant vertices of $K_{1,n}$ and let V be the vertex of $K_{1,n}$ adjacent to v_i for $1 \leq i \leq n$ and clearly deg (v) = n.

ie $V(K_{1,n}) = \{V\} \cup \{ v_i : 1 \leq i \leq n \}$ and $E(K_{1,n}) = \{ e_i : 1 \leq i \leq n \}$ between the vertices vv_i for i = 1,2,.......n. In $K_{1,n}$, there is no incident lines.

Let us consider the transformation graph $G^{++-} (K_{1,n})$. By the definition of the Transformation graph G^{++-}, the vertex set is defined as

$V [G^{++-} (K_{1,n})] = \{ v \} \cup \{ v_i : 1 \leq i \leq n \} \cup \{ e_i : 1 \leq i \leq n \}$.

Here the vertices of this transformation graph admits an acyclic coloring of order n+1.Consider the color class $C = \{ c_1, c_2, \ldots. \ldots c_{n+1}\}$.Assign the following n+1 coloring for $G^{++-} (K_{1,n})$ as achromatic.

Case – 1: Assign a proper vertex coloring to all the vertices. For $1 \leq i \leq n$, assign the color $c_{n+1}$to all v_i and assign the color c_i to e_i for i = 1,2,...n and also assign the color c_1 to the root vertex v. Now this coloring will accommodate all the pairs of the color class. Hence the coloring is achromatic and it is a maximal one.

Therefore $\psi [G^{++-} (K_{1,n})] = n+1$

Case - 2: Now assign a proper vertex coloring to all the vertices. First assign c_1 to the root vertex and assign the set of colors c_{i+1} to both vertices v_i and e_i for $1 \leq i \leq n$ Since v_i is not adjacent with e_i for i = 1,2,...n

This produces a maximal coloring which satisfies the procedure of achromatic coloring

Hence $\psi [G^{++-} (K_{1,n})] = n+1$

Theorem 2.4: If G is W_n then clearly $\psi [G^{++-} (W_n)] = n+3$, for $n \geq 6$

Proof : Let W_n be the Wheel graph on n-vertices $v_1, v_2, \ldots\ldots v_n$ where v_n is the hub. A Wheel graph W_n of order n is a graph that contains a Cycle of order n-1 and for which every graph vertex in the Cycle is connected to one other graph vertex.

ie., $V(W_n) = \{ v_1, v_2, \ldots\ldots v_n\}$ and $E(W_n) = \{ e_1, e_2, \ldots\ldots e_{2n-2}\}$

In Wheel graph $W_{n,}$ each vertex v_i is adjacent with vertices of the Cycle graph C_{n+3}.

Consider the Transformation graph $G^{++-} (W_n)$. By the definition of Transformation graph G^{++-} , the vertex set is defined as

$V[G^{++-} (W_n)] = \{ v_i : 1 \leq i \leq n \} \cup \{ e_i : 1 \leq i \leq 2n-2 \}$.

Assign the following n+3 coloring for $G^{++-} (W_n)$ as achromatic:

Consider the color class $C = \{ c_1, c_2, \ldots. \ldots c_{n+3}\}$.

Assign the color c_i to v_i for $1 \leq i \leq n$.

Assign the color c_{n+2} to $e_{1.}$

Assign the color c_{i+1} to e_i for $2 \leq i \leq n-2$.

Assign the color c_{i+2} to e_{n-1} and e_n for i = n-1 and i = n+1.

Assign the color c_n to e_{n+1} and c_{n+3} to $e_{n+2.}$ Also assign the color c_{n+2} to $e_{n+3,}$ c_{n+1} to e_{n+4} and c_n to e_{n+5} respectively.

Assign the color c_{n+2} to e_i for i = 2n-4 and assign c_{n+2} and c_{n+1} to e_{2n-3} and e_{2n-2} respectively.

Now the coloring makes the non-adjacency condition is possible. Thus by the procedure of achromatic coloring, the coloring accommodates all the pairs of the color class and hence it is maximal. Hence by the very construction, the above said coloring is achromatic.

Therefore $\psi [G^{++-} (W_n)] = n+3$, for $n \geq 6$.

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